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APPLIED MECHANICS. STATIC SYSTEMS

Examples and tasks

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This handbook is intended to present basic practical issues in theoretical mechanics (statics), as well as the strength of materials and machine parts. Each chapter contains a concise theoretical introduction, basic calculation formulas, examples of typical calculations and tasks for individual work. Finally, there are reference tables and a list of used and recommended literature.

The handbook was created in cooperation between the Jan Długosz University in Czestochowa (Poland) and Kryvyi Rih State University of Economics and Technology (Ukraine). It is intended for engineering students, including those majoring in safety engineering, innovative technologies and modern materials or any related fields.

Contents

PREFACE

Applied mechanics consists mainly of three interrelated departments: theoretical mechanics, strength of materials and machine parts. The main purpose of this course is to provide engineering students with knowledge and skills that will enable them to solve engineering problems concerning individual parts of structures and machines in practice, taking into account their reliability and efficiency. The computational examples presented in the handbook make it possible to optimize the design of both the entire machine and its specific parts, enabling to minimize the occasional contradictions between reliability and efficiency that may arise during designing process.

The script sequentially presents the basic theoretical issues for each section and then shows several examples of solutions to typical problems in mechanics, strength of materials and machine design. After each topic, the student is given the opportunity to choose and solve practical tasks. Tasks are constructed in a manner to present multiple choices, which promotes understanding and consolidating knowledge of previously learned topics. The tasks, due to the formulated solution schemes, can be used for individual practical computational work of the student both in and out of class. The handbook deals with static systems, which are based mainly on the laws of solid mechanics.

CHAPTER I
THEORETICAL MECHANICS

1.1. Flat covering force system

General information

A convergent force system - is a system of forces whose lines of action intersect at a single point, called the point of convergence. There is a **planar** convergent force system when the lines of action of all these forces lie in the same plane, and a **spatial** convergent force system when the lines of action of the forces lie in different planes.

A system of forces whose lines of action lie in the same plane and intersect at a single point is called a **planar convergent force system**.

Basic calculation formulae

Equally acting convergent force system in the geometric method of determination

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i$$

Analytical (computational) method for determining the equivalent force in a converging force system

$$\begin{cases} \vec{R}_x = \vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x}; \text{in general form } \vec{R}_x = \sum\limits_{i=1}^{n} \vec{F}_{ix} \\ \vec{R}_y = \vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y}; \text{in general form } \vec{R}_y = \sum\limits_{i=1}^{n} \vec{F}_{iy} \end{cases}$$

or

$$\begin{cases} R\cos\alpha = F_1\cos\alpha_1 + F_2\cos\alpha_2 + F_3\cos\alpha_3 \\ R\cos\beta = F_1\cos\beta_1 + F_2\cos\beta_2 + F_3\cos\beta_3 \end{cases}$$

The value of the modulus (absolute value) of the resultant force vector is determined by Pythagoras' theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

The direction of the resultant force vector $\vec{R}$ is determined by its so-called directional cosines (cosines of the angles of this vector $\vec{R}$ with the coordinate axes):

$$cos\,\alpha = \frac{R_x}{R}; cos\,\beta = \frac{R_y}{R}$$

Equilibrium conditions of a converging force system

Geometric condition for the equilibrium of a converging force system

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_i = 0 \text{ or } \vec{R} = \sum_{i=1}^{n} \vec{F}_i = 0$$

The equivalent of such a force system will be zero when the force polygon is closed, i.e. the origin of the first force vector coincides with the end of the last force vector (Fig. 1.1).

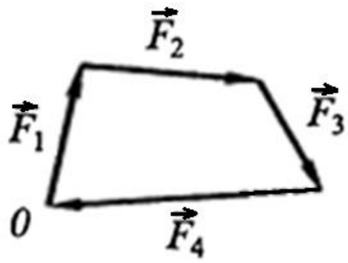


Fig. 1.1. Force balance

The geometric equilibrium condition can be used to solve some statics problems using a graphical method.

The way to proceed in such a case is as follows:

1. select the body whose equilibrium will be considered;
2. discard the bilges, replacing them with reactions;
3. using the equilibrium condition, create a closed polygon of forces, determining unknown quantities (in most cases the reactions of the ties).

Analytical equilibrium condition for a converging force system

$$\begin{cases} \vec{R}_x = \sum_{i=1}^{n} \vec{F}_{ix} = 0 \\ \vec{R}_y = \sum_{i=1}^{n} \vec{F}_{iy} = 0 \end{cases}$$

In an abbreviated form, the equilibrium conditions of a convergent force system in the plane are written as follows:

$$\begin{cases} \sum X = 0 \\ \sum Y = 0 \end{cases}$$

A system of such forces in space has three equations in equilibrium:

$$\begin{cases} \sum X = 0 \\ \sum Y = 0 \\ \sum Z = 0 \end{cases}$$

When solving static problems using the analytical method, the following sequence of steps should be followed:

1. select the body (point) whose equilibrium we are considering;
2. discard the bilges, turning them into reactions;
3. introduce the coordinate system and arrange the equilibrium equations;
4. using the equilibrium condition, determine the unknown quantities (in most cases the reactions of the nodes);
5. perform a verification of the results using an equation that was not used in the solution, for example by creating an equilibrium equation along a different coordinate axis or using a geometric method.

When choosing the position of the axes of the coordinate system, it is advisable to arrange them in such a way that as many unknowns as possible are perpendicular to their axes.

Examples of calculations

Example 1.1. Determine the reactions in the bars (Fig. 1.2) at F = 50 kN.

Data:
F = 50 kN

Searched for:
N_1, N_2 - ?

Solution

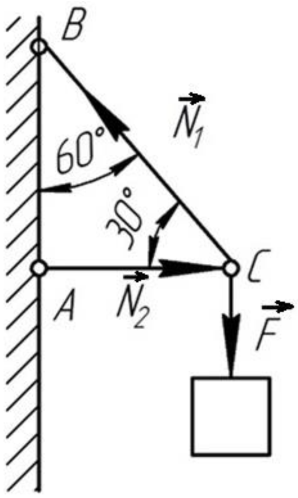


Fig. 1.2. Reactions forces in bars

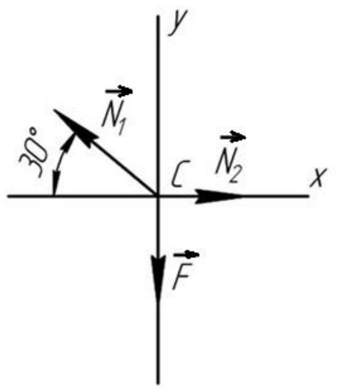


Fig. 1.3. Force distribution in the implemented coordinate system

1. We get rid of the nodes by replacing them with reactions (Fig. 1.2). Reactions in bars occur along the bar and their direction is chosen according to the deformation. The rod BC under force F is in tension, so the reaction N_1 will be towards the support in the direction opposite to the deformation, the rod AC under force F is in compression, so the reaction N_2 will be towards the direction away from the support.

2. We introduce the coordinate system and decompose the forces acting on it (Fig. 1.3).

3. Write down the equilibrium conditions for the given system of forces.

$$\sum F_y = 0;$$
$$N_1 \cos 60° - F = 0$$
$$N_1 = \frac{F}{\cos 60°} = \frac{50 \cdot 10^3}{0,5} = 100\ kN;$$
$$\sum F_x = 0;$$
$$-N_1 \cos 30° + N_2 = 0;$$
$$N_2 = N_1 \cos 30° = 100 \cdot 10^3 \cdot 0,87 = 87kN.$$

Verification

Examples of ways to verify calculations:

Option 1

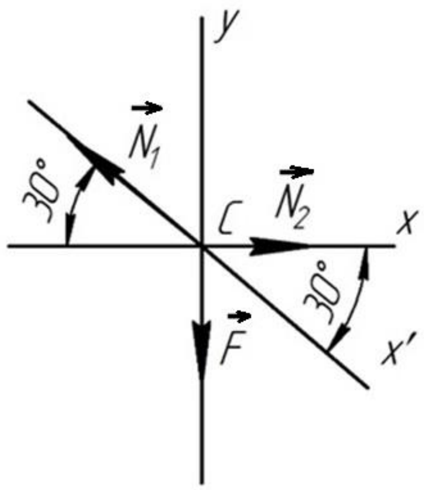


Fig. 1.4. Force projections on the *x*' axis

We determine the projections of forces on another axis, for example, x'(Fig. 1.4)

$$\sum F_{x'} = 0$$
$$-N_1 + N_2 \cos 30° + F \cos 60° = 0$$
$$-100 + 87 \cdot 0.87 + 50 \cdot 0.5 = 0$$
$$0 = 0$$

The reactions were calculated correctly.

Option 2

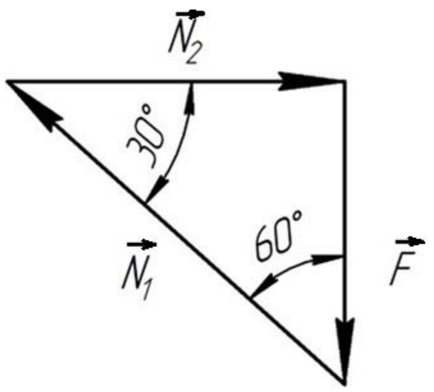


Fig. 1.5. Force triangle

We draw a force triangle (Fig. 1.5) and, using the ratio of the sides of the triangle, determine the reactions in the bars.

$$\cos 60° = \frac{F}{N_1}$$
$$N_1 = \frac{F}{\cos 60°} = \frac{50 \cdot 10^3}{0.5} = 100\ kN$$
$$\sin 60° = \frac{N_2}{N_1}$$
$$N_2 = N_1 \sin 60° = 100 \cdot 10^3 \cdot 0.87 = 87\ \text{kN}$$

The reactions were calculated correctly.

We will apply the sine theorem (Snellius theorem) - the sides of a triangle are proportional to the sines of the opposite angles, so:

$$\frac{N_1}{\sin 90°} = \frac{N_2}{\sin 60°} = \frac{F}{\sin 30°}$$
$$N_1 = \frac{F \sin 90°}{\sin 30°} = \frac{50 \cdot 10^3 \cdot 1}{0.5} = 100\ \text{kN}$$
$$N_2 = \frac{F \sin 60°}{\sin 30°} = \frac{50 \cdot 10^3 \cdot 0.87}{0.5} = 87\ \text{kN}$$

Reactions were determined correctly.

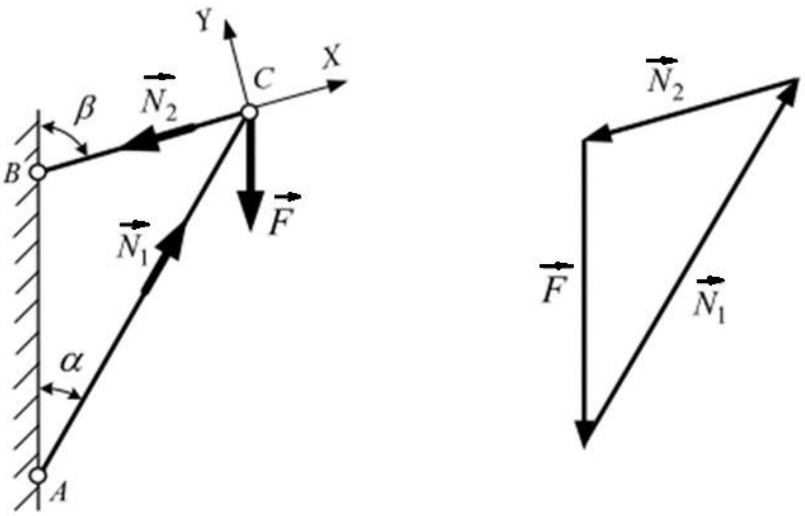
Answer: N_1 = 100 kN; N_2 = 87 kN.

Example 1.2. Determine the reactions in the cantilever bars (Fig. 1.6, *a*) when:

F = 3 kN; α = 30 °; β = 65 °.

Data:
F = 3 kN
α = 30 °
β = 65 °

Searched for:
N_1, N_2-?



a *b*

Fig. 1.6. Bracket: *a* - distribution of forces in the bracket; *b* - force triangle

Solution

1. Graphical method

Plot the force vector $\vec{F}$ on a scale of

$$\mu_F = \frac{F}{\overline{F}} = 0.1\,\frac{\mathrm{kN}}{\mathrm{mm}}$$

So

$$\vec{F} = \frac{F}{\mu_F} = \frac{3\ \mathrm{kN}}{0.1\,\frac{\mathrm{N}}{\mathrm{mm}}} = 30\ \mathrm{mm}$$

Bar reactions are always directed along the bars. After analysing the deformation of the bars (tension or compression), choose the direction of the reactions. We then create a force triangle (Fig. 1.6, *b*). We measure the vectors $\vec{N}_1$ i $\vec{N}_2$ and, taking into account the scale factor, calculate the reaction values N_1 and N_2 :

$$N_1 = 4.73\ \mathrm{kN}, N_2 = 2.62\ \mathrm{kN}.$$

For greater accuracy, the use of graphic software is recommended.

2. Analytical method

Choose the X and Y axes. For convenience, we orient the X-axis along the N_2 force (Fig. 1.6, a)

We set up two equilibrium equations and determined the unknown reactions:

$$\sum F_y = 0,\ \ N_1 \cos 55° - F \cos 25° = 0,$$

$$N_1 = \frac{F \cos 25°}{\cos 55°} = 4.74 \text{ kN},$$

$$\sum F_x = 0, -N_2 + N_1 \cos 35° - F \cos 65° = 0,$$

$$N_2 = 4.74 \cos 35° - 3 \cos 65° = 2.62 \text{ kN}.$$

Answer: N_1 = 4.74 kN, N_2 = 2.62 kN.

Example 1.3. a weight of G = 60 kN is suspended by a rope, thrown over block a and leading to winch D. Determine the reactions in bars AC and BA of the crane (Fig. 1.7).

Data:
G = 60 kN

Searched for:
S_1 -? S_2 - ?

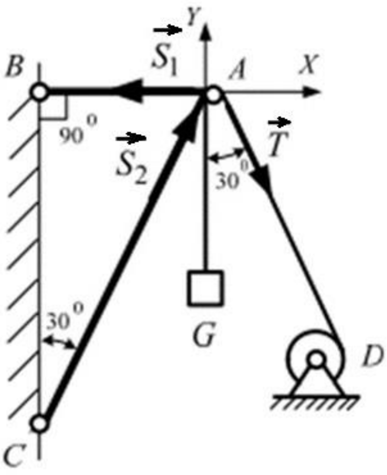


Fig. 1.7. Force distribution in the calculated system

Solution

1. The reactions of bars AB and AC are directed along the bar.

It is clear from the bar load analysis that bar AB is in tension, so the reaction S_1 is directed from point a to point B.

The rod AC is compressed, so the reaction S_2 is directed from point C to point A.

The rod AC is compressed, so the reaction S_2 is directed from point C to point A.

The force in the rope T is directed along the rope from point a to point D as the rope is stretched by the load G. It is obvious that $T = G$.

2. We choose to arrange the X and Y axes in such a way that one of the reactions (for example S_1) is directed along one of the axes.

Two equilibrium systems can be arranged for such a system:

$$\sum F_y = 0$$

$$S_2 \cos 30° - G - T \cos 30° = 0$$

$$S_2 = \frac{G + T \cos 30°}{\cos 30°} = \frac{60 \cdot 10^3 + 60 \cdot 10^3 \cdot 0.87}{0,87} = 129 \text{ kN}$$

$$\sum F_x = 0$$

$$-S_1 + S_2 \cos 60° + T \cos 60° = 0$$

$$S_1 = S_2 \cos 60° + T \cos 60° = 129 \cdot 10^3 \cdot 0.5 + 60 \cdot 10^3 \cdot 0.5 = 94.5 \text{ kN}$$

Answer: S_1 = 129 kN, S_2 = 94.5 kN.

Individual tasks

(calculation)

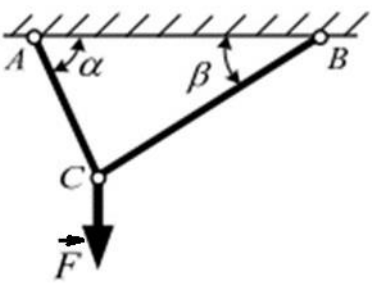
Task 1.1. Determine the reactions in the cantilever rods under force $\vec{F}$. The data for the task is shown in Table 1.1.

Table 1.1. Initial data for Task 1.1

Var. no	Scheme	α °	β °	F kN	Variant	Scheme	α °	β °	F kN	Wariant	Schemat	α °	β °	F kN
1	1	10	20	20	17	5	45	40	95	33	9	70	30	42
2		20	15	30	18		60	10	100	34		60	40	44
3		30	50	40	19		65	15	15	35		65	25	46
4		40	25	50	20		40	30	18	36		50	45	48
5	2	15	80	60	21	6	60	20	16	37	10	30	35	52
6		25	70	70	22		70	25	14	38		40	45	54
7		55	45	80	23		80	30	12	39		50	50	56
8		20	75	90	24		65	35	10	40		35	40	58
9	3	5	80	100	25	7	30	45	22	41	11	30	95	62
10		10	70	25	26		40	35	24	42		20	110	64
11		45	50	35	27		45	40	26	43		02	120	66
12		30	60	45	28		50	30	28	44		15	115	68
13	4	30	40	55	29	8	10	100	32	45	12	30	45	72
14		20	30	65	30		15	95	34	46		20	60	74
15		15	20	75	31		20	110	36	47		30	50	76
16		25	25	85	32		25	105	38	48		40	45	78

Schemes to Task 1.1

1



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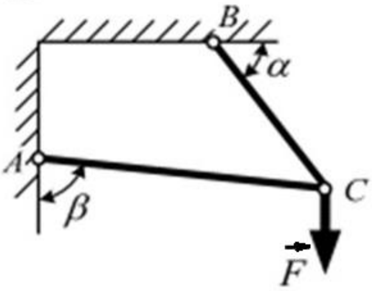

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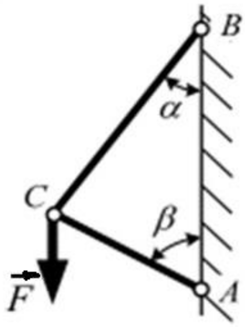

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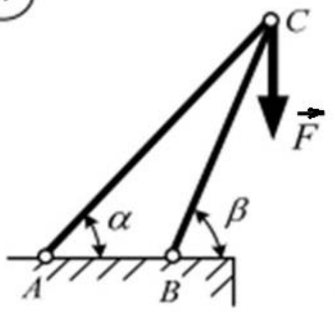

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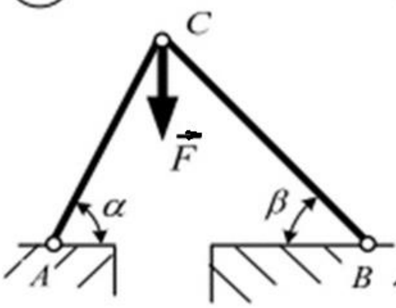

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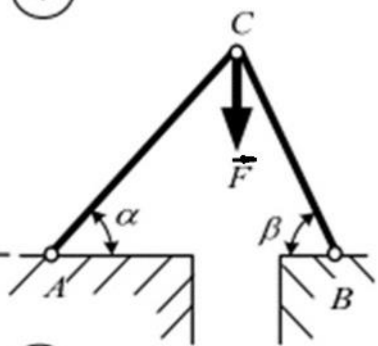

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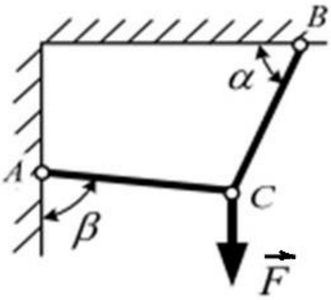

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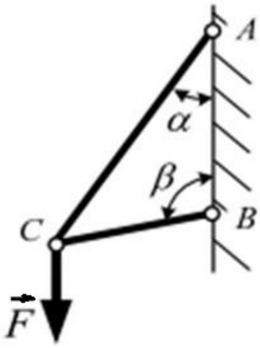

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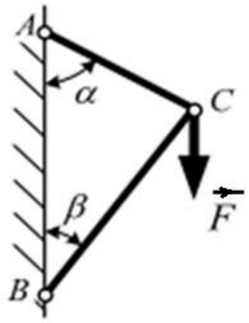

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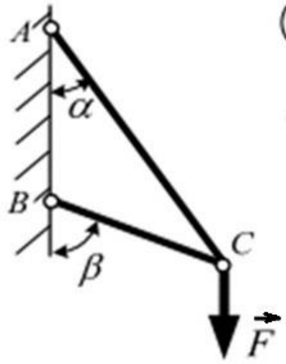

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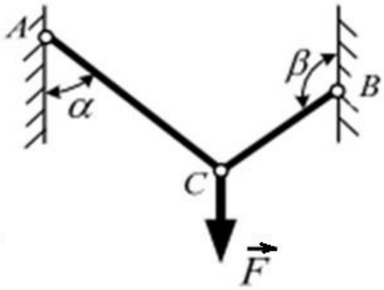

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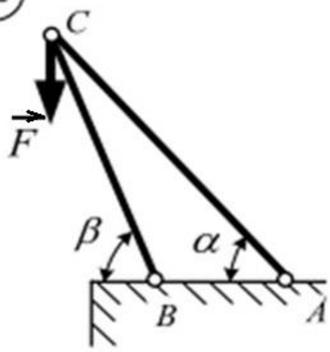
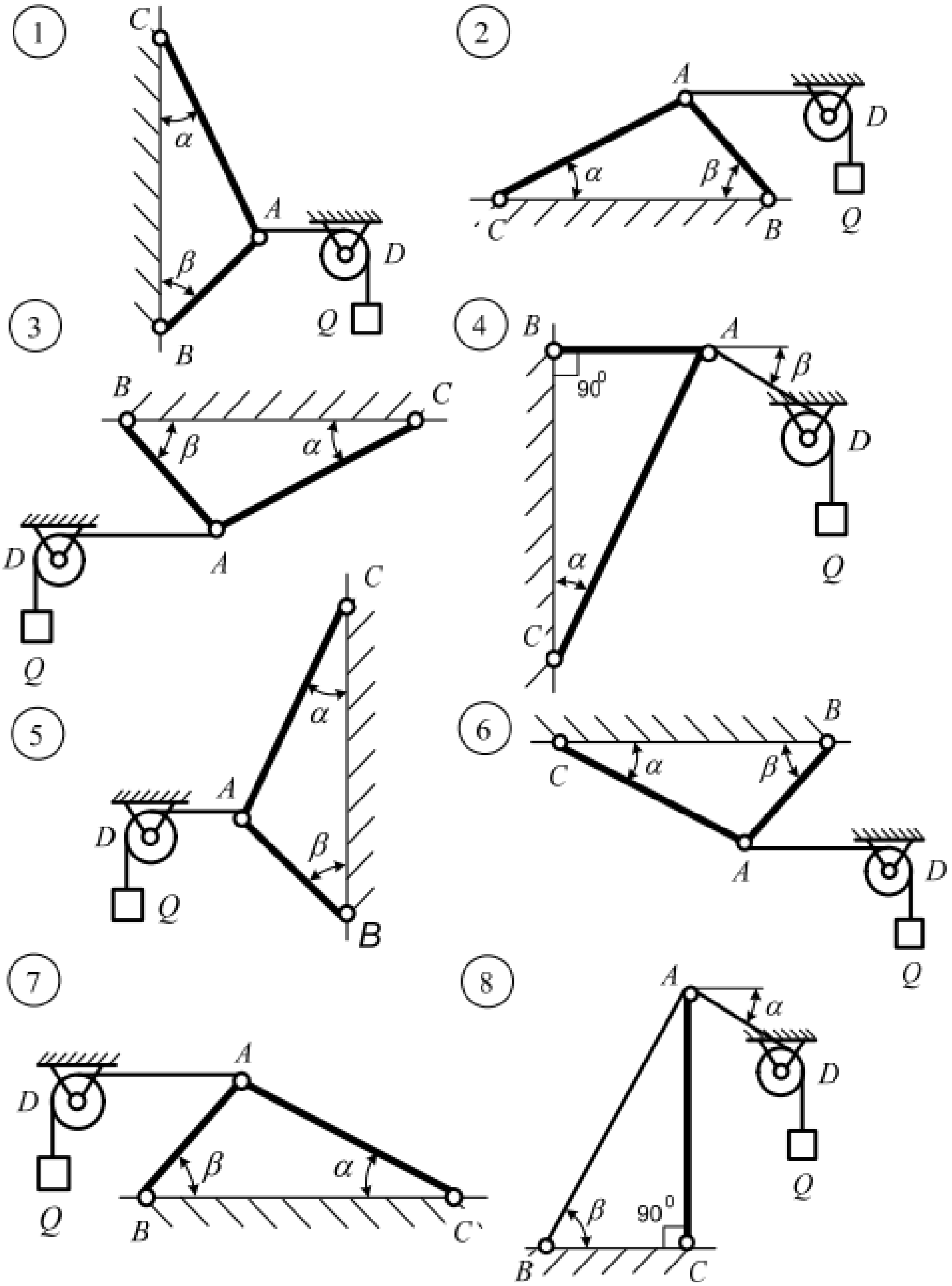


Task 1.2. Determine the values and direction of the node reactions for the schemes shown below. The data for the task is shown in Table 1.2.

Table 1.2. Initial data for Task 1.2

Var. no	Scheme	α °	β °	Q kN	Var. no	Scheme	α, °	β °	Q kN
1	1	30	50	20	17	5	30	50	20
2		40	45	22	18		35	50	25
3		25	60	30	19		25	60	10
4		45	30	25	20		10	70	40
5	2	15	60	40	21	6	5	30	28
6		20	50	50	22		10	35	16
7		25	55	48	23		15	40	2
8		30	45	30	24		10	45	40
9	3	50	20	32	25	7	10	60	32
10		55	20	46	26		15	55	40
11		40	30	28	27		20	50	50
12		45	25	30	28		5	65	40
13	4	10	70	60	29	8	45	40	15
14		15	70	30	30		50	35	12
15		35	40	10	31		55	30	40
16		20	45	15	32		60	20	45

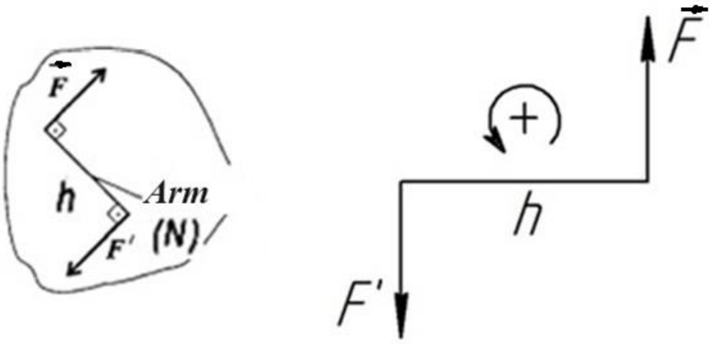
Schemes to Task 1.2



1.2. Force pairs and moment of a force about a point

General information

Pair of forces - two equal and parallel forces directed in opposite directions that do not lie in a straight line (Fig. 1.8).



a *b*

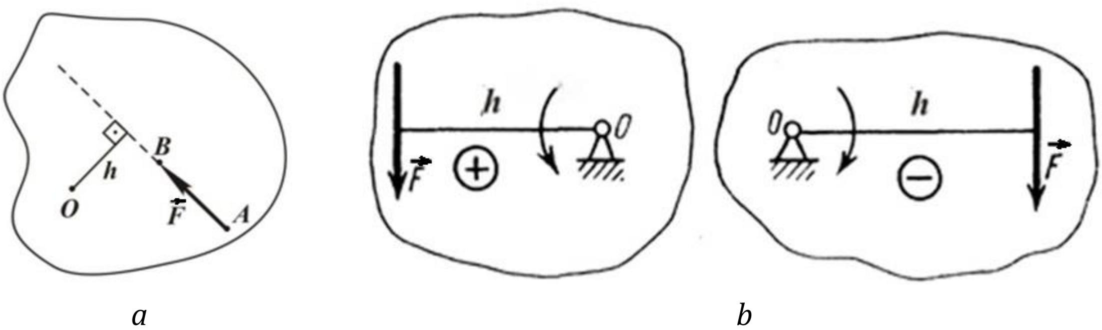
Fig. 1.8. Pair of forces:
a - determining the arm; *b* - determining the sign of the moment of the force pair

Moment of force with about a point

A force that does not pass through the point of attachment of the body causes the body to rotate around that point and the effect of such a force on the body is assessed by a **moment**.

The moment of force about a point (pole) is called the vector $\vec{M}_0$ multiplied by the distance between the direction of this vector and the point at which we want to calculate this moment (Fig. 1.9, a).

$$\vec{M}_0 = \vec{r} \cdot \vec{F}$$



a *b*

Fig. 1.9. Moment of force relative to a point:
a - arm; *b* - determining the sign of the moment

A perpendicular line drawn from a point to the line of action of the force is called **the force arm** *h*.

Basic calculation formulae

Power couple moment

$$M(F, F') = Fh$$

The arm *h* of a force pair is the shortest distance between the lines of action of the force pair.

The moment is considered positive if the force pair rotates the body in a counterclockwise direction (Fig. 1.9, *b*). The unit of measure of the moment of a force pair *M(F, F')* is Nm. The moment of a force pair is equal to the algebraic sum of the moments of the pairs forming the system:

$$M = \sum_{i=1}^{n} M_i$$

For pair equilibrium it is necessary and sufficient that the algebraic sum of the moments of the pairs of the system is equal to zero (pair equilibrium condition):

$$M = \sum_{i=1}^{n} M_i = 0$$

To balance a force pair system, it is necessary to apply a pair of forces of equal modulus and directed in opposite directions. Such a pair of forces is called **balanced**.

Several pairs of forces can be applied to a body. Two pairs of forces are **equivalent** if, given other equivalent conditions, their action on the body is the same. Since a pair of forces is characterised by a pair moment, pairs of forces lying in the same plane will be **equivalent** if they have the same moment (same magnitude and direction).

The moment of force *F* relative to point 0 is denoted M_0 *(F)*:

$$M_0(F) = Fh$$

The unit of measure $M_0(F)$ is Nm.

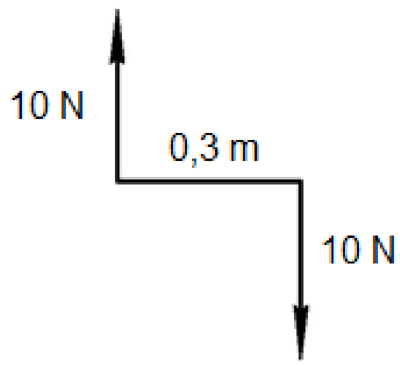
The sign principle of moments. a moment is considered positive if the force attempts to rotate the body relative to a given point in a counterclockwise direction and negative if it is clockwise (Fig. 1.9, *b*).

Varignon's theorem. The moment concerning any point O of the resultant of two forces is equal to the sum of the moments of these forces concerning point O.

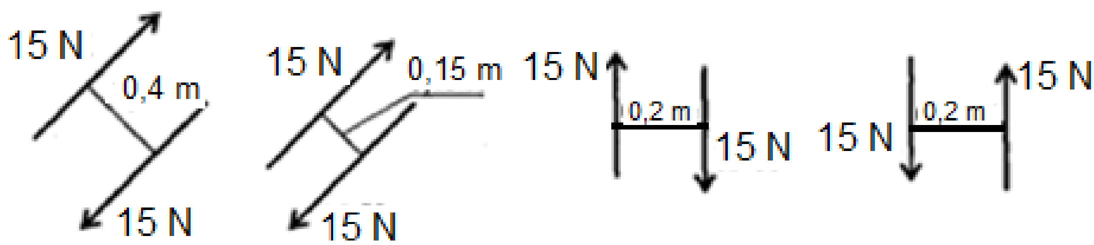
$$M_0(R) = \sum_{i=1}^{n} M_0(F_i)$$

Examples of calculations

Example 1.4. Determine the pair of equivalent forces given (Fig. 1.10).



Answer choices



a *b* *c* *d*

Fig. 1.10. Variants for defining a pair of equivalent forces

Solution

Since a pair of forces is characterised by a moment of force, pairs of forces lying in the same plane will be **equivalent** if they have the same moment (same magnitude and direction).

The moment of a given pair is:

$$M(F) = F \cdot h = 15 \cdot 0.2 = 3 \text{ kNm}$$

so the *c* variant is correct. For this pair, the direction and moment coincide with the given

$$M(F) = F \cdot h = 15 \cdot 0.2 = 3 \text{ kNm}$$

Answer: c.

Example 1.5. Determine the moments of forces acting on bar AC (Fig. 1.11) relative to points A, B, C, if: F_1 = 10 N; F_2 = 20 N; F_3 = 30 N; l_{AB} = 1 m; l_{BC} = 2 m.

Data:
F_1 = 10 N
F_2 = 20 N
F_3 = 30 N
l_{AB} = 1 m
l_{BC} = 2 m

Searched for:
ΣM_A - ? ΣM_B - ?
ΣM_C - ?

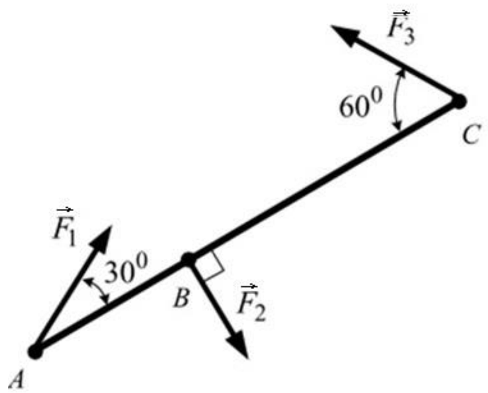


Fig. 1.11. Force distribution in the bar

Solution

Write down the equations of moments relative to points A, B, C:

$$\sum M_A = -F_2 \cdot l_{AB} + F_3 \cdot \sin 60° \cdot l_{AC} =$$
$$= -20 \cdot 1 + 30 \cdot 0.866 \cdot 3 = 57.9 \text{ Nm},$$

$$\sum M_B = -F_1 \cdot \sin 30° \cdot l_{AB} + F_3 \cdot \sin 60° \cdot l_{BC} =$$
$$-10 \cdot 0.5 \cdot 1 + 30 \cdot 0.866 \cdot 2 = 46.96 \text{ Nm},$$

$$\sum M_C = -F_1 \cdot \sin 30° \cdot l_{AC} + F_2 \cdot l_{BC} = -10 \cdot 0.5 \cdot 3 + 20 \cdot 2 = 25 \text{ Nm}.$$

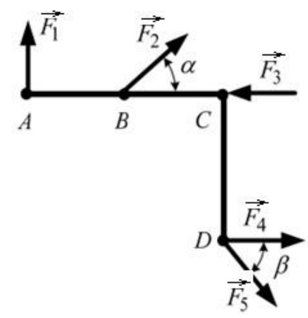
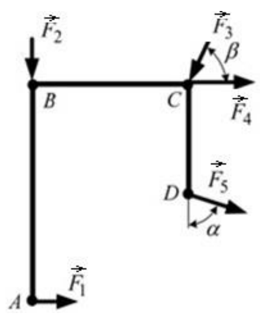
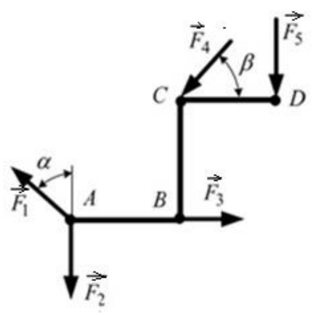
Answer: ΣM_A = 57.9 Nm, ΣM_B = 46.96 Nm, ΣM_C = 25 Nm.

Individual tasks

(calculation)

Task 1.3. Determine the values of moments of forces acting on bars *ABCD* concerning points *A*, *B*, *C*, *D.* Data for the task is shown in Table 1.3.

Table 1.3. Initial data for Task 1.3

Var. no	F_1 N	F_2 *N*	F_3 *N*	F_4 *N*	F_5 *N*	α °	β °	*AB* m	*BC* m	*CD* m	Calculation scheme
1	5	1	2	8	4	30	60	1	1	2	
2	10	2	4	6	5	45	30	2	1	2	
3	15	3	6	12	6	60	45	1	2	1	
4	8	4	8	20	7	30	30	1	1	2	
5	6	5	10	15	8	45	60	2	1	2	
6	12	6	12	4	10	60	50	1	2	1	
7	20	7	15	2	12	30	45	1	1	2	
8	15	8	18	1	5	45	30	2	1	2	
9	4	10	14	2	10	60	50	1	2	1	
10	2	12	20	3	15	50	60	1	1	2	
11	1	5	1	4	8	30	60	1	1	2	
12	2	10	2	5	6	45	30	2	1	2	
13	3	15	3	6	12	60	45	1	2	1	
14	4	8	4	7	20	30	30	1	1	2	
15	5	6	5	8	15	45	60	2	1	2	
16	6	12	6	10	4	60	50	1	2	1	
17	7	20	7	12	2	30	45	1	1	2	
18	8	15	8	20	7	45	30	2	1	2	
19	10	4	10	15	8	60	50	1	2	1	
20	12	2	12	4	10	50	60	1	1	2	
21	20	7	15	8	4	30	45	1	1	2	
22	15	8	18	6	5	45	30	2	1	2	
23	4	10	14	12	6	60	50	1	2	1	
24	2	12	20	20	7	50	60	1	1	2	
25	1	5	1	15	8	30	60	1	1	2	
26	2	10	2	4	10	45	30	2	1	2	
27	3	15	3	2	12	60	45	1	2	1	
28	4	8	4	1	5	30	30	1	1	2	
29	5	6	5	2	10	45	60	2	1	2	
30	6	12	6	3	15	60	50	1	2	1	

1.3. Flat arbitrary force system. Determination of reactions at the supports

General information

Flat arbitrary force system - is a system of forces applied to a body whose lines of action are arbitrarily distributed in the same plane (they do not intersect at a single point).

Parallel force transfer theorem. The equilibrium of a solid body will not be disturbed if a force acting on the body is transferred parallel to itself to any point of the body, or by adding a pair of forces whose moment is equal to the moment of that force relative to the point to which the force is transferred.

The parallel force-displacement theorem is a fundamental statistical theorem for the reduction of any system of forces acting on a solid body to a force and force pair.

Reduction of a plane system of arbitrarily distributed forces

Reducing a force system involves replacing it with another system that is equivalent to the first, but simpler.

Theorem. a system of forces can be reduced to an equivalent system consisting of one force applied at an arbitrary pole of reduction O and a pair of forces of torque M. a flat system of arbitrary forces is equivalent to one force, **the principal vector of the system**, which is added at the centre of the system, and one pair of forces, **the principal moment of the system (Fig. 1.13)**.

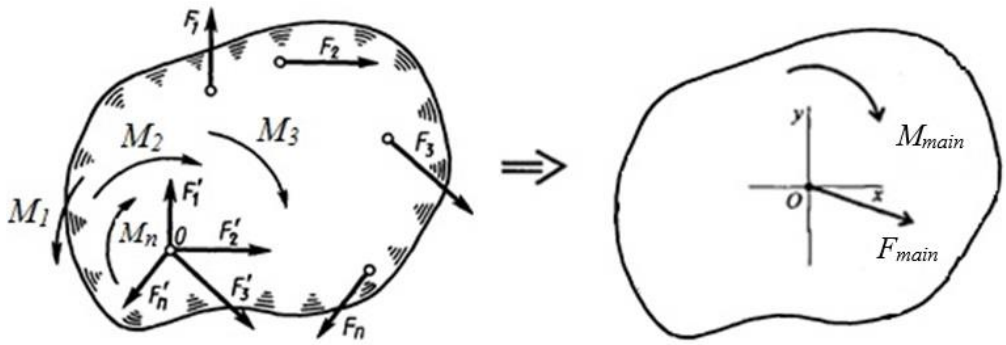


Fig. 1.12. The main torque of the system

Basic calculation formulae

Principal vector of an arbitrary plane force system

The geometric sum of the forces of the system is called the principal vector of the system.

$$\vec{F}_{main} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \ldots = \sum_{i=1}^{n} \overrightarrow{F_i}$$

Principal vector modulus of a plane system of arbitrary forces

$$F_{main} = F_{1_x} + F_{2_x} + F_{3_x} + \cdots = \sum_{i=1}^{n} F_{i_x}$$

$$F_{main} = F_{1_y} + F_{2_y} + F_{3_y} + \ldots = \sum_{i=1}^{n} F_{i_y}$$

then $F_{main} = \sqrt{F_{main_X}^2 + F_{main_y}^2}$

Direction of **the principal vector modulus of a plane system of arbitrary** forces

$$\cos(F_{main}, x) = \frac{F_{main_x}}{F_{main}}$$

Main moment of a plane system of arbitrary forces

$$M_{main} = M_1 + M_2 + M_3 + \ldots = \sum_{i=1}^{n} M_i = \sum_{i=1}^{n} M_0(F_i)$$

Conditions of equilibrium of a plane system of arbitrary forces

Geometrical equilibrium conditions

$$F_{main} = 0, M_{main} = 0.$$

Analytical equilibrium conditions

The first form of the equilibrium condition for a plane system of arbitrary forces

$$\sum_{i=1}^{n} F_{i_x} = 0; \sum_{i=1}^{n} F_{i_y} = 0;$$

$$\sum_{i=1}^{n} M_i = \sum_{i=1}^{n} M_0(F_i) = 0$$

In short, it can be written down:

$$\sum F_{i_x} = 0, \sum F_{i_y} = 0, \sum M_0 = 0$$

Other variants of the notation of the equilibrium condition can be found in the literature, for example:

$$\sum X = 0, \sum Y = 0, \sum M_A = 0,$$

where $\sum X$ and $\sum Y$ – the sum of the projections of the active and reactive forces of the system on the coordinate axes (i.e. all the external forces);

ΣM_A – the sum of the moments of all the external forces of the system (active and reactive) relative to any point A.

The second form of the equilibrium condition for a plane system of arbitrary forces

$$\sum M_A = 0, \sum M_B = 0, \sum M_C = 0$$

where A, B, C – arbitrary reference points of the system's moments of force;

The third form of the equilibrium condition for a plane system of arbitrary forces

$$\sum M_A = 0, \sum M_B = 0, \sum F_{i_x} = 0$$

Conditions of equilibrium of a plane system of parallel forces

The first form of the equilibrium condition

$$\sum F_{i_y} = 0, \sum M_O = 0$$

The second and third form of the equilibrium condition

$$\sum M_A = 0, \sum M_B = 0$$

Recommended sequence of operations when solving a planar system of arbitrarily distributed forces

1. Determine which body's equilibrium should be taken into account in this task.

2. Treating this body as free, apply to it all the forces and reactions of the bonds acting on the body.

3. Arrange the equilibrium conditions using the form of the conditions that lead to the simplest solution and determine the unknowns.

4. Check calculations using equilibrium conditions not used in this task.

For simpler equations it is worthwhile:

1. When determining the equation of projection, draw a coordinate axis perpendicular to one of the unknown forces.

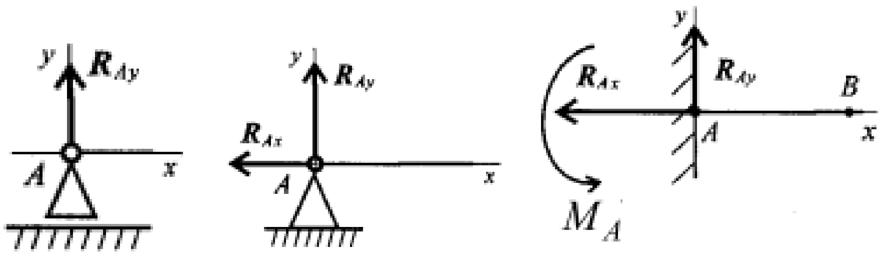
2. When determining the equation of moments, choose the point where the most forces intersect.

Determination of reactions in the support

The basic three types of support:

Sliding articulated support (Fig. 1.13, *a*). This support gives only one reaction - R_{Ay} , which is directed along the normal to the resistance surface;

Non-sliding articulated support (Fig. 1.13, *b*). The support allows rotation about the joint and can be replaced by two-component forces acting along the coordinate axis;



a *b* *c*

Fig. 1.13. Determination of reactions in supports:
a - articulated sliding; *b* - articulated non-sliding; *c* - in the support

Bracket (Fig. 1.13, *c*). No displacements are possible. Under the influence of external forces, two reactions R_{Ax}, R_{Ay} and a reaction moment M_A occur at the restraint point, preventing rotation.

Conditions of equilibrium

$$\sum F_{i_x} = 0, \sum F_{i_y} = 0, \sum M_A = 0$$

Each equation has one unknown and is solved without substitution.

To check the correctness of the solution, an additional equation of moments is used concerning an arbitrary point on the beam, for example, B:

$$\sum M_B = 0$$

$$\sum M_A = 0, \sum M_B = 0, \sum F_{i_x} = 0$$

The equations of moments are defined by the anchor points of the beam. Since the moment of the force passing through the anchor point is 0, one unknown force remains in the equation.

From the equation $\sum M_A = 0$ the reaction R_{By} is determined.

From the equation $\sum M_B = 0$ the reaction R_{Ay} is determined.

From the equation $\sum F_{i_x} = 0$ the reaction R_{Bx} is determined.

To check the correctness of the solution, an additional equation is used

$$\sum F_{i_y} = 0$$

In solid equilibrium, where three points can be chosen that do not lie on the same line, it is convenient to use a system of equations of the second form.

$$\sum M_A = 0, \sum M_B = 0, \sum M_C = 0$$

Examples of equations

Example 1.6. Determine the principal vector of the force system and the principal moment of the system concerning point *B* (Fig. 1.14), if:

F_1 = 10 kN; F_2 = 16 kN; F_3 = 12 kN; M = 60 kNm.

Data:	*Searched for:*
F_1 = 10 kN	F_{main} - ?
F_2 = 16 kN	M_{main} - ?
F_3 = 12 kN	
M = 60 kNm	

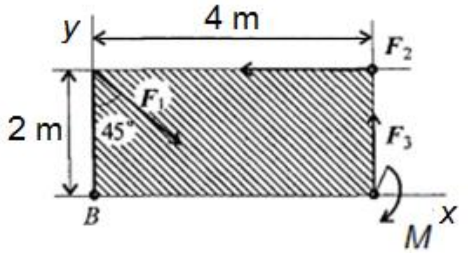


Fig. 1.14. The system of forces and moments

Solution

1. Determine the principal vector of the force system.

The principal vector is equal to the geometric sum of the vectors of the force system:

$$\vec{F}_{main_x} = \vec{F}_{1_x} + \vec{F}_{2_x} + \vec{F}_{3_x} + \ldots = \sum_{i=1}^{n} \vec{F}_{i_x}$$

$$F_{main_x} = F_{1_x} \cos 45° - F_{2_x} = 10 \cdot 0{,}71 - 16 = -8.9 \text{ kN};$$

$$F_{main_y} = F_{1_y} + F_{2_y} + F_{3_y} + \ldots = \textstyle\sum_{i=1}^{n} F_{i_y};$$

$$F_{main_y} = -F_{1_y} \cos 45° + F_{3_y} = -10 \cdot 0{,}71 + 12 = 4.9 \text{ kN};$$

$$F_{main} = \sqrt{F_{main_X}^2 + F_{main_y}^2} = \sqrt{(-8{,}9)^2 + 4{,}9^2} \approx 10 \text{ kN}.$$

2. Determine the principal moment of the system of forces about point *B.*

The principal moment of a force system is equal to the algebraic sum of the moments of all the forces of the system concerning the reference point:

$$M_{main} = \sum_{i=1}^{n} M_B(F_i)$$

$$\sum M_B = -F_1 \cos 45° \cdot 2 + F_2 \cdot 2 + F_3 \cdot 4 - M$$

$$\sum M_B = -10 \cdot 0.71 \cdot 2 + 16 \cdot 2 + 12 \cdot 4 - 60 = 5.8 \text{ kNm}.$$

Answer: F_{main} = 10 kN; M_{main} = 5.8 kNm.

Example 1.7. Determine the reactions in the supports (Fig. 1.15), if: F_1 = 10 kN; F_2 = 30 kN, M = 5 kNm, a = 2 m, b = 4 m, α = 30 °, β = 45 °

Data:
F_1 = 10 kN
F_2 = 30 kN
M = 5 kNm
a = 2 m
b = 4 m
α = 30 °
β = 45 °

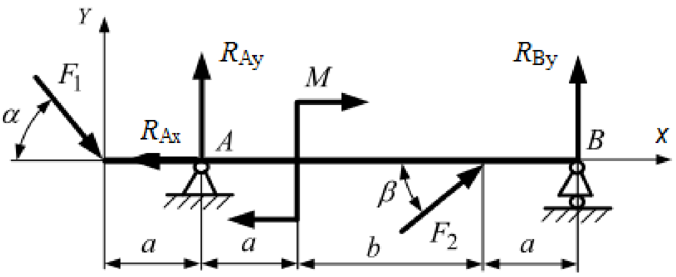


Fig. 1.15. The system of forces and moments

Searched for:
R_{Ax} - ? R_{Ay} - ?
R_{By} - ?

Solution

1. Introduce the coordinate system and mark it on the scheme (Fig. 1.15).

2. Convert the supports into the corresponding reactions and mark them on the scheme (Fig. 1.15). We choose the direction of the reactions arbitrarily.

3. Determine the reactions in the supports.

To determine the reactions in the supports, we use the third form of the equilibrium condition for a plane system of arbitrary forces.

$$\sum M_A = 0, \sum M_B = 0, \sum F_{i_x} = 0$$

To check the correctness of the solution, we use the following additional equation

$$\sum F_{i_y} = 0$$

Adopt the sign principle, counterclockwise torque is positive.

$$\sum M_A = 0;$$

$$F_1 \sin\alpha \cdot a - M + F_2 \sin\beta \cdot (a+b) + R_{By}(2a+b) = 0;$$

$$R_{By} = \frac{-F_1 \sin\alpha \cdot a + M - F_2 \sin\beta \cdot (a+b)}{2a+b} = \frac{-10 \cdot sin\, 30° \cdot 2 + 5 - 30 \cdot \sin 45° \cdot (2+4)}{2 \cdot 2 + 4}$$
$$= -16.6 \text{ kN}.$$

A minus sign indicates that the reaction is in the opposite direction.

$$\sum M_B = 0;$$

$$F_1 \sin\alpha \cdot (3a+b) - M - F_2 \sin\beta \cdot \mathrm{a} - R_{Ay}(2a+b) = 0;$$

$$\sum F_{i_x} = 0;$$
$$F_1 \cos\alpha - R_{Ax} + F_2 \cos\beta = 0;$$
$$R_{Ax} = F_1 \cos\alpha + F_2 \cos\beta = 10 \cdot \cos 30° + 30 \cdot \cos 45° = 30 \text{ kN}.$$

Verification

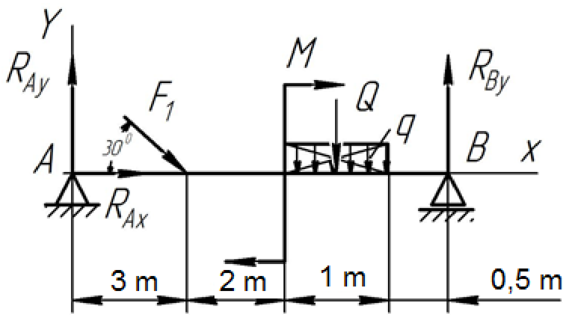
$$\sum F_{i_y} = 0;$$
$$-F_1 \sin\alpha + R_{Ay} + F_2 \sin\beta + R_{By} = 0;$$
$$-10 \cdot 0.5 + 0.3 + 30 \cdot 0.71 - 16.6 = 0;$$
$$0 = 0.$$

Reactions were determined correctly.

Answer: R_{Ax} = 30 kN; R_{Ay} = 0.3 kN; R_{By} = −16.6 kN.

Example 1.8. Determine the reactions in the supports (Fig. 1.16), if:
F_1 = 10 kN, q = 15 kN/m, M = 20 kNm.

Data:
F_1 = 10 kN
q = 15 kN/m
M = 20 kNm



Searched for:
R_{Ax} - ? R_{Ay} - ?
R_{By} - ?

Fig. 1.16. The system of forces and moments

Solution

1. Enter the coordinate system and mark it on the diagram (Figure 1.16).

2. Convert the supports into the corresponding reactions and mark them on the diagram (Fig. 1.16). Choose the direction of the reactions arbitrarily.

3. Substitute the distributed load for the equivalent force that is applied at the centre of the diagram (Fig. 1.16):

$$Q = q \cdot l = 15 \cdot 1 = 15 \text{ kN}$$

4. Determine the reactions in the supports.

To determine the reactions in the supports we use the third form of the equilibrium condition for a plane system of arbitrary forces

$$\sum M_A = 0, \sum M_B = 0, \sum F_{i_x} = 0$$

To check the correctness of the solution, we use the following additional equation

$$\sum F_{i_y} = 0$$

Adopt the sign principle. The counterclockwise torque is positive.

$$\sum M_A = 0;$$

$$-F_1 \sin 30° \cdot 3 - M - Q \cdot 5.5 + R_{By} \cdot 6.5 = 0;$$

$$R_{By} = \frac{F_1 \sin 30° \cdot 3 + M + Q \cdot 5.5}{6.5} = \frac{10 \cdot 0.5 \cdot 3 + 20 + 15 \cdot 5.5}{6.5} = 18 \text{ kN};$$

$$\sum M_B = 0;$$

$$-R_{Ay} \cdot 6.5 + F_1 \sin 30° \cdot 3.5 - M + Q1 = 0;$$

$$R_{Ay} = \frac{F_1 \sin 30° \cdot 3.5 - M + Q1}{6.5} = \frac{10 \cdot 0.5 \cdot 3.5 - 20 + 15 \cdot 1}{6.5} \approx 2 \text{ kN};$$

$$\sum F_{i_x} = 0;$$

$$F_1 \cos 30° + R_{Ax} = 0;$$

$$R_{Ax} = -F_1 \cos 30° - F_1 \cos 30° = -10 \cdot 0.87 = -8.7 \text{ kN}.$$

A minus sign indicates that the reaction is in the opposite direction.

Verification

$$\sum F_{i_y} = 0; -F_1 \sin 30° + R_{Ay} - Q + R_{By} = 0;$$

$$-10 \cdot 0{,}5 + 2 - 15 + 18 = 0;$$

$$0 = 0.$$

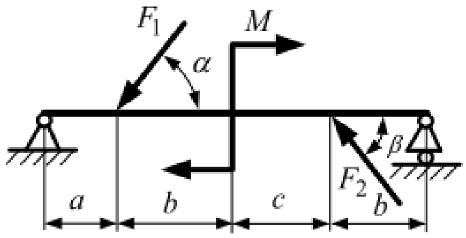
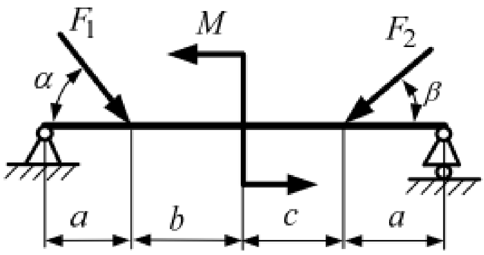
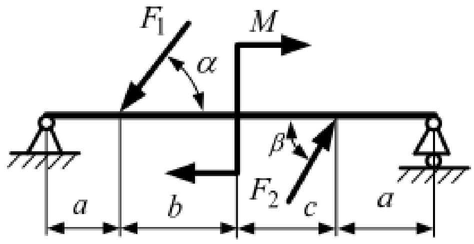
Reactions were determined correctly.

Answer: R_{Ax} = –8.7 kN; R_{Ay} = 2 kN; R_{By} = 18 kN.

Individual tasks
(calculation)

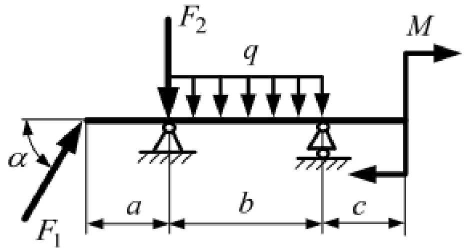
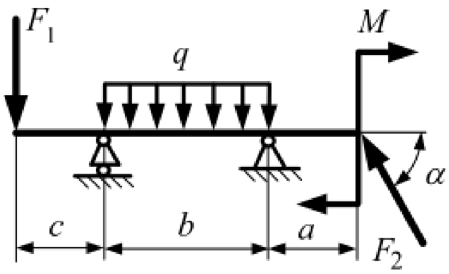
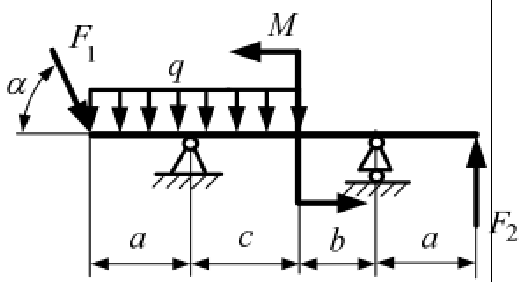
Task 1.4. Determine the reactions in the supports is shown in Table 1.4

Table 1.4. Initial data for Task 1.4

Var. no	F_1, kN	F_2, kN	M, kNm	α, °	β, °	a, m	b, m	c, m	Calculation scheme
1	5	1	2	30	60	1	1	2	
2	10	2	4	45	30	2	1	2	
3	15	3	6	60	45	1	2	1	
4	8	4	8	30	30	1	1	2	
5	6	5	10	45	60	2	1	2	
6	12	6	12	60	50	1	2	1	
7	20	7	15	30	45	1	1	2	
8	15	8	18	45	30	2	1	2	
9	4	10	14	60	50	1	2	1	
10	2	12	20	50	60	1	1	2	
11	1	5	1	30	60	1	1	2	
12	2	10	2	45	30	2	1	2	
13	3	15	3	60	45	1	2	1	
14	4	8	4	30	30	1	1	2	
15	5	6	5	45	60	2	1	2	
16	6	12	6	60	50	1	2	1	
17	7	20	7	30	45	1	1	2	
18	8	15	8	45	30	2	1	2	
19	10	4	10	60	50	1	2	1	
20	12	2	12	50	60	1	1	2	
21	20	7	15	30	45	1	1	2	
22	15	8	18	45	30	2	1	2	
23	4	10	14	60	50	1	2	1	
24	2	12	20	50	60	1	1	2	
25	1	5	1	30	60	1	1	2	
26	2	10	2	45	30	2	1	2	
27	3	15	3	60	45	1	2	1	
28	4	8	4	30	30	1	1	2	
29	5	6	5	45	60	2	1	2	
30	6	12	6	60	50	1	2	1	

Task 1.5. Determine the reactions in the supports. Initial data is shown in Table 1.5.

Table 1.5. Initial data for Task 1.5

Var. No	F_1, kN	F_2, kN	M, kNm	q, kN/m	α, °	a, m	b, m	c, m	Calculation scheme
1	5	1	2	1	30	1	1	2	
2	10	2	4	2	45	2	1	2	
3	15	3	6	3	60	1	2	1	
4	8	4	8	4	30	1	1	2	
5	6	5	10	5	45	2	1	2	
6	12	6	12	6	60	1	2	1	
7	20	7	15	7	30	1	1	2	
8	15	8	18	8	45	2	1	2	
9	4	10	14	10	60	1	2	1	
10	2	12	20	12	50	1	1	2	
11	1	5	1	5	30	1	1	2	
12	2	10	2	10	45	2	1	2	
13	3	15	3	15	60	1	2	1	
14	4	8	4	8	30	1	1	2	
15	5	6	5	6	45	2	1	2	
16	6	12	6	12	60	1	2	1	
17	7	20	7	20	30	1	1	2	
18	8	15	8	15	45	2	1	2	
19	10	4	10	4	60	1	2	1	
20	12	2	12	2	50	1	1	2	
21	20	7	15	1	30	1	1	2	
22	15	8	18	2	45	2	1	2	
23	4	10	14	3	60	1	2	1	
24	2	12	20	4	50	1	1	2	
25	1	5	1	5	30	1	1	2	
26	2	10	2	6	45	2	1	2	
27	3	15	3	7	60	1	2	1	
28	4	8	4	8	30	1	1	2	
29	5	6	5	10	45	2	1	2	
30	6	12	6	12	60	1	2	1	

1.4. Determination of the centre of gravity of flat shapes

General information

Material bodies consist of elementary particles whose position in space is determined by their coordinates. The gravitational forces of each particle concerning the Earth can be considered as a system of parallel forces, and the equivalent of these forces is called **the gravity of the** body or the **weight of the** body.

The centre of gravity of a body is the centre of parallel forces of attraction of all elementary particles of the body. The centre of gravity is the geometric point of application of the force of gravity, which may be outside the body (e.g. a disc with a hole, a torus, an angle, a hollow sphere, etc.).

Basic calculation formulae

Centre of gravity of plane bodies and geometrical plane figures

$$x_c = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} = \frac{A_1 x_1 + A_2 x_2 + \cdots + A_n x_n}{A_1 + A_2 + \cdots + A_n}$$

$$y_c = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i} = \frac{A_1 y_1 + A_2 y_2 + \cdots + A_n y_n}{A_1 + A_2 + \cdots + A_n}$$

where A_i – area of the part of the figure, mm^2;

x_i, y_i – coordinates of the centre of gravity of the part of the figure, mm.

Formulae $x_C = \sum_{i=1}^{n} A_i x_i$ is called **the static moment** (S_y) of a plane cross-section (figure). The static moment of the area of a plane body concerning an axis in the plane of the body is a geometrical characteristic that is equal to the product of the area of the body and the distance from its centre of gravity to that axis. Then the coordinates of the centre of gravity of the section can be expressed in terms of the static moment:

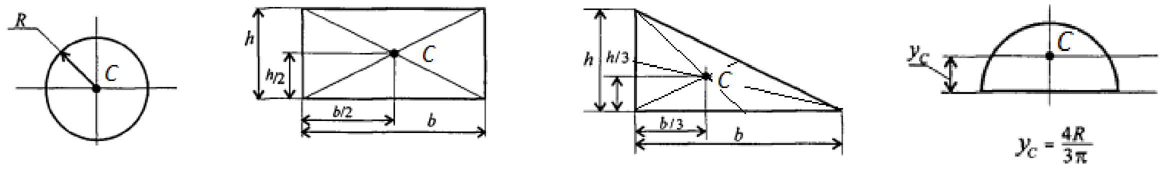
$$\sum_{i=1}^{n} A_i x_i = S_y; \; x_C = \frac{S_y}{A}$$

$$\sum_{i=1}^{n} A_i y_i = S_{yx}; \; y_C = \frac{S_x}{A}$$

The axes that intersect the centre of gravity are called **central axes.** The static moment about the central axis **is zero**.

Center of gravity some simple figures

The position of the centres of gravity of simple geometric shapes can be calculated using the relevant formulae (Fig. 1.17).



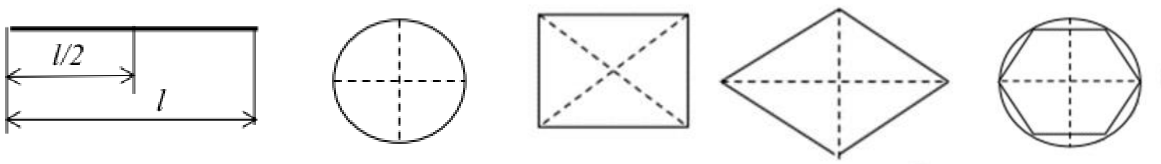
a *b* *c* *d*

Fig. 1.17. Position of the centres of gravity of the figures:
a - circle; *b* - square, rectangle; *c* - triangle; *d* - semicircle

Methods for determining the position of the centre of gravity

Analytical methods

Symmetry method. If a homogeneous body has a plane, axis or centre of symmetry, the centre of gravity lies either on the plane of symmetry, the axis of symmetry or the centre of symmetry, respectively. This property reduces the number of coordinates of the centre of gravity that need to be determined. Given this property, the number of the centre of gravity coordinates to be determined is reduced.

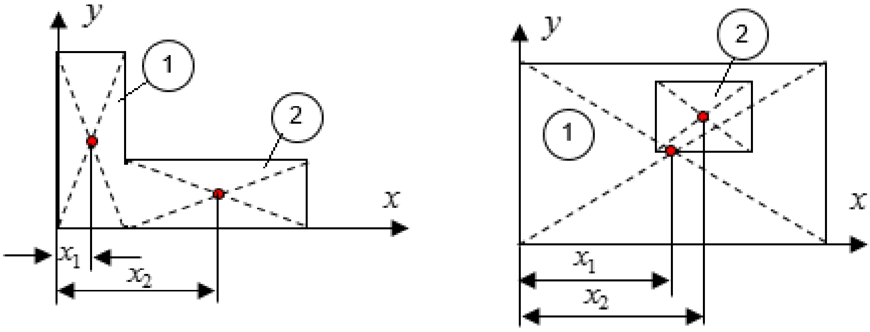


a *b* *c* *d*

Fig. 1.18. The centre of gravity of:
a - a segment of length *l*; *b* - a circle; *c* - a parallelogram, rhombus or parallelogram;
d - a regular polygon

The centre of gravity of a segment of length *l* is at its midpoint (Fig. 1.18, *a*). The centre of gravity of a circle or a circle of radius *R is* at its centre, i.e. at the point of intersection of the diameters (Fig. 1.18, *b*). The centre of gravity of a parallelogram, rhombus or parallelepiped is at the point of intersection of the diagonals (Fig. 1.18, *c*). The centre of gravity of a regular polygon is at the centre of the inscribed or circumscribed circle (Fig. 1.18, *d*).

Method of division (subdivision). A composite figure is divided into a series of simple figures for which the position of the centre of gravity is known or easy to determine (Fig. 1.19, *a*).



a *b*

Fig. 1.19. Determination of the centre of gravity of figures:
a - by the method of division (subdivision); *b* - by the method of negative areas

Then the position of the centre of gravity of the whole figure is determined according to the formulae

$$x_c = \frac{\sum(A_i x_i)}{\sum A_i}$$

$$y_c = \frac{\sum(A_i y)}{\sum A_i}$$

A for the figure shown in Fig. 1.20, *a* the centre of gravity

$$x_c = \frac{A_1 \cdot x_1 + A_2 \cdot x_2}{A_1 + A_2}$$

$$y_c = \frac{A_1 \cdot y_1 + A_2 \cdot y_2}{A_1 + A_2}$$

Here x_i, y_i – coordinates of the simple figure, A_i – its area.

The negative (positive) area method – is a special case of the division method. As in the subdivision method, a complex shape is subdivided into a set of simple shapes for which the position of the centre of gravity is known or easy to determine, but where holes or voids exist, it is convenient to represent in terms of a "negative" cross-sectional area. For example, the figure in (Fig. 1.19, *b*) can be represented as two rectangles, one of which has a negative cross-sectional area. The centre of gravity is then determined as follows

$$x_c = \frac{A_1 \cdot x_1 - A_2 \cdot x_2}{A_1 - A_2}$$

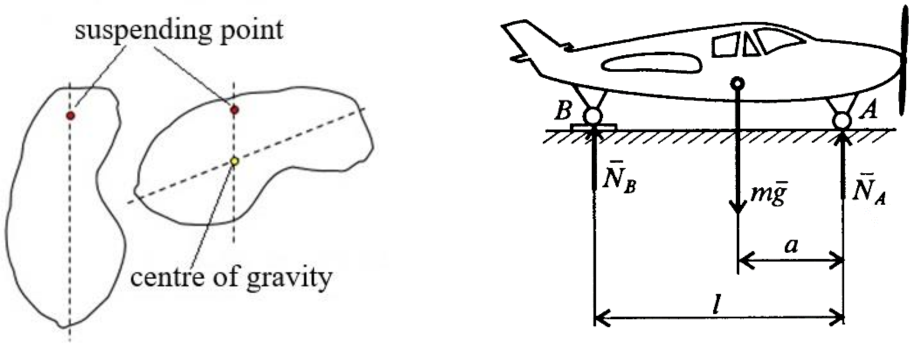
$$y_c = \frac{A_1 \cdot y_1 - A_2 \cdot y_2}{A_1 - A_2}$$

The integration method. It is used in cases where the first three methods cannot be used to determine the centre of gravity. If the figure has a fairly simple contour described by a well-known equation (circle, parabola, etc.), an elementary place or band is selected and analytical integration is performed. If analytical integration is difficult, numerical integration methods are used.

Experimental methods

Experimental methods are used when bodies have a complex shape, configuration, large size and mass for which other methods are not suitable due to complexity and cumbersomeness. For example, various machines or their parts (aircraft, cars, etc.).

Suspension method. It consists of the fact that when a body or figure is suspended at any point, the centre of gravity is at the same vertical as the point of suspension. To determine the position of the centre of gravity of a plane figure, it is sufficient to suspend it alternately at any two points and draw the corresponding verticals, for example, by lines, and the point of intersection of these lines corresponds to the position of the figure's centre of gravity (Fig. 1.20, *a*).



a *b*

Fig. 1.20. Determination of the centre of gravity:
a - suspension method; *b* - weighing method

Weighting method. Requires measuring the weight of the whole body as well as the separate parts of the body. If the mass is known (for example, of an aeroplane), the rear wheels are placed on a scale (Fig. 1.20, *b*) and the reaction N_B is determined using the weight readings. Then one of the equations of equilibrium is laid out; the most convenient is to determine the sum of moments concerning point A:

$$\sum_{i=1}^{n} M_A(F_i) = 0$$

$$m \cdot g \cdot a - N_B \cdot l = 0$$

From here, the unknown value of a, the position of the aircraft's centre of gravity, is determined:

$$a = \frac{N_B l}{mg}$$

In this way, the experimental method is faster and more convenient, especially when it is necessary to determine the centre of gravity of a plane figure that is difficult to divide into simpler elements. However, this method is less accurate than the analytical method, which is more accurate but more difficult and time-consuming.

Examples of calculation

Example 1.9. Determine the positions of the figure's centre of gravity (Fig. 1.21).

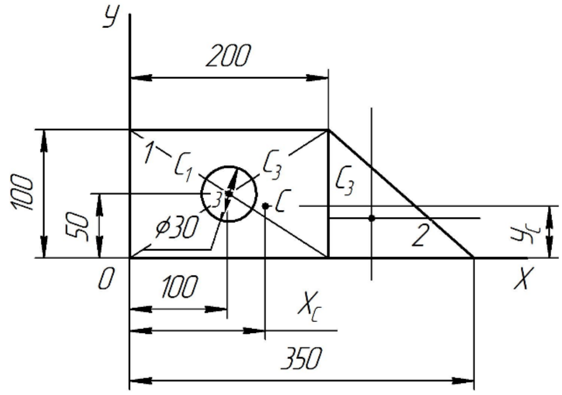


Fig. 1.21. Section to Example 1.9

Solution

1. To determine the centre of gravity of a figure we will use the analytical method of division and negative areas. We divide the complex figure into simple components: a rectangle, a triangle and a circle. We give them numbers and place them on the shape of the figure (Fig. 1.21).

2. Create an XY coordinate system and determine the centres of gravity of the components of the composed figure:

1 – a rectangle – a symmetrical figure whose centre of gravity is at the point of intersection of the diagonals, its coordinates are:

$$x_1 = \frac{200}{2} = 100 \text{ mm};\ y_1 = \frac{100}{2} = 50 \text{ mm};\ \mathrm{C}_1(100; 50)$$

2 – a triangle – the centre of gravity is either at the point of intersection of its midlines or at the point of intersection of the lines, located at a distance of 1/3 from the perpendiculars, its coordinates are:

$$x_2 = 200 + \frac{350 - 200}{3} = 250 \text{ mm};\ y_2 = \frac{100}{3} = 33 \text{ mm};\ \mathrm{C}_2(250; 33)$$

3 – a circle – a symmetrical figure, the centre of gravity of which is at its centre, its coordinates are:

$$x_3 = \frac{200}{2} = 100 \text{ mm};\ y_3 = \frac{100}{2} = 50 \text{ mm};\ \mathrm{C}_3(100; 50)$$

The specified coordinates and points of the centres of gravity of the components of the plane figure are plotted in Fig. 1.21.

3. Determine the cross-sectional area of the components of the plane figure:

1 – rectangle

$$A_1 = 100 \cdot 200 = 20000 \ \text{mm}^2$$

2 – triangle

$$A = 0.5\big(100 \cdot (350 - 200)\big) = 7500 \, \text{mm}^2$$

3 – circle

$$A_3 = \frac{\pi d^2}{4} = 3.14 \cdot \frac{30^2}{4} = 707 \text{ mm}^2$$

4. Determine the coordinates of the figure's centre of gravity.

As the circle represents the negative part of a plane figure, it takes the value of the area with a minus.

$$x_c = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3} = \frac{20000 \cdot 100 + 7500 \cdot 250 - 707 \cdot 100}{20000 + 7500 - 707} = 142 \text{ mm}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3} = \frac{20000 \cdot 50 + 7500 \cdot 33 - 707 \cdot 50}{20000 + 7500 - 707} = 45 \text{ mm}$$

The coordinates of the centre of gravity $C(142; 45)$, let us mark them on the figure (Fig. 1.21).

Example 1.10. Determine the centre of gravity of the composite section (Fig. 1.22), which contains: a 5×100 mm plate and rolled products: C-bar C10 and I-section I16.

Comment. Often frames are welded from different profiles to form the required profile. This reduces material consumption and results in a high-strength structure. For standard rolled profiles, the geometrical characteristics are known and are regulated by the relevant standards.

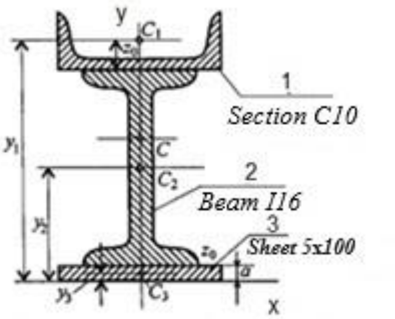


Fig. 1.22. Section for Example 1.10

Data:
5 × 100 mm sheet
C-bar C10
I-beam I16

Searched for:
C - ?

Solution

1. Define the *XY* coordinate system, label the figures with numbers and take all the data from tables D.58 and D.59:

1 – channel section (C-bar) C10; height h_1 = 100 mm; width b_1 = 46 mm; z_0 = 14.4 mm;

2 – I-beam I16; height h_2 = 160 mm; width b_2 = 81 mm;

3 – metal sheet: heigth h_3 = 5 mm; width b_3 = 100 mm.

2. Determine the coordinates of the centres of gravity of each figure:

1 – C-bar C10:

$$x_1 = 0 \text{ mm};\ y_1 = \text{a} + h_2 + z_0 = 5 + 160 + 14.4 = 179.4 \text{ mm};$$
$$\text{C}_1(0; 179.4)$$

2 – I-beam I16:

$$x_2 = 0 \text{ mm};\ y_2 = a + \frac{h_2}{2} = 5 + \frac{160}{2} = 85 \text{ mm}$$
$$\text{C}_2(0; 85)$$

3 – metal sheet:

$$x_3 = 0 \text{ mm};\ y_3 = \frac{a}{2} = \frac{5}{2} = 2.5 \text{ mm}$$
$$\text{C}_3(0; 2.5)$$

Specified coordinates and centres of gravity points of each part of the plane figure are marked in Fig. 1.22.

3. Determine the areas of each figure:

From the tables D.58 and D.59:

1 – C-bar C10:

$$A_1 = 1090 \text{ mm}^2;$$

2 – I-beam I16:

$$A_2 = 2020 \text{ mm}^2;$$

3 – metal sheet:

$$A_3 = a \cdot b_3 = 5 \cdot 100 = 500 \text{ mm}^2.$$

4. Determine the coordinates of the figure's centre of gravity:

$$x_c = 0 \text{ mm},$$
$$y_c = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{1090 \cdot 179.4 + 2020 \cdot 85 + 500 \cdot 2.5}{1090 + 2020 + 500} = 102 \text{ mm}.$$

The coordinates of the figure's centre of gravity $C(0; 102)$ are marked on the figure in Fig. 1.22.

Example 1.11. Determine the centre of gravity of a plane figure (Fig. 1.23) using an experimental method.

Solution

We put holes *A*, *B* and *D* on the flat figure at arbitrary points (preferably at the greatest distance from each other). The flat Figure 1 is suspended on needle 2 first at point *A* and then at points, *B* and *D*. With the help of weight 3, fixed at point 2, a vertical line is marked on the figure, which repeats the position of the thread. The centre of gravity of Figure 1 will lie at the point of intersection of the vertical lines drawn when the figure is suspended at points *A*, *B* and *D* (Fig. 1.23).

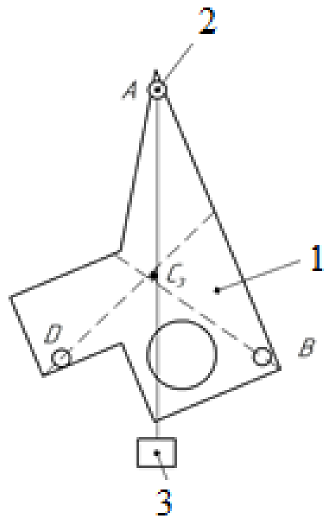


Fig. 1.23. Section to Example 1.11

As a result of the experiment, we have obtained three lines that do not intersect at a single point, but form a triangle. To draw the centre of gravity C_e of the flat figure, determined by the experimental method, we will draw three centre lines and at the point of their intersection, we will mark the point C_e, and then use a ruler to determine its coordinates: x_{ce} =78 mm; y_{ce} = 45 mm, then C_e (78;45).

Individual tasks

(calculation)

Task 1.6. Determine the centre of gravity of the composite section shown in Table 1.6.

Table 1.6. Initial data for Task 1.6

<table>
<tr><th rowspan="2">Var. no</th><th colspan="2">Rolled product range</th><th rowspan="2">Sections</th></tr>
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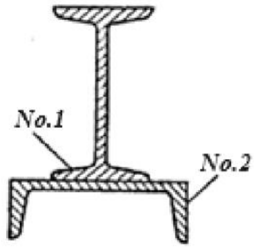
<tr><td>1</td><td>10</td><td>18</td><td rowspan="5"></td></tr>
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<tr><td>3</td><td>14</td><td>14</td></tr>
<tr><td>4</td><td>16</td><td>12</td></tr>
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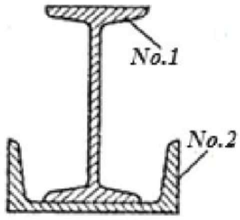
<tr><td>6</td><td>20</td><td>10</td><td rowspan="5"></td></tr>
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<tr><td>8</td><td>16</td><td>14</td></tr>
<tr><td>9</td><td>14</td><td>16</td></tr>
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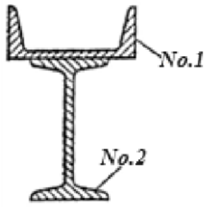
<tr><td>11</td><td>10</td><td>30</td><td rowspan="5"></td></tr>
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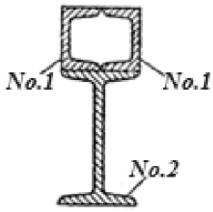
<tr><td>16</td><td>10</td><td>22</td><td rowspan="5"></td></tr>
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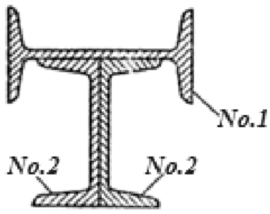
<tr><td>21</td><td>10</td><td>30</td><td rowspan="5"></td></tr>
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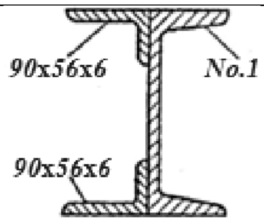
<tr><td>26</td><td>20</td><td>–</td><td rowspan="5"></td></tr>
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<tr><td>30</td><td>24</td><td>–</td></tr>
</table>

CHAPTER II
STRENGTH of MATERIALS

2.1. Tension and compression

Tension and compression are among the simplest and most common deformations of a solid body. They can occur in elements of almost all engineering and construction structures. Cables, screws, axial turbine blades, and compressors are subjected to tension while building columns are subjected to compression.

A rod is subjected to tension or compression by balanced external forces acting along its axis. Under the action of these forces, the cross-section of the rod experiences only one internal force, the longitudinal (normal) force (N). Its value is equal to the algebraic sum of all axial external forces acting on one side of the cross-section. Forces directed away from the cross-section are considered positive, while those directed towards the cross-section are considered negative. a positive force (N) corresponds to tension, while a negative force (N) corresponds to compression.

Calculation of normal stresses in the cross-section of the rod

Fig. 2.1. Normal stress in the rod

Normal stresses at all points in the cross-section of the rod are the same and are determined by the formula:

$$\sigma = \frac{N}{A},$$

where N – is the longitudinal force in the cross-section;

A – is the cross-sectional area of the rod (Fig. 2.1). The sign of corresponds to the sign of N (positive for tension and negative for compression). The unit of stress is Pascal (Pa).

1 Pa = 1 N/m^2; 1 MPa = 1·10^6 N/m^2.

Formulas for calculating deformations and displacements in the cross-section of a rod

Absolute deformation – elongation in tension (Fig. 2.2a) and shortening in compression (Fig. 2.2b) – in the elastic deformation range is calculated according to Hooke's law:

$$\Delta l = \frac{N \cdot l}{E \cdot A},$$

where l – is the length of the deformed region (Fig. 2.2, *a*, *b*);
E – is the longitudinal modulus of elasticity (Young's modulus), which is one of the elastic properties of the material.

The product EA is called the stiffness of the cross-section in tension (compression).

The *deformation area* is the part of the rod where the values of N, a and E are constant or change according to the same laws. The boundaries of the regions are the end sections and sections where at least one of these values changes.

If the rod has n sections, its absolute deformation is equal to the algebraic sum of the deformations of all sections:

$$\Delta l = \sum_{i=1}^{n} \frac{N_i \cdot l_i}{E_i \cdot A_i}$$

The displacement δ of one region relative to another is equal to the deformation of the rod section between these cross-sections.

The relative *longitudinal deformation of* the section is given by:

$$\varepsilon = \frac{\Delta l}{l}$$

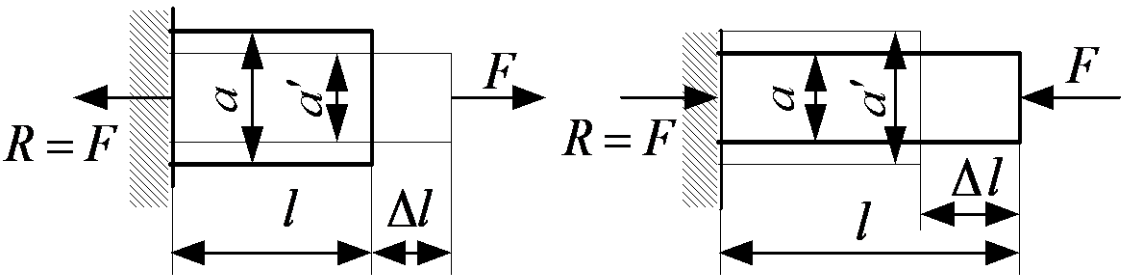
In the elastic deformation range, there is a relationship between stress and relative deformation:

$$\sigma = \varepsilon \cdot E$$

The relative transverse deformation (narrowing or widening) is given by:

$$\varepsilon' = -\frac{\Delta a}{a},$$

where $\Delta a = a - a'$ – is the change in the cross-sectional area (Fig. 2.2).



A *b*

Fig. 2.2. Deformation in uniaxial state of stress:
a - elongation with contraction; *b* - shortening with widening

The relationship between relative transverse and longitudinal deformations is given by:

$$\varepsilon' = -\nu \cdot \varepsilon$$

where ν- is Poisson's ratio, an elastic property of the material.

For isotropic materials $0 < \nu \leq 0.5$.

Strength calculations

Strength conditions:

$$\sigma_{max} = \frac{N}{A} \leq k$$

where N – axial force in the region of highest normal stresses;

A – cross-sectional area of the rod in this region;

k – allowable stress of the material.

For a deformable material, the allowable tensile k_+ and compressive k_- stresses are the same: $k_+ = k_- = k$

$$k = \frac{R_e}{[n]}$$

where R_e– yield strength of the material; $[n]$ – safety factor. For deformable materials $[n] = 1.4 \div 1.6$.

For a brittle material, k_+ and k_- are different. Therefore, the strength conditions for tension and compression are written separately:

$$\sigma_{r_{max}} = \frac{N}{A} \leq k_r$$

$$\sigma_{c_{max}} = \frac{N}{A} \leq k_c$$

$$k_r = \frac{R_m}{[n]}$$

$$k_c = \frac{R_c}{[n]}$$

where k_r and k_c – ultimate strength limits of the material for tension and compression, respectively;

$[n]$ – safety factor.

For brittle materials, $[n] = 2.5 \div 3.0$.

Three types of problems were addressed using strength of materials conditions

Strength verification. We calculate σ_{max} and compare it with k:

$$\sigma_{max} = \frac{N}{A} \leftrightarrow k$$

If $\sigma_{max} \leq k$, the strength of the component is ensured.

Determination of the rod's load-bearing capacity.

(a) calculate the permissible axial force:

$$[N] \leq A \cdot k$$

(b) establish the relationship between axial forces and external forces, and based on this relationship, determine their permissible values ($[F]$ or $[q]$).

Selection of cross-section. Required cross-sectional dimension:

$$A \geq \frac{N}{k}$$

When determining the dimensions of a geometrically constant cross-section, the area a should be expressed in terms of one of its dimensions. Rolled sections are selected according to standard tables of profiles.

Stiffness calculation

The calculation uses the stiffness condition:

$$\Delta = [\Delta],$$

where Δ– the actual deformation of the rod or displacement in the structure;

$[\Delta]$ – the permissible value of deformation, which is accepted based on the conditions of normal operation of the structure.

Examples of calculations

Example 2.1. For the rod shown in the drawing (Fig. 2.3, a), plot the normal forces and, based on the strength condition, determine the dimensions of the cross-sections in all areas. It is given that the cross-sections are circular, the rod is made from greycast iron EN GJL-150, with a safety factor n = 3. The internal forces and lengths are as follows: P_1 = 50 kN, P_2 = 80 kN, P_3 = 40 kN, a = 0.5 m, b = 1.0 m, c = 1 m. Draw a sketch of the rod and the force and displacement diagrams for the rod's cross-sections.

Procedure:

1. Determine the permissible stresses: calculate the allowable stress based on the material properties and safety factor.
2. Determine the axial Force (N) in each segment: use the section method to calculate the axial forces in the different segments of the rod.
3. Plot the axial force diagram: draw the axial force diagram to scale.
4. Select the cross-section: choose the cross-sectional dimensions for these results.
5. Draw the displacement diagram: to plot the displacement diagram, first determine the absolute deformation in each segment according to Hooke's Law.
6. Determine the displacement of key sections: calculate the displacement for the significant cross-sections of the rod and plot the displacement diagram based on the obtained data.

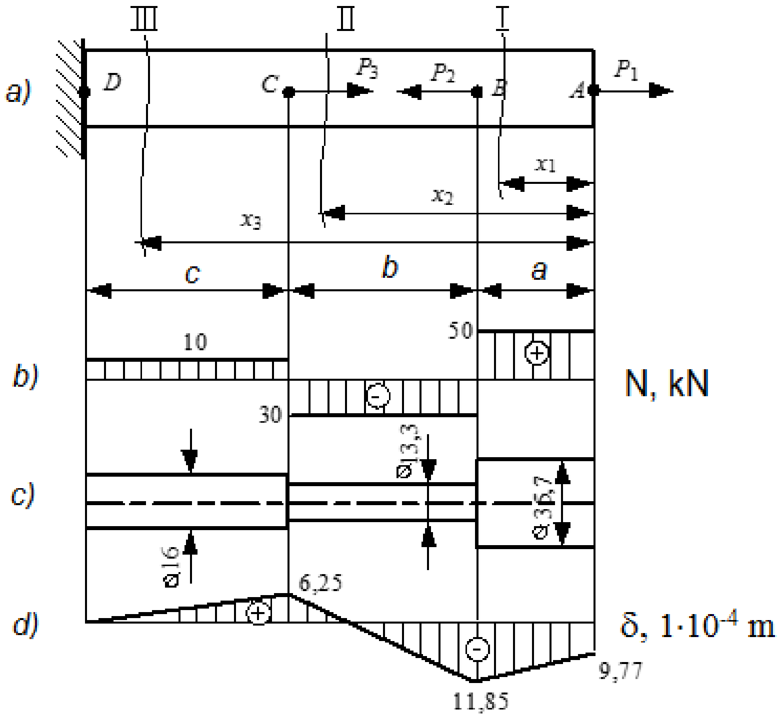


Fig. 2.3. Diagram of the bar to Example 2.1:
a - system of forces in the bar; *b* - force diagram; *c* - cross-section of the bar in its individual compartments; *d* - displacement diagram of the bar cross-sections

Solution

1. Determine the allowable stresses:

From Appendix D, select the values for the grey cast iron EN-GJL-150, necessary for the calculation: R_m= 150 MPa, R_c = 650 MPa, E=1.2·10^5 MPa.

$$k_r = \frac{R_m}{n} = \frac{150}{3} = 50 \text{ MPa;}$$

$$k_c = \frac{R_c}{n} = \frac{650}{3} = 217 \text{ MPa.}$$

2. Determine the Axial **:**

Force N in the Rod's Cross-Sections Based on the Section Method: We do not need to determine the support reactions since the rod is fixed only at the leftmost cross-section. Consequently, the forces will be determined based on the known forces on the right side: $N = \sum P_r$.

Define the segments: segment I – *AB*, segment II – *BC*, segment III – *CD*.

Determine the axial forces *N* for each segment:

Segment I $0 \le x_1 \le a$

$$N_1 = P_1 = 50 \text{ kN}$$

Segment II $a \le x_2 \le a + b$

$$N_2 = P_1 - P_2 = 50 - 80 = \ -30 \text{ kN}$$

Segment III $a+b \leq x_2 \leq a+b+c$

$$N_3 = P_1 - P_2 + P_3 = 50 - 80 + 40 = \ 10 \text{ kN}$$

The calculations indicate that in segments I and III, the rod is subjected to tensile forces, while in segment II, it is subjected to compressive forces.

It is important to note that the mass of the rod was not considered in the calculations due to its negligible value compared to the external loads.

3. We make a graph of N at any scale (Fig. 2.3, b).

4. Using the strength conditions, we calculate the diameter of the bar at each segment:

$$\sigma_{max} = \frac{N}{A} \leq k, \text{so } A \geq \frac{N}{k} \text{and } d = \sqrt{\frac{4A}{\pi}} \approx 1.13\sqrt{A}$$

$$A_1 = \frac{N_1}{k_r} = \frac{50 \cdot 10^3}{50 \cdot 10^6} = 1 \cdot 10^{-3} \text{ m}^2 = 10 \text{ cm}^2;\ d_1 = 3.57 \text{ cm}$$

$$A_2 = \frac{N_2}{k_r} = \frac{30 \cdot 10^3}{217 \cdot 10^6} = 0.138 \cdot 10^{-3} \text{ m}^2 = 1.38 \text{ cm}^2;\ d_1 = 1.33 \text{ cm}$$

$$A_3 = \frac{N_3}{k_r} = \frac{10 \cdot 10^3}{50 \cdot 10^6} = 0{,}2 \cdot 10^{-3} \text{ m}^2 = 2 \text{ cm}^2;\ d_1 = 1.6 \text{ cm}$$

Draw a diagram of the forces acting in the rod (Fig. 2.3, c).

5. To draw a displacement diagram, using Hooke's law, you need to determine the absolute strain in each area:

$$\Delta l_i = \frac{N_i \cdot l_i}{E \cdot A_i}$$

Extension of the It segment

$$\Delta l_1 = \frac{N_1 \cdot a}{E \cdot A_1} = \frac{50 \cdot 10^3 \cdot 0{,}5}{1.2 \cdot 10^{11} \cdot 1.0 \cdot 10^{-3}} = 0.0208 \cdot 10^{-2} \text{ m} = 0.0208 \text{ cm}$$

Compression of the II segment

$$\Delta l_2 = \frac{N_2 \cdot b}{E \cdot A_2} = \frac{-30 \cdot 10^3 \cdot 1{,}0}{1.2 \cdot 10^{11} \cdot 0.138 \cdot 10^{-3}} = -0.181 \cdot 10^{-2} \text{ m} = -0.181 \text{ cm.}$$

Extension of the III segment

$$\Delta l_3 = \frac{N_3 \cdot c}{E \cdot A_3} = \frac{10 \cdot 10^3 \cdot 1.5}{1.2 \cdot 10^{11} \cdot 0.2 \cdot 10^{-3}} = 0.0625 \cdot 10^{-2} \text{ m} = 0.0625 \text{ cm.}$$

6. Determine the displacements of individual sections and from the data obtained, we draw displacement diagrams.

The vertical displacement of any section is equal to the change in the length of the part of the rod that is between the given section and the inelastic support (starting point). Determine the displacements of individual areas of the rod:

$$\delta_D = 0$$

$$\delta_C = \Delta l_3 = 0.0625 \text{ cm}$$

$$\delta_B = \Delta l_3 - \Delta l_2 = 0.0625 - 0{,}181 = 0.1185 \text{ cm}$$

$$\delta_A = \Delta l_3 - \Delta l_2 + \Delta l_1 = 0.0625 - 0.181 + 0.208 = -0.0977 \text{ cm}$$

From the results, we draw a displacement diagram (Fig. 2.3, *d*).

The displacement of the section A is equal to the absolute deformation of the whole rod

$$\delta_A = \Delta l = -0.977 \text{ mm}$$

Example 2.2. Select the dimensions of the AB beam on which there is a continuous load (Fig. 2.4). Material – steel S215.

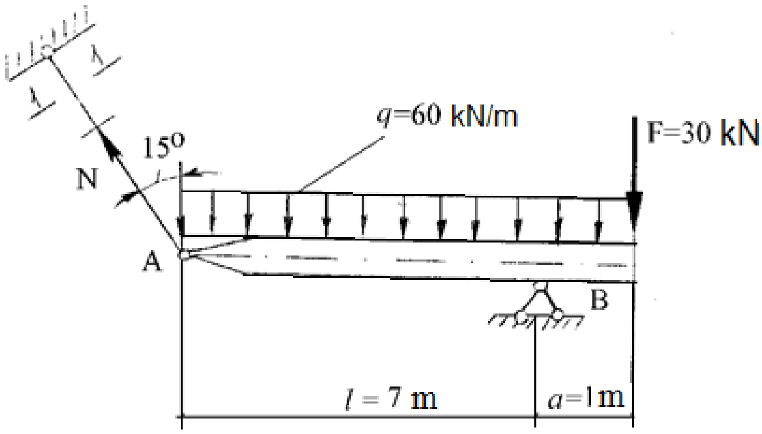


Fig. 2.4. Forces acting on the beam to Example 2.2

Procedure:

1. Determine the reaction of the support.
2. Determine the required cross-sectional area of the bar from the strength conditions.
3. Determine the required profile from the cross-sectional area.
4. Check the strength of the adopted cross-section.

Solution

1. Convert the continuous load to a concentrated force N.
2. Establish the equilibrium equation

$$\sum M_B = 0;$$

$$-N \cdot \sin 75° \cdot 7 + q \cdot 7 \cdot 3.5 - q \cdot 1 \cdot 0.5 - F \cdot 1 = 0$$

$$N = \frac{60 \cdot 7 \cdot 3.5 - 60 \cdot 1 \cdot 0.5 - 30 \cdot 1}{\sin 75° \cdot 7} = 208.6 \text{ kN}$$

3. Determine the required cross-section of the rod from the formula:

$$A \geq \frac{N}{k}$$

$$A \geq \frac{208{,}6 \cdot 10^3}{160 \cdot 10^6} = 1.3 \cdot 10^{-9} \text{ m}^2 = 13 \text{ cm}^2$$

For S215 k = 160 MPa (table D.2).

4. Knowing the cross-sectional area, we determine the required shape of the profile. Two angle bars should have a cross-sectional area of 13 cm^2, therefore one A_1 = 6.5 cm^2. From Table D.60, we take 2 angle bars 70 × 5 with cross-sectional area A_1 = 6.86 cm^2 each.

$$A = 2 \cdot 6.86 = 13.72 \text{ cm}^2$$

5. Check the strength of the adopted section:

$$\sigma \geq \frac{N}{A}$$

$$A \geq \frac{208.6 \cdot 10^3}{13.72 \cdot 10^{-4}} \approx 152 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 150 \text{ MPa} < 160$$

The strength condition has been met.

Answer: For the rod, a cross-section of two 70 × 5 angles was adopted.

In the case of a circular rod section, we use the formula:

$$A = \frac{\pi d^2}{4} \Rightarrow d = \sqrt{\frac{4A}{\pi}} = 1{,}13\sqrt{A} = 1{,}13\sqrt{9.93} = 3{,}56 \text{ cm}$$

Round the calculated value to the standard value from Table D.63 d = 36 mm = 3.6 cm.

Verification*:*

1) $A = \frac{\pi d^2}{4} = 3.14 \cdot \frac{3{,}6^2}{4} = 10.17 \text{ cm}^2$.

2) $\frac{N}{A} = \frac{208.6 \cdot 10^3}{10.17 \cdot 10^{-4}} = 205.1 \text{ MPa } < 210 \text{ MPa}$.

Durability has been assured.

Individual tasks
(calculation)

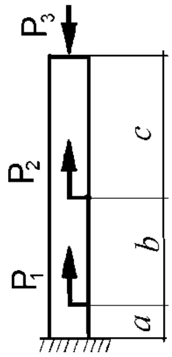
Task 2.1. Carry out strength calculations and determine tensile and compressive deformations (schemes for the task), the rod was made of grey cast iron EN-GJL-150 for which k_r = 150 MPa, k_c = 650 MPa, E = $1.2 \cdot 10^5$ MPa. Ignore the mass of the rod. The data for the calculations are shown in Table 2.1

Table 2.1. Initial data for Task 2.1

Var. no	P_1	P_2	P_3	a	b	c	d	e
	kN			m				
1	2.5	4	2	1	2	1	2	1
2	1	6	4	2	1	2	2	1
3	2	5	8	2	1	3	2	1
4	10	24	15	2	4	1	2	2
5	7	2	4	1	2	2	2	1
6	11	6	8.5	1	2	1	1	1
7	34	11	6	2	1	2	1	1
8	2	6	5	1	3	1	2	1
9	1	2.5	1.5	3	1	1	3	1
10	8	5	7	1	2	2	1	1

Schemes for Task 2.1

1



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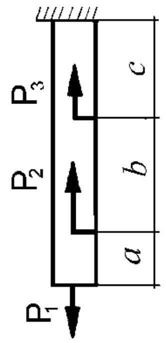

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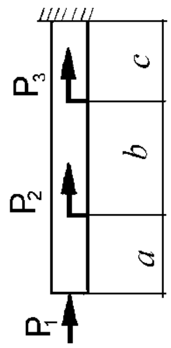

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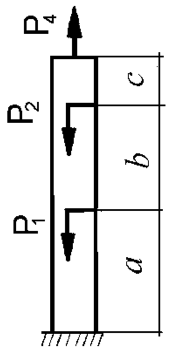

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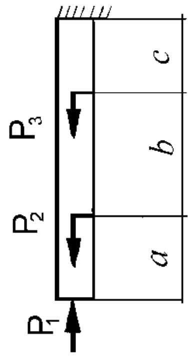

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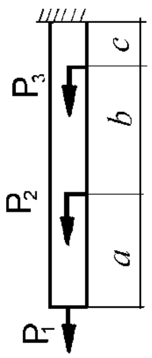

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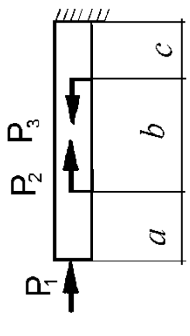

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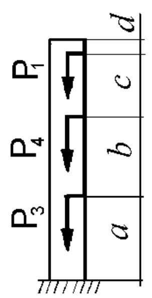

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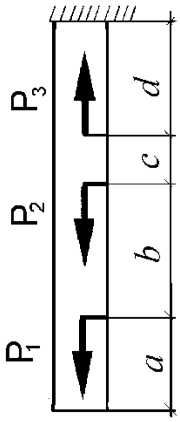

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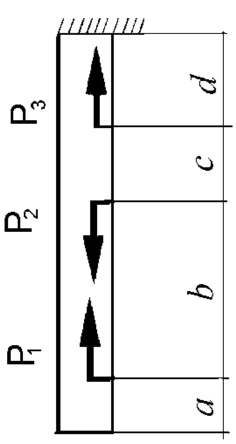

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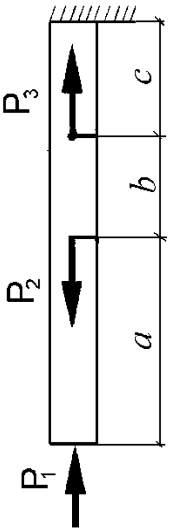

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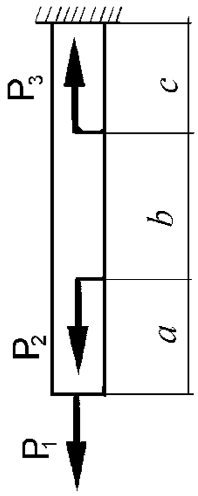

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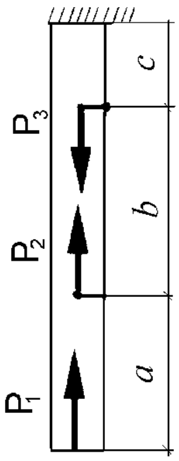

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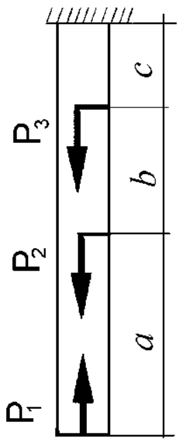

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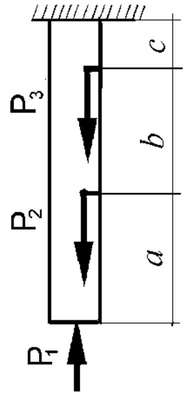

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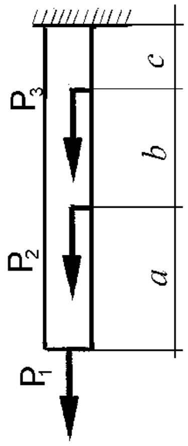

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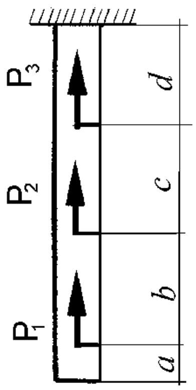

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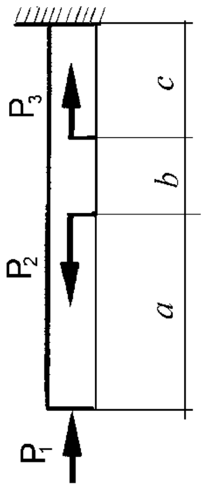

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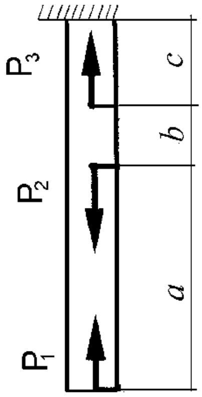

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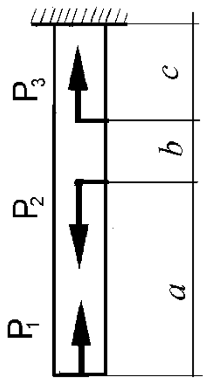

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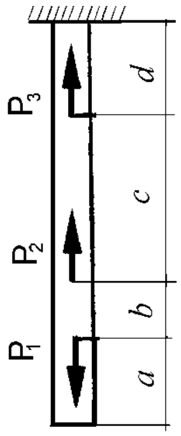

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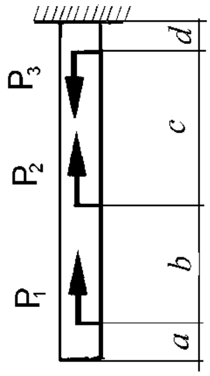

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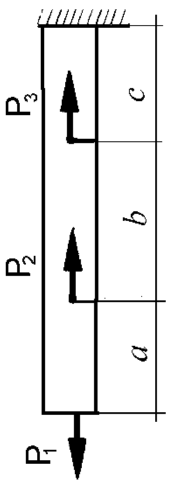

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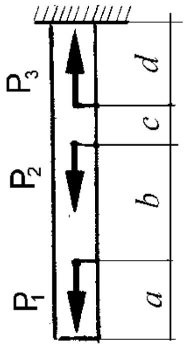

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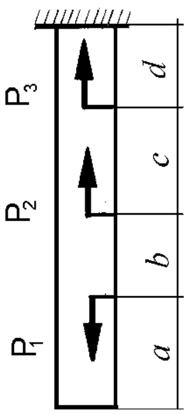

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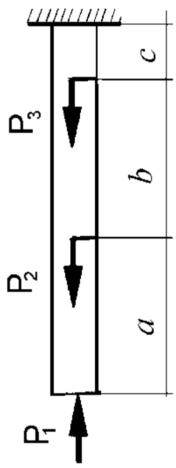

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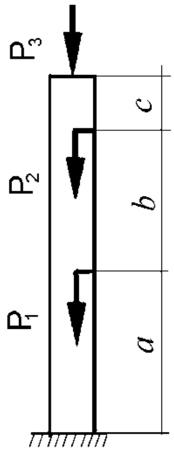

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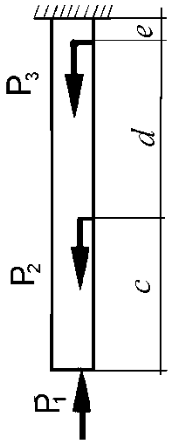

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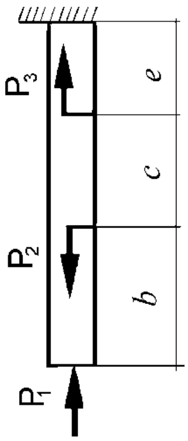

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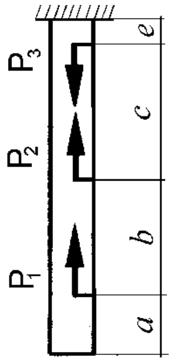

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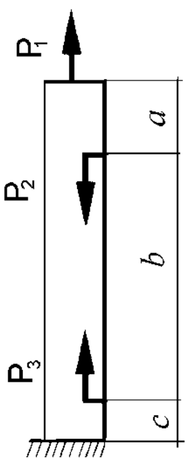

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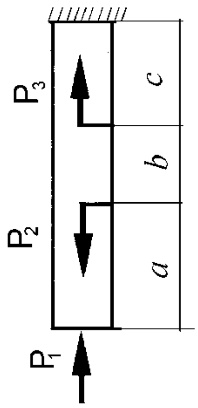

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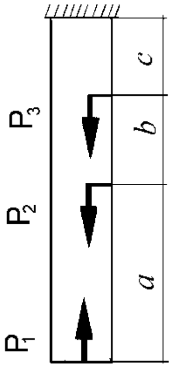

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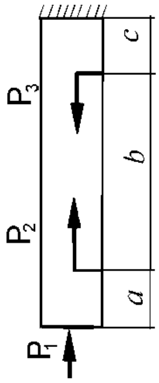

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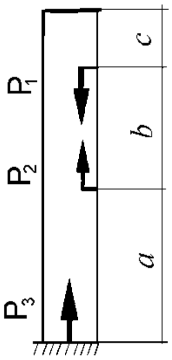

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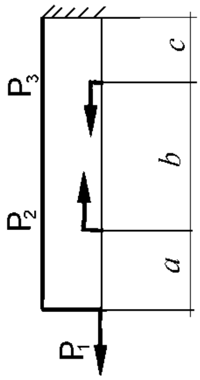

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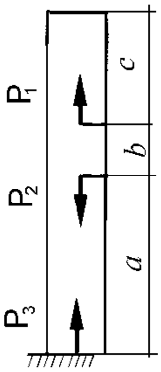

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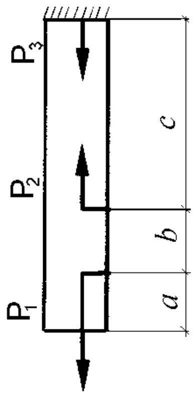

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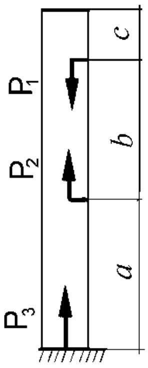

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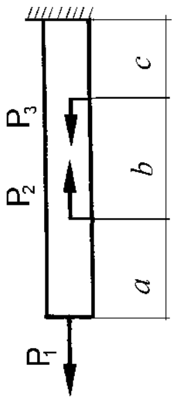

Task 2.2. Select the cross-section of the rod holding the crossbar (see the schemes for the calculation). Material - steel S215.

Schemes for Task 2.2

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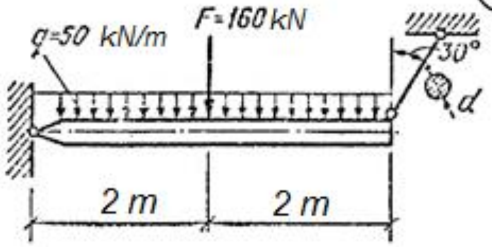

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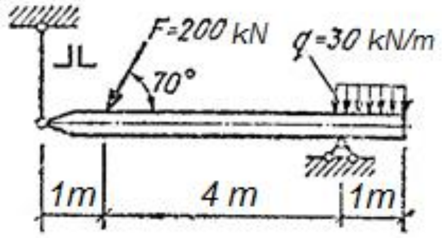

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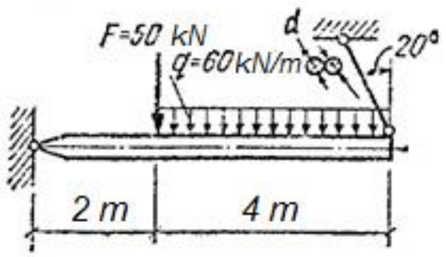

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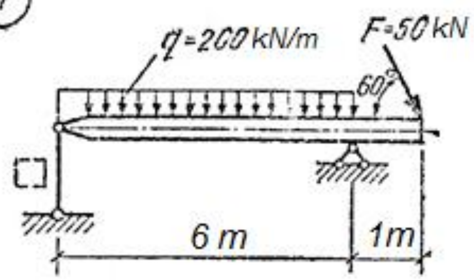

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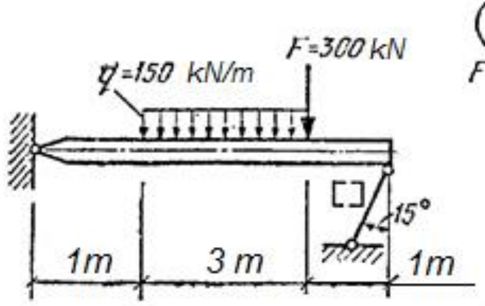

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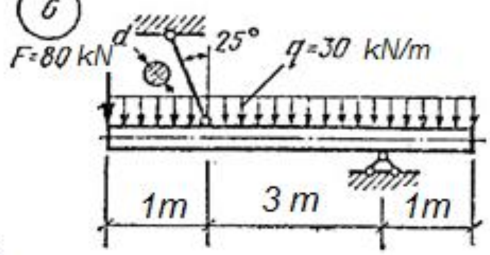

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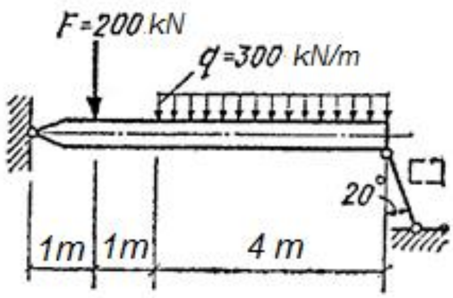

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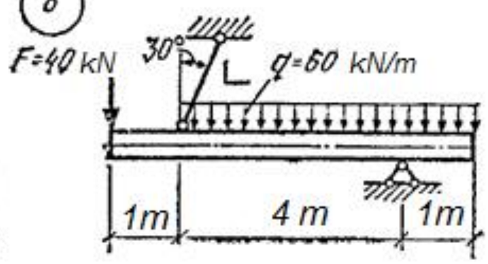

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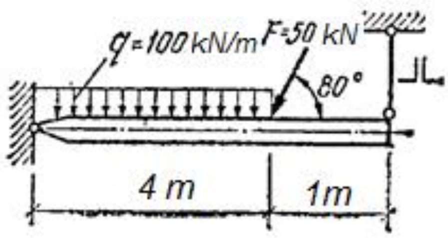

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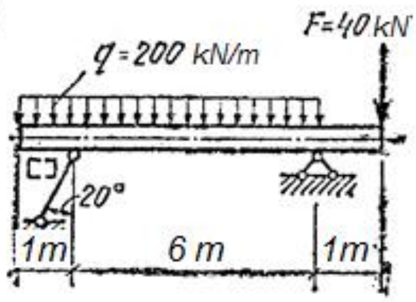

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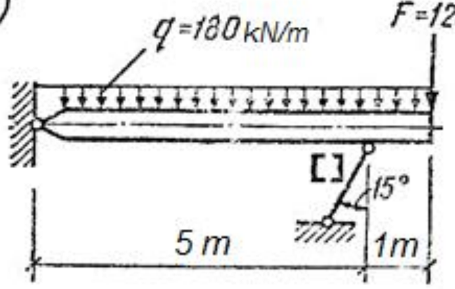

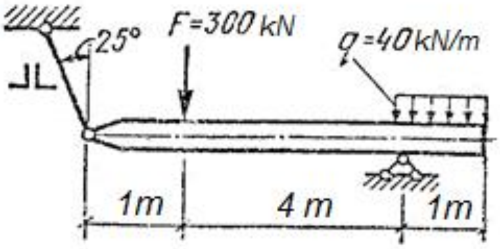

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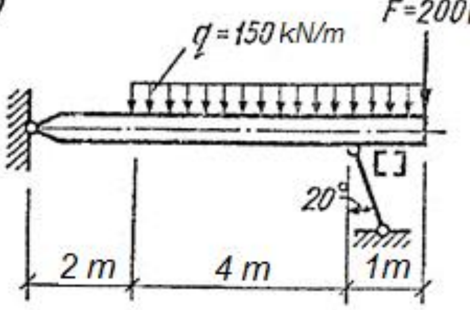

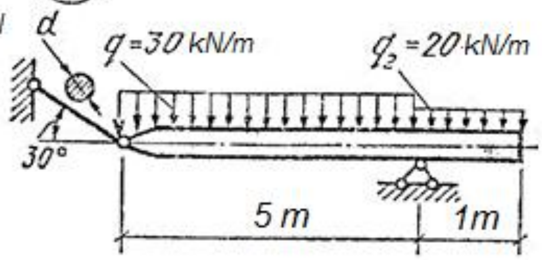

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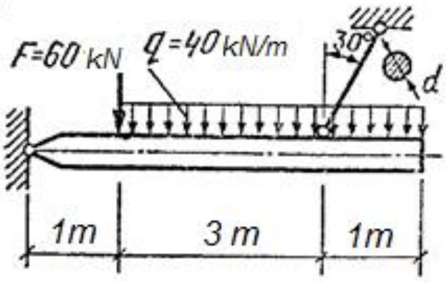

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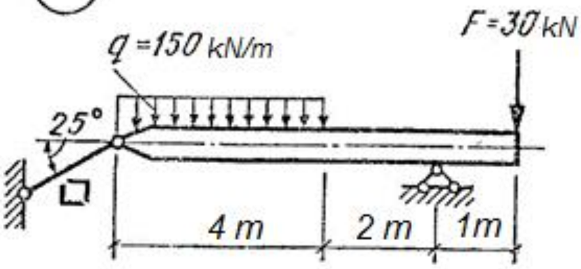

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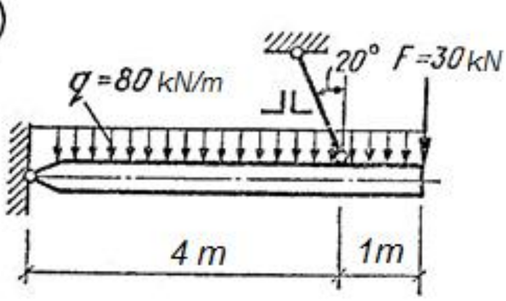

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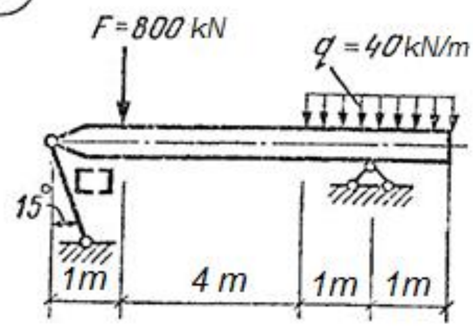

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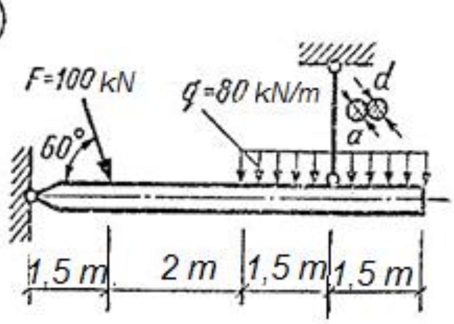

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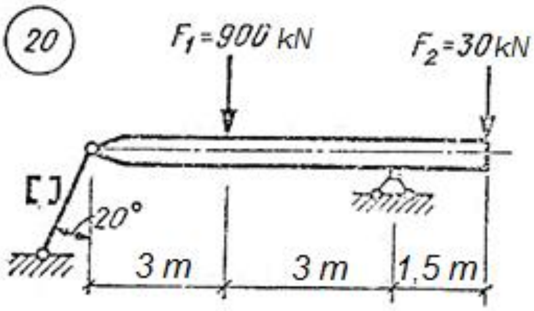

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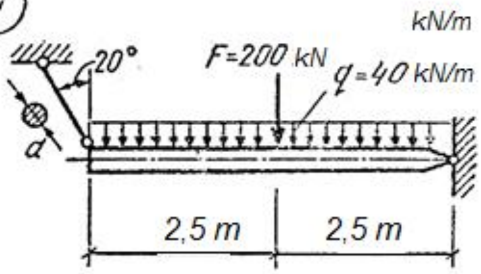

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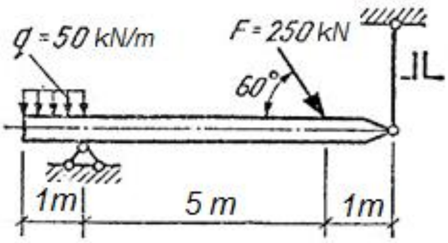

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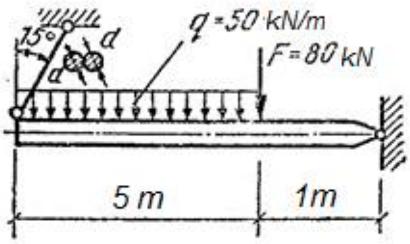

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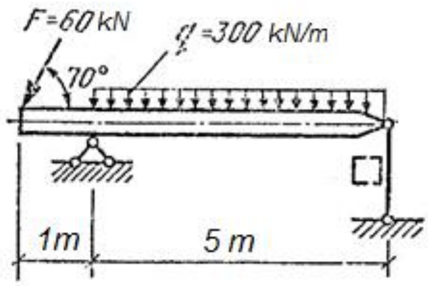

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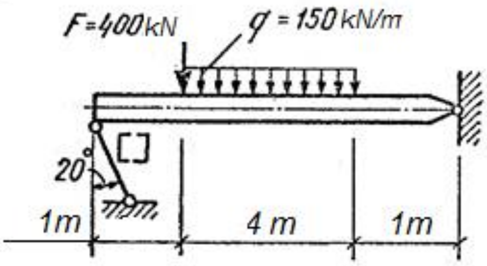

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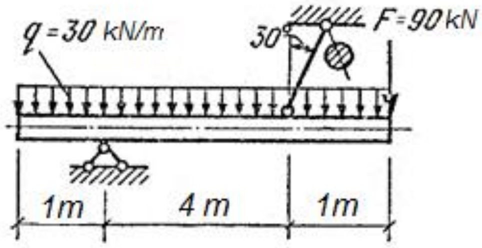

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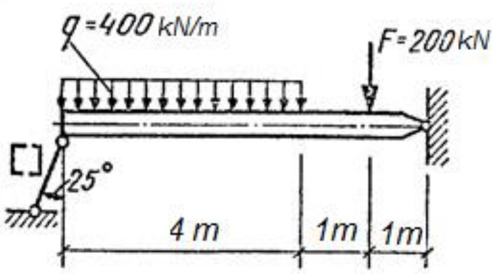

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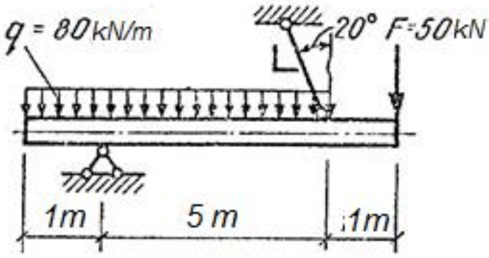

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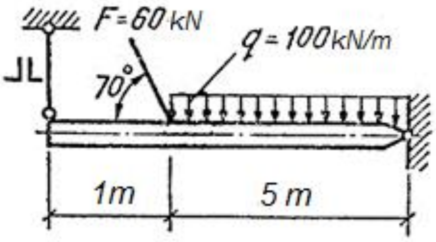

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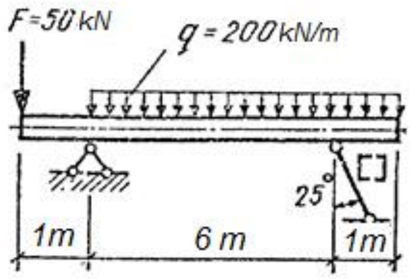

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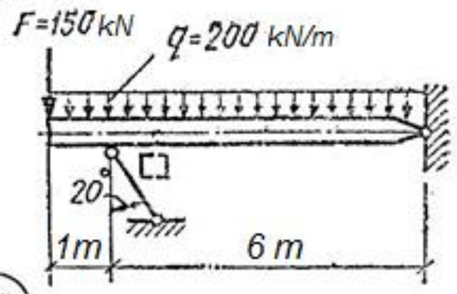

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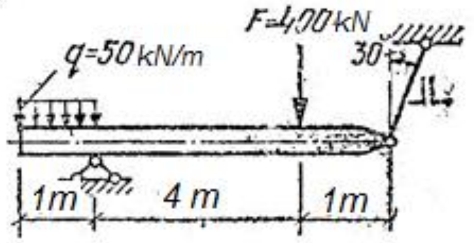

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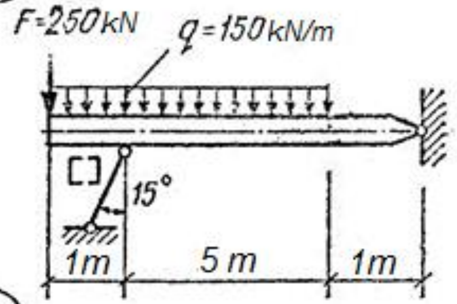

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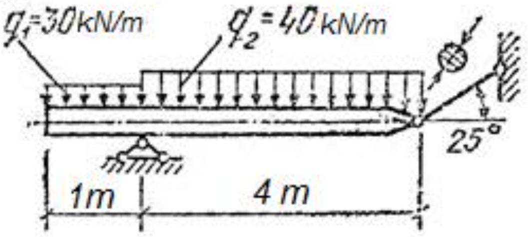

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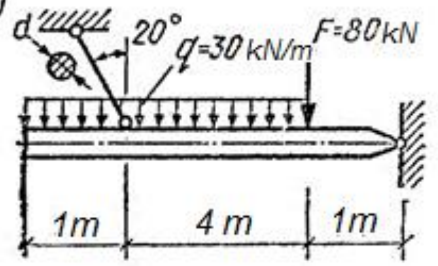

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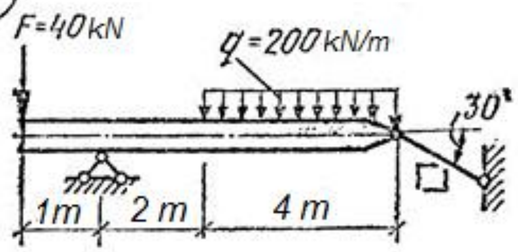


2.2. Statically non-determinable structures

General information

There are many structures in which the internal forces cannot be determined using the equations of statics alone because the number of unknown forces in these structures is greater than the number of equilibrium equations. These tasks are called *statically indeterminate.*

The difference between the number of unknowns and the number of equations of statics determines the number of redundant unknowns or the degree of static uncertainty. When there is one redundant unknown the element is called statically indeterminate once, when there are two, it is called statically indeterminate twice, and so on.

General methods for solving statically non-equivocal systems have been developed: the static equilibrium equations are supplemented with additional displacement equations based on the commonality of deformations reflecting the specificity of the structure's action and with relations expressing the dependence of the displacements of structural elements on forces. It is convenient to follow the following sequence:

Static aspect of the problem. Arrange the equilibrium equations for a single structural element, taking one of the forces as statically indeterminate.

Geometric aspect of the issue. Determine the relationship between the deformation of individual structural elements based on the total deformation. The equations obtained are the equations of total deformation.

Physical aspect of the issue. Based on Hooke's law, we express the deformation of structural elements by statically indeterminate forces acting on them.

Synthesis. By solving the static, geometric and physical equations together, we determine the unknown forces.

Examples of calculations

Example 2.3. For a statically indeterminate rods system (Fig. 2.5), determine the dimensions of the cross-sections of the rods from the strength condition, if it is known that the ratio of their cross-sections $A_1 : A_2 = \beta$, the bars are made of steel S215; the factor of safety n = 1.6. The horizontal beam AD is completely rigid, the external forces and geometric dimensions are:

$$F = 10\ \text{kN};\ q = 20\ \frac{\text{kN}}{\text{m}};\ \alpha = 30\ °; a = 2\ \text{m};\ b = 0{,}5\ \text{m};\ \beta = 0{,}5;\ l_1 = 2\ \text{m};\ l_2 = 3{,}4\ \text{m}$$

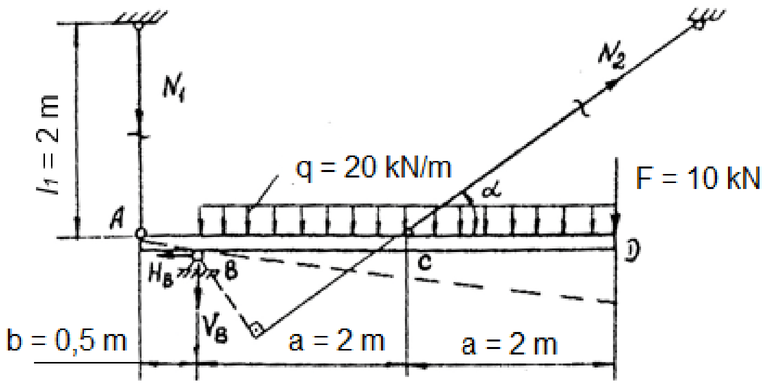


Fig. 2.5. Diagram of a statically indeterminate bar system to Example 2.3

Solution

1. For steel S215 in Table D.1 we find the values of the mechanical properties of the material, necessary for calculations: E = 2·10⁵ MPa and R_e = 240 MPa.

The allowable stresses will be:

$$k = \frac{R_e}{n} = \frac{240}{1.5} = 1.5\ \text{MPa}$$

2. Using the section method, we determine the unknown N_1 and N_2 forces in the rods, aligning their direction with an arbitrary deformation character of the rod system (for example, as shown by the dashed line in Fig. 2.5, the first rod will be in compression and the second in tension). To determine the forces N_1 and N_2, we will consider the equilibrium of the system with the interaction of internal forces and reactions in the supports:

$$\sum M_B = 0$$

$$N_1 \cdot l + N_2 \cdot a \cdot \sin\alpha - q \cdot 2 \cdot a^2 - F \cdot 2 \cdot a = 0 \qquad (2.1)$$

$$N_1 \cdot 0.5 + N_2 \cdot 2 \cdot 0.5 - 20 \cdot 4 \cdot 2 - 10 \cdot 4 = 0$$

$$0.5 \cdot N_1 + N_2 = 200 \qquad (2.2)$$

The use of two other equilibrium equations ($\sum X = 0$ and $\sum Y$) makes no sense since they contain unknown reactions H_B and V_B, which need not be determined. Thus, the system is one time statically indeterminate, because using the equations of statics, three equilibrium equations can be determined, and four quantities are unknown: N_1, N_2, H_B and V_B.

3. Write an auxiliary equation for the deformed state of the system (Fig. 2.6).

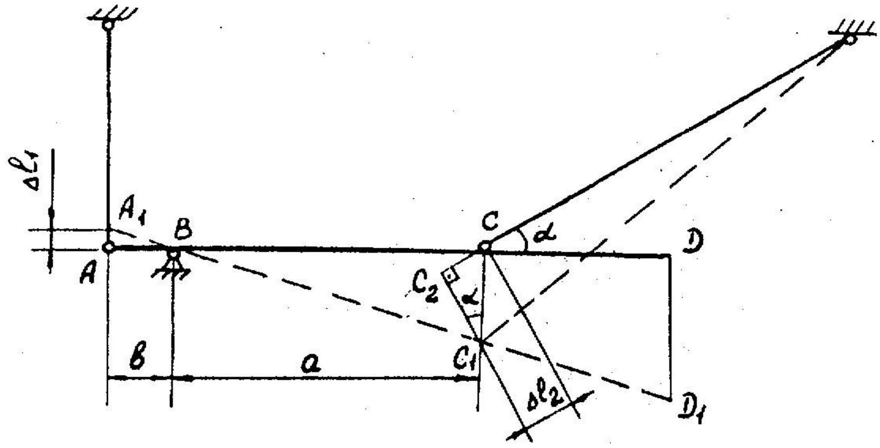


Fig. 2.6. Deformed state of the system

From the similarity ΔAA_1B and ΔCC_1B we have $\frac{\Delta l_1}{CC_1} = \frac{b}{a}$, from where CC_1 id determined from ΔCC_1C_2

$$CC_1 = \frac{\Delta l}{\sin\alpha}$$

The compatibility equation will then be of the form:

$$\frac{\Delta l_1}{\Delta l_2/\sin\alpha} = \frac{a}{b} \qquad (2.3)$$

According to Hooke's law, we express deformations by unknown forces:

$$\Delta l_1 = \frac{N_1 \cdot l_1}{E \cdot A_1}$$

$$\Delta l_2 = \frac{N_2 \cdot l_2}{E \cdot A_2}$$

After transformations, we get:

$$N_1 \cdot l_1 \cdot \sin\alpha \cdot -N_2 \cdot l_2 \cdot \beta \cdot b = 0$$

$$N_1 \cdot 2 \cdot 0.5 \cdot 2 - N_2 \cdot 3.4 \cdot 0.5 \cdot 0.5 = 0$$

$$2N_1 - 0{,}85N_2 = 0 \qquad (2.4)$$

4. Solving the equilibrium equations (2.2) and the deformation equations (2.4), we obtain the normal forces in the rods:

$$N_1 = 70 \text{ kN}; \qquad N_2 = 165 \text{ kN}$$

5. According to the strength conditions, we determine the required cross-sectional areas of the two rods:

$$A_1 \geq \frac{N_1}{k} = \frac{70 \cdot 10^3}{160 \cdot 10^6} = 0.438 \cdot 10^{-3} \text{ m}^2 \tag{2.5}$$

$$A_2 \geq \frac{N_2}{k} = \frac{165 \cdot 10^3}{160 \cdot 10^6} = 1.03 \cdot 10^{-3} \text{ m}^2 \tag{2.6}$$

According to the task, the ratio of the cross-sectional areas should be:

$$\beta = A_1 : A_2 = 0.5$$

According to the condition (2.5) $A_1 = 0.438 \cdot 10^{-3}$ m^2 and knowing the value β, we calculate

$$A_2 = \frac{A_1}{\beta} = 0.876 \cdot 10^{-3} \text{m}^2,$$

which does not satisfy condition (2.6). Therefore, to satisfy both conditions, we will take $A_2 = 1.03 \cdot 10^{-3}$ m^2 from (2.6).

Then $A_1 = \beta \cdot A_2 = 0.525 \cdot 10^{-3} \text{m}^2$ instead of $0.438 \cdot 10^{-3}$ m^2.

In this case, the stresses acting in the two bars will equal:

$$\sigma_1 = \frac{N_1}{A_1} = \frac{70 \cdot 10^3}{0.515 \cdot 10^{-3}} = 136 \text{ MPa} < k = 160 \text{ MPa}$$

$$\sigma_2 = \frac{N_2}{A_2} = \frac{165 \cdot 10^3}{1.03 \cdot 10^{-3}} = 160 \text{ MPa} = k = 160 \text{ MPa}$$

Individual tasks

(calculation)

Task 2.3. A rigid beam is supported by an articulated fixed support and attached to two elastic rods (see diagrams for the task). From the strength calculations, determine the value of the load *P*. The data for the calculations can be found in Table 2.2.

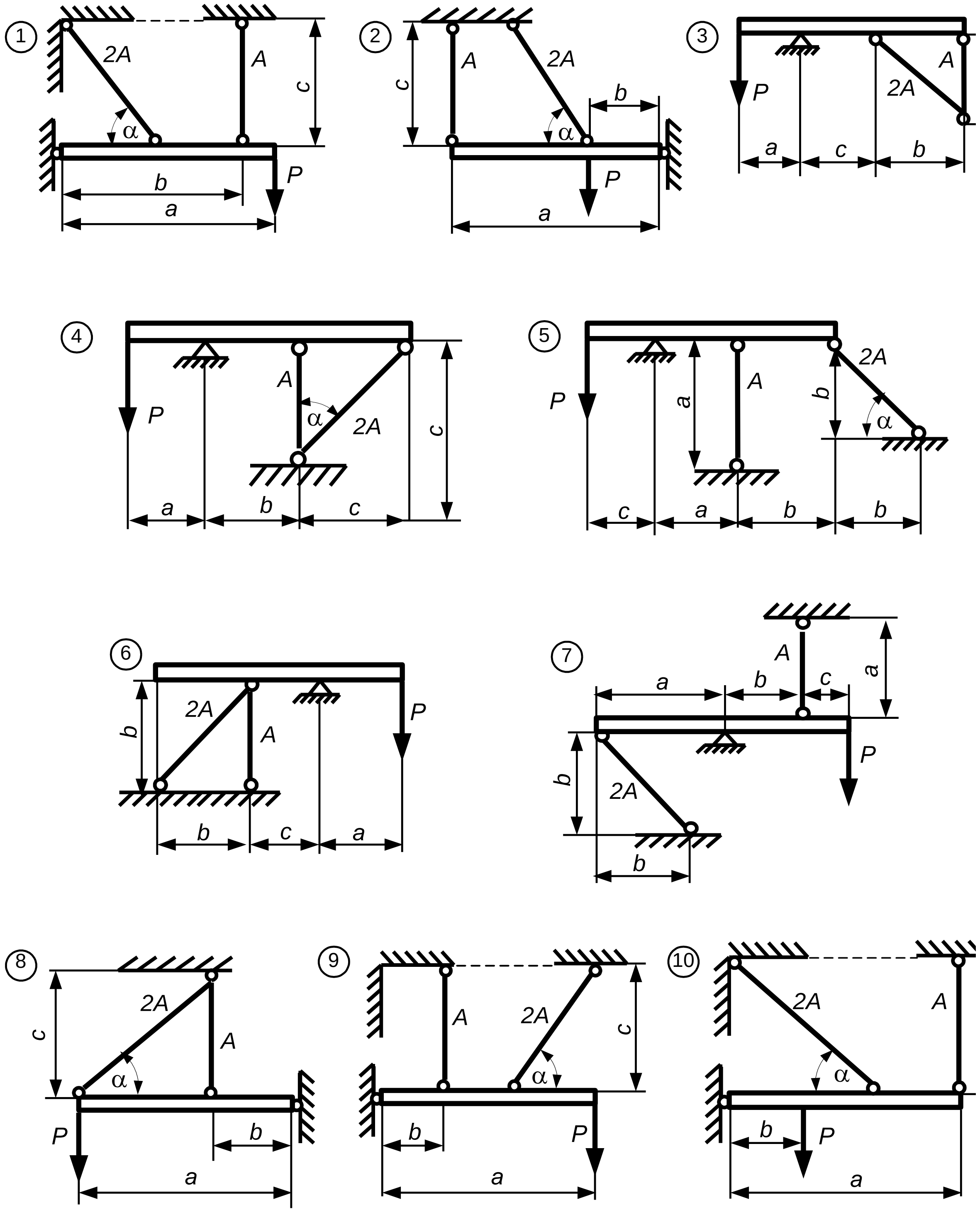
Procedure:

1. Using the section method, determine the forces acting in the sections.
2. To determine the forces, consider the equilibrium of the system taking into account the loads applied to the beam and the reactions in the supports.
3. Arrange the auxiliary deformation compatibility equation by considering the deformed state system of the system and the ratio of forces.
4. From the strength condition, determine the maximum value of the load *P*.

Table 2.2. Initial data for Task 2.3

Var. no	Scheme no	A, cm^2	a, m	b, m	c, m	α, °	k, MPa	$E \cdot 10^5$, MPa
1	1	16	10	8	2.5	30	160	2
2	7	14	7	2	2	45	180	2
3	13	15	5	4	1.5	45	160	2
4	11	16	12	6	4	45	180	2
5	12	15	7	2	2	45	160	2
6	14	16	3	4	2	45	160	2
7	15	16	10	5	3	45	180	2
8	9	15	8	2	4	45	180	2
9	4	16	10	8	4	45	180	2
10	3	16	4	2	3	45	180	2
11	2	17	7	3	2.5	30	150	2
12	5	14	7	2	4	45	140	2
13	16	18	5	8	2	45	160	2
14	17	15	10	6	4	45	140	2
15	8	12	5.5	2	2	30	120	2
16	18	16	9	6	2	45	150	2
17	10	14	8	2	4	45	160	2
18	6	15	7	2.5	4	45	140	2
19	7	18	10	3	2.5	45	150	2
20	14	16	2.8	2.8	1.8	45	120	2
21	1	13	10	7	2	30	140	2
22	5	16	8	3.5	5	45	150	2
23	13	14	3	5	2	45	160	2
24	17	16	5	3	2	45	180	2
25	12	15	8	2	3	45	160	2
26	18	16	6	2.3	1.3	45	180	2
27	3	15	7	2	3	45	160	2
28	9	16	7	2	3	45	160	2
29	4	16	6	3	2.5	45	180	2
30	3	16.5	3	4	4	45	140	2
31	15	16	5	2.5	1.5	45	150	2
32	6	16	8	2	4	45	160	2
33	8	18	7.4	3	2.5	30	160	2
34	16	20	2.5	7.5	1.5	45	140	2
35	10	15	7	1	3	45	160	2
36	11	12	9	4	2	45	150	2
37	17	16	6	4	2	45	170	2
38	14	14	3	4	2	45	180	2
39	9	18	9	2	3	30	140	2
40	7	16	9	2	3	45	150	2

Schemes to Task 2.3



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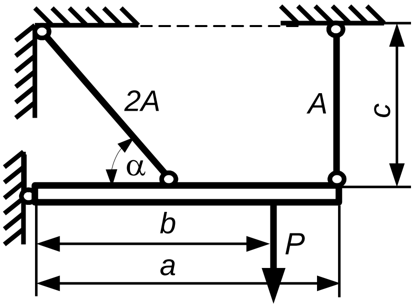

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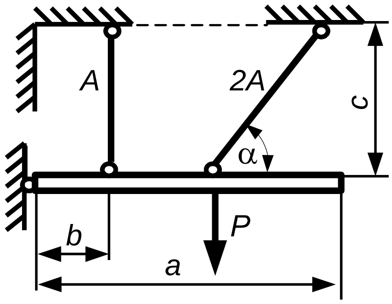

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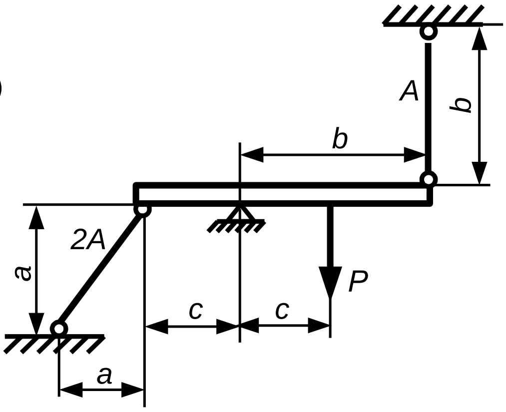

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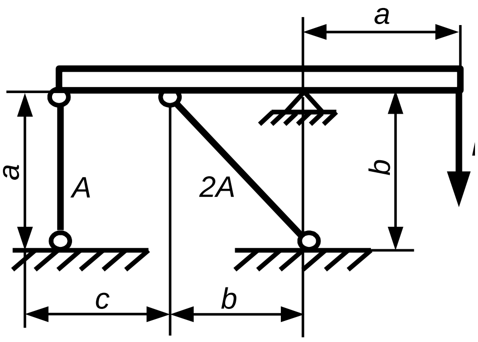

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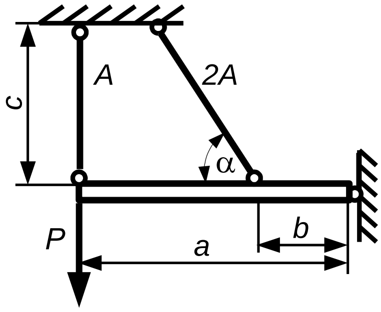

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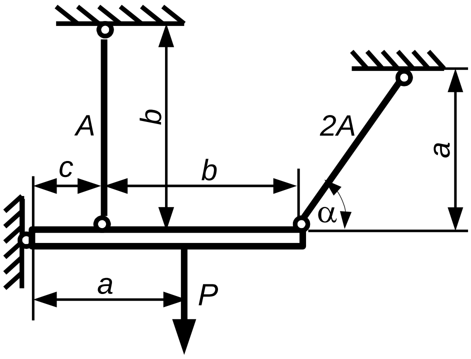

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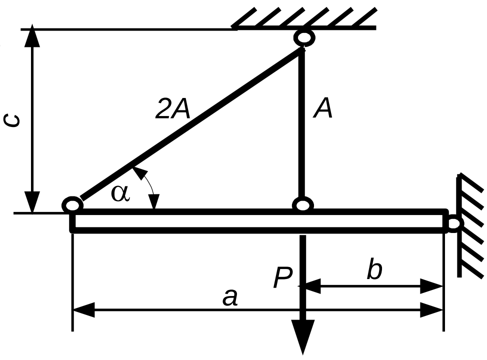

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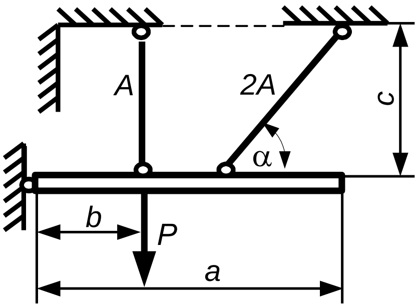


2.3. Geometric characteristics of cross-sections

General information

For uniaxial tension, compression and shear, the geometrical characteristic of the cross-section of a member was the cross-sectional area, which fully determined the member's resistance to deformation. However, for bending, torsion and complex deformations, this characteristic is not sufficient.

The strength and stiffness of a beam for a given material and length depend on the dimensions and shapes of the cross-section. The geometrical characteristics of the cross-sections are used to quantify this relationship.

The ability to determine the required geometrical characteristics is essential for calculating the strength and stiffness of beams under different types of deformation.

Geometrical characteristics of flat sections

Moments of interia

The static moment of inertia of a planar section concerning any axis that lies with it in one section is the sum of the products of the elementary areas dA of the whole section and their distances to this axis (Fig. 2.7), i.e.:

$$S_x = \int_A y \cdot dA, \qquad S_y = \int_A x \cdot dA$$

Static moment unit - [m³].

Applying the theorem on the sum of moments of systems of forces:

$$S_x = \int_A y \cdot dA = A \cdot y_c$$

$$S_y = \int_A x \cdot dA = A \cdot x_c,$$

where a – is the area of the entire section;

x_c, y_c – coordinates of the centre of gravity of the section.

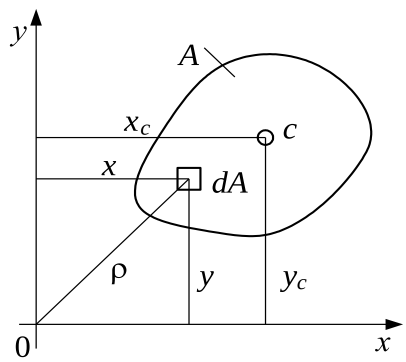


Fig. 2.7. Cross-section of a bar with an xy system

The static moment can take on **positive** or **negative** values. If the axis concerning which the static moment is determined passes through the centre of gravity of the section $x_c = 0$ and $y_c = 0$, then its static moment is **zero**:

$$S_x = A \cdot 0 = 0$$

$$S_y = A \cdot 0 = 0$$

The static moments of the composite section are expressed by the formulae:

$$S_x = \sum A_i \cdot y_i$$

$$S_y = \sum A_i \cdot y_i$$

where A_i – areas of the components of the composite section;

x_i, y_i – coordinates of the centre of gravity of the components of the section relative to the x and y axes.

The determination of the coordinates of the centre of gravity of the composite section is carried out using the formulae:

$$x_c = \frac{\sum A_i x_i}{\sum A_i}$$

$$y_c = \frac{\sum A_i y_i}{\sum A_i}$$

If a section has two axes of symmetry, then the centre of gravity is located at their intersection. If the section has one axis of symmetry, then the centre of gravity lies on this axis and only one coordinate is required to determine its position.

The axial moment of inertia of a planar section concerning any axis that lies in the same plane is the sum of the products of the elementary surfaces dA by the square of their distances from the axis (Fig. 2.7). It is calculated from the formula:

$$J_x = \int_A y^2 \, dA$$

$$J_y = \int_A x^2 \, dA,$$

where x, y – distance from the axes relative to which the moment of inertia is determined. Unit of moment of inertia – [m^4].

Axial moments of inertia are always **positive and cannot be zero.**

The polar moment of inertia of a planar cross-section J_o relative to any pole '0' that lies in the plane of the cross-section is the sum of the products of the elemental areas *dA* by the square of their distances from the pole, i.e.:

$$J_o = \int_A \rho^2 \, dA$$

where ρ - is the distance of the elementary surface *dA* from the pole (Fig. 2.7).

If the pole coincides with the origin of the coordinate system, then the relation is satisfied:

$$J_o = J_x + J_y$$

Unit of polar moment of inertia – [m^4].

The polar moment of inertia is always **positive and cannot be zero.**

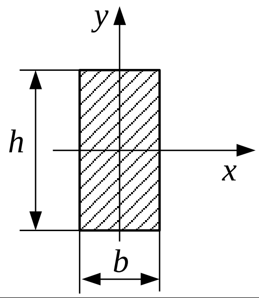
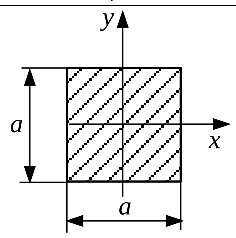
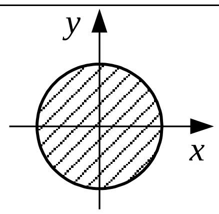
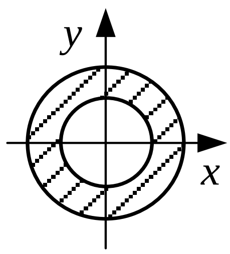
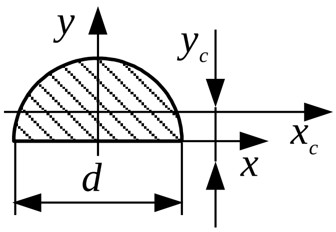
The centrifugal moment of inertia of a cross-section is the sum of the products of the elemental areas *dA* and their distances from the axes *x* and *y*. It is calculated from the formula:

$$J_{xy} = \int_A x \cdot y \, dA$$

The centrifugal moment can be **positive, negative or zero**. If at least one of the two mutually perpendicular axes is an axis of symmetry, then the centrifugal moment of inertia of the section about such axes is zero.

Table 2.3 may be used to calculate the geometrical characteristics of simple shapes concerning their central axes and, for sections, these are taken from the tables in Appendix.

Table 2.3. Moments of inertia of selected plane figures

Rectangle 	$A = h \cdot b$ $J_x = \frac{bh^3}{12};\ J_y = \frac{hb^3}{12}$
Square 	$A = a^2$ $J_x = J_y = \frac{a^4}{12}$
Circle 	$A = \frac{\pi d^2}{4}$ $J_x = J_y = \frac{\pi d^4}{64} = 0.05d^4$
Ring 	$A = \frac{\pi D^2}{4} - \frac{\pi d^2}{4}$ $J_x = J_y = \frac{\pi D^4}{64} - \frac{\pi d^4}{64} = \frac{\pi}{64}(D^4 - d^4) = (D^4 - d^4)$
Semicircle 	$A = \frac{\pi d^2}{8}$ $y_c = 0.424r$ $J_x = J_y$ $J_{x_c} = 0.00686d^4, J_{y_c} = 0.025d^4$

Moments of inertia with displacement of the coordinate system

The axes passing through the centre of gravity of the section are called the **central axes** (x_c, y_c) and the moments of inertia concerning them are called the **central moments of inertia**.

If the moments of inertia J_{xc}, J_{yc}, J_{xcyc} concerning the central axes x_c, y_c (Fig. 2.8) are known, the moments of inertia concerning the axes x and y, which are displaced parallel to the central axes, are determined according to the formulae (Steiner's theorem):

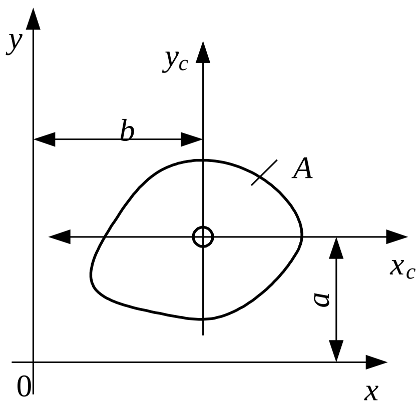


Fig. 2.8. Cross-section of a bar with an xy system

$$J_x = J_{x_c} + a^2 \cdot A \qquad J_y = J_y + b^2 \cdot A \qquad J_{xy} = J_{x_c y_c} + a \cdot b \cdot A$$

where a, b – coordinates of the centre of gravity (x, y).

The centrifugal moment of inertia should take into account the signs of the a and b coordinates of the centre of gravity.

Calculation of moments of inertia of compound sections

The moments of inertia of a composite section are calculated as the sum of the moments of its parts:

$$J_x = \sum J^i_{x_c} \qquad J_y = \sum J^i_{y_c} \qquad J_{xy} = \sum J^i_{x_c y_c}$$

If the cross-section has an opening, it is convenient to consider it as part of a figure with a “negative” area.

Moments of inertia under rotation of the coordinate system

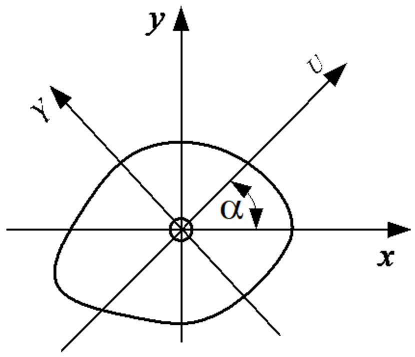


Fig. 2.9. Cross-section of a bar in a rotated coordinate system

The moments of inertia of the section concerning the x', y' axes rotated by an angle α from the Input x, y axes (Fig. 2.9) are calculated using the formulae:

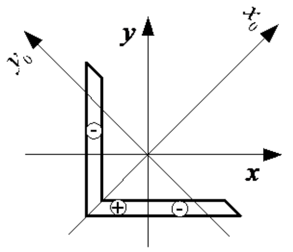
$$J_{x'} = J_x \cos^2 \alpha + J_x \sin^2 \alpha - J_{xy} \sin \alpha$$

$$J_{y'} = J_y \sin^2 \alpha + J_y \cos^2 \alpha - J_{xy} \sin \alpha$$

$$J_{x'y'} = \frac{J_x - J_y}{2} \cdot \sin 2\alpha + J_{xy} \cdot \cos 2\alpha$$

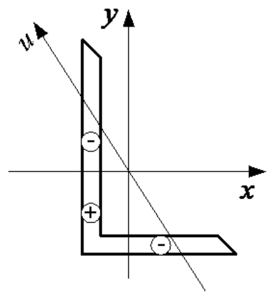
The positive reference direction of the angle is clockwise.

The centrifugal moment of inertia of angles concerning parallel offset axes:



Isosceles angle

$$J_{xy} = \pm \frac{J_{x_0} - J_{y_0}}{2}$$



Non-armed angle

$$J_{xy} = \pm \sqrt{(J_x - J_u)(J_y - J_u)}$$

The “+” and “-“ signs depend on the position of the angle section in the coordinate system.

The geometrical characteristics of the sections for rolled sections are taken from the catalogue tables.

Principal axes and principal moments of inertia

Principal axes are axes for which the centrifugal moment of inertia is zero and the axial moments of inertia reach extreme values.

The angle α, which determines the position of the principal axes, is calculated according to the formula:

$$tg2\alpha = \frac{2J_{xy}}{J_x - J_y}$$

The moments of inertia about the principal axes are called principal moments of inertia and are calculated according to the formula:

$$J_{\substack{max \\ min}} = \frac{J_x + J_y}{2} \pm \sqrt{\left(\frac{J_x - J_y}{2}\right)^2 + J_{xy}^2}$$

One of the principal axes is rotated by an angle α to the x-axis, and the other is perpendicular to it.

Principal axes passing through the centre of gravity of a section are of practical importance. They are called the *central principal axes*. The moments of inertia concerning these axes are called *principal central moments of inertia*. They are used in calculations.

Section strength indices

The bending strength indices of sections are calculated about the principal central axes according to the formula:

$$W_x = \frac{J_x}{y_{max}}, W_y = \frac{J_y}{x_{max}},$$

where W_x, W_y – indices of section bending strength;

x_{max}, y_{max} – distance of the furthest point of the cross-section from its major central axis.

Unit of strength index - [cm^3]. **The indices are not additive!**

Formulas for flexural strength indices for simple sections:

rectangle: $$W_x = \frac{bh^2}{2}, W_y = \frac{hb^2}{2}$$

square: $$W_x = W_y = \frac{a^3}{6}$$

circle: $$W_\rho = \frac{\pi d^3}{16}, W_x = W_y = \frac{\pi d^3}{32} = 0{,}1d^3$$

ring: $$W_\rho = \frac{\pi D_z^3}{16}(1 - \alpha^4) = 0{,}2D_z^3(1 - \alpha^4), \alpha = \frac{d_w}{D_z}$$

D_z – outer diameter, d_w – inner diameter.

Examples of calculation

Example 2.5. Determine the principal moments of inertia and the strength indices of the section, which consists of two angles of dimension 56 × 56 × 4 and C-bar C18 (Fig. 2.10).

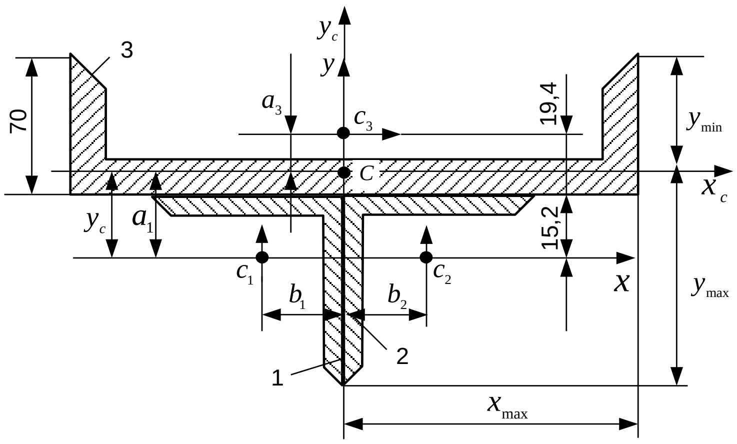


Fig. 2.10. Calculation scheme to Example 2.5

Solution

1. Divide the section into rolling profiles (Fig. 2.10). It consists of two angles 56 × 56 × 4 and C-bar C18, we label them 1, 2, 3.

2. Determine the centres of gravity C_1, C_2, C_3, using data from the table corresponding to each profile.

3. Enter the coordinate system. The y-axis coincides with the axis of symmetry, and we will take the x-axis through the centre of gravity of the angles.

4. Determine the coordinates of the centre of gravity of the entire section.

Since the y-axis coincides with the axis of symmetry it passes through the centres of gravity of the section, therefore $x_c = 0$.

Determine the y_c coordinate using the formula:

$$y_c = \frac{\sum A_i \cdot y_i}{\sum A_i} = \frac{A_1 \cdot y_1 + A_2 \cdot y_2 + A_3 \cdot y_3}{A_1 + A_2 + A_3}$$

Using the tables in the appendix, we determine the area of each profile and their coordinates of the centre of gravity.

$A_1 = 4.38$ cm^2, $y_1 = 0$;

$A_2 = 4.38$ cm^2, $y_2 = 0$;

$A_3 = 20.7$ cm^2, $y_3 = z_0$ (angle) + z_0 (C-bar) = 1.52 + 1.94 = 3.46 cm.

The coordinates of y_1 and y_2 are equal to zero because the x axis passes through the centres of gravity of the angles.

Substitute the values obtained into the formula for calculating y_c:

$$y_c = \frac{2 \cdot 4.38 \cdot 0 + 20.7 \cdot 3.46}{2 \cdot 4.38 + 20.7} = 2.43 \text{ cm}$$

Denote the centre of gravity by the letter c in Fig. 2.10.

Draw the main central axes. Connect the y_c axis to the axis of symmetry, and take the x_c axis through the centre of the c section perpendicular to the y_c axis. The axes y_c and y_3 coincide.

From tables D.58 and D.60:

C-bar C18: a = 20.7 cm^2, z_0 = 1.94 cm, J_x = 86 cm^4, J_y= 1090 cm^4.

Angle bar 56 × 56 × 4: a = 4.38 cm^2, z_0 = 1.52 cm, $J_x = J_y$ = 13.1 cm^4,

$$J_{x_0} = 20.8 \text{ cm}^4, J_{y_0} = 5.41 \text{ cm}^4$$

Determine the principal moment of inertia about the x_c axis of the whole section (using Steiner's theorem)

$$J_{x_c} = \sum J_{x_c}^i = J_{x_c}^1 + J_{x_c}^2 + J_{x_c}^3$$

$$J_{x_c}^1 = J_{xc1} + a_1^2 A_1 = 13{,}1 + 2{,}43^2 \cdot 4.38 = 38.96 \text{cm}^4$$

where $a_1 = y_c$ = 2.43 cm.

From Fig. 2.10, it follows that $A_1 = A_2$ =4.38 cm and $a_1 = a_2 = y_c$ = 2.43 cm,

so $J_{x_c}^1 = J_{x_c}^2 = 38.96 \text{ cm}^4$

$$J_{x_c}^3 = J_{xc3} + a_3^2 \cdot A_3 = 86 + 1.03^2 \cdot 20.7 = 107.76 \text{cm}^4,$$

where

$$a_3 = z_{0_{C-bar}} + z_{0_{angle}} - y_0 = 1.94 + 1.52 - 2{,}43 = 1.03 \text{ cm}$$

there

$$J_{x_c} = 2 \cdot 38.96 + 23.48 = 185.88 \text{cm}^4$$

Determine the principal moment of inertia about the y_c axis of the entire section:

$$J_{y_c} = \sum J_{y_c}^i = J_{y_c}^1 + J_{y_c}^2 + J_{y_c}^3$$

$$J_{y_c}^1 = J_{y_{c1}} + b_1^2 A_1 = 13{,}1 + (-1.52)^2 \cdot 4.38 = 23.22 \text{ cm}^4$$

where $b_1 = -z_{0_{angle}} = -1.52$ cm

$$J_{y_c}^2 = J_{y_{c2}} + b_2^2 F_2 = 13.1 + 1.52^2 \cdot 4.38 = 23.22 \text{ cm}^4$$

where $b_2 = z_{0_{angle}} = 1{,}52$ cm

$$J_{y_c}^3 = J_{y_{c3}} + b_3^2 F_3 = 1090 + 0^2 \cdot 20.7 = 1090 \text{cm}^4$$

where b_3 = 0 ($y_{c\text{ C-beam}}$ is coincident with y_c).

Then:

$$J_{y_c} = 2 \cdot 23.22 + 1090 = 1136.44 \text{ cm}^4$$

Determine the strength index relative to the x_c axis of the entire section:

$$W_x = \frac{J_x}{y_{max}}$$

$$W_{x_c} = \frac{J_{x_c}}{y_{max}} = \frac{185.88}{6.51} = 28.55 \text{ cm}^3$$

$$W_{x_c} = \frac{J_{x_c}}{y_{min}} = \frac{185.88}{6.09} = 30.52 \text{ cm}^4$$

where

$$y_{max} = 5.6 - z_{0_{angle}} + y_c = 5.6 - 1.52 + 2.43 = 6.51 \text{ cm}$$

$$y_{min} = 7 + 5.6 - 6.51 = 6.09 \text{ cm}$$

Determine the strength index relative to the y_c axis of the entire section:

$$W_y = \frac{J_y}{x_{max}} = \frac{1136.44}{9} = 126.27 \text{cm}^3$$

where $x_{\max} = 18/2 = 9$ cm.

Example 2.6. For a given cross-section, determine the position of the principal central axes and the values of the principal central moments of inertia (Fig. 2.11).

Data:
strip 240 × 10
isosceles angle 110 × 110 × 8
C-bar C20

Solution

1. Draw the cross-section to any scale and enter the coordinate systems passing through the centre of gravity of each of its components x_1, y_1; x_2, y_2; x_3, y_3 (Fig. 2.11).

2. Calculate and extract the geometrical characteristics of the components from the assortment tables.

Strip: area $A_1 = 24 \cdot 1 = 24$ cm^2;

moments of inertia:

$$J_{x_1} = \frac{24 \cdot 1^3}{12} = 2 \text{ cm}^4, J_{y_1} = \frac{24^3 \cdot 1}{12} = 1152 \text{ cm}^4$$

Angle 110 × 110 × 8 from table D.60: area A_2= 17.2 cm^2,
moments of inertia:

$$J_{x_2} = J_{y_2} = 98 \text{ cm}^4,$$
$$J_{max} = 315 \text{ cm}^4,$$

$$J_{min} = 81.8 \text{ cm}^4$$

$$J_{x_2y_2} = \frac{J_{max} - J_{min}}{2} \sin 2(-45°) = \frac{315 - 81{,}8}{2} = -116.6 \text{ cm}^4$$

C-bar C20 from table D.58: area A_3= 23.4 cm^2;

moments of inertia:

$$J_{x_3} = 1520 \text{ cm}^4, J_{y_3} = 113 \text{ cm}^4, J_{x_3y_3} = 0$$

3. Introduce an additional coordinate system x_0 , y_0 in such a way that the whole section is in the first quadrant (this is not obligatory, but it is convenient since with such a choice of axes the coordinates of the centres of gravity of the components of the section have positive values) and determine using formulae the positions of the centres of gravity concerning these axes:

$$x_c = \frac{A_1 \cdot x_1 + A_2 \cdot x_2 + A_3 \cdot x_3}{A_1 + A_2 + A_3} = \frac{24 \cdot 12 + 17.2 \cdot 3 + 23.4 \cdot 21.93}{24 + 17.2 + 23.4} = 13.2 \text{ cm}$$

$$y_c = \frac{A_1 \cdot y_1 + A_2 \cdot y_2 + A_3 \cdot y_3}{A_1 + A_2 + A_3} = \frac{24 \cdot 0.5 + 17.2 \cdot 4 + 23.4 \cdot 11}{24 + 17.2 + 23.4} = 5.24 \text{ cm}$$

We visually check that the position of the specified centre of gravity is correct: it is in the area of the triangle $C_1C_2C_3$, so the calculation can be continued.

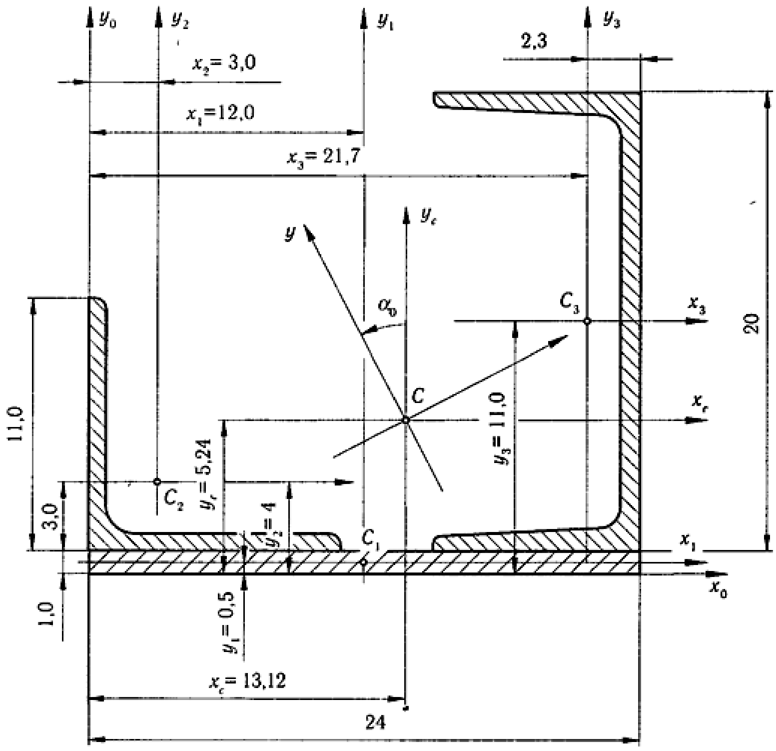


Fig. 2.11. Calculation scheme to Example 2.6

4. Through the calculated centre of gravity C pass the central axes of the sections x_c and y_c and concerning these axes we determine the coordinates of the centres of gravity of the component figures.

To check the correctness of the determination of the centre of gravity of the section and the coordinates of the centres of gravity of the simple components concerning the central axes x_c, y_c we calculate the static moments of the section concerning the axes x_c and y_c:

$$S_{x_c} = A_1(y_1 - y_c) + A_2(y_2 - y_c) + A_3(y_3 - y_c) = 24(0.5 - 5.24) +$$
$$+17.2(4 - 5.24) + 23.4(11 - 5.24) = -135.088 + 134.784 = -0.304 \text{ cm}$$

$$S_{y_c} = A_1(x_1 - x_c) + A_2(x_2 - x_c) + A_3(x_3 - x_c) = 24(12 - 13.2) +$$
$$+17.2(3 - 13.2) + 23.4(21.93 - 13.2) = -204.24 + 204.282 = 0.042 \text{ cm}$$

The values of the static moments are close to zero. This shows that the coordinates of the centre of gravity were calculated with a high degree of accuracy. So the position of the central axes was determined correctly.

5. We determine the axial moments and the centrifugal moment of inertia concerning the central axes x_c and y_c:

$$J_{y_c} = \sum_{i=1}^{3}[J_{y_c}] = \sum_{i=1}^{3}[J_{y_1} + (x_i - x_c)^2 A_i]$$
$$= J_{y_1} + (x_1 - x_c)^2 \cdot A_1 + J_{y_2} + (x_2 - x_c)^2 \cdot A_2 + J_{y_3} + (x_3 - x_c)^2 \cdot A_3$$
$$= 1152 + (12 - 13.2)^2 \cdot 24 + 198 + (3 - 13.2)^2 \cdot 17.2 + 113 + (21.93 - 13.2)^2$$
$$\cdot 23.4 = 5070.43 \text{ cm}^4$$

$$J_{x_c} = \sum_{i=1}^{3}[J_{x_c}] = \sum_{i=1}^{3}[J_{x_1} + (y_i - y_c)^2 A_i]$$
$$= J_{x_1} + (y_1 - y_c)^2 \cdot A_1 + J_{x_2} + (y_2 - y_c)^2 \cdot A_2 + J_{x_3} + (y_3 - y_c)^2 \cdot A_3$$
$$= 2 + (0.5 - 5.24)^2 \cdot 24 + 198 + (4 - 5.24)^2 \cdot 17.2 + 1520 + (11 - 5.24)^2 \cdot 23.4$$
$$= 3062.02 \text{ cm}^4$$

Centrifugal moment of the entire section:

$$J_{x_c y_c} = \sum_{i=1}^{3}[J_{x_c y_c}] = \sum_{i=1}^{3}[J_{x_i y_i} + (x_i - x_c)(y_i - y_c)A_i] = J_{x_1 y_1} +$$
$$+(x_1 - x_c)(y_1 - y_c) \cdot A_1 + J_{x_2 y_2} + (x_2 - x_c)(y_2 - y_c) \cdot A_2 + J_{x_3 y_3} +$$
$$+(x_3 - x_c)(y_3 - y_c) \cdot A_3 = 0 + (12 - 13.2)(0.5 - 5.24) \cdot 24 - 116.6 +$$
$$+(3 - 13.2)(4 - 5.24) \cdot 17.2 + 0 + (21.93 - 13.2)(11 - 5.24) \cdot 23.4 = 1414.12 \text{cm}^4$$

6. Calculate the position of the principal central axes and determine the values of the principal central moments of inertia:

$$\text{tg}2\alpha_0 = \frac{2J_{x_c y_c}}{J_{y_c} - J_{x_c}} = \frac{2 \cdot 1414.12}{5070.43 - 3062.02} = 1.408$$

$$2\alpha_0 = 54.62° \Rightarrow \alpha_0 = 27.31°$$

Since $J_{x_c} < J_{y_c}$, the axis of least stiffness x ($J_x = J_{min}$) is inclined at $\alpha_0 < 45°$ to the x_c axis, the axis of greatest stiffness y ($J_y = J_{max}$) is perpendicular to it. The angle α_0 is postponed counterclockwise ($\alpha_0 > 0$).

$$J_{\frac{max}{min}} = \frac{J_{x_c} + J_{y_c}}{2} \pm \sqrt{\left(\frac{J_{x_c} - J_{y_c}}{2}\right)^2 + J_{x_c y_c}^2}$$
$$= \frac{3062.02 + 5070.43}{2} \pm \sqrt{\left(\frac{3062.02 - 5070.43}{2}\right)^2 + 1414.12^2}$$
$$= (4066.225 \pm 1734.406)\ \text{cm}^4$$

$$J_{y_{max}} = 5800.63\ , J_{x_{min}} = 2331.82\ \text{cm}^4$$

7. Verification.

For verification, we check the following conditions:

1) the sum of moments about any pair of central axes of the section should be constant:

$$J_x + J_y = 5800.63 + 2331.82 = 8132.45\text{cm}^4$$
$$J_{x_c} + J_{y_c} = 3062.02 + 5070.43 = 8132.45\text{cm}^4$$

Condition (1) is met:

$$J_x + J_y = J_{x_c} + J_{y_c}$$

2) the centrifugal moment of inertia of the section concerning the principal central axes must be zero. We calculate the centrifugal moment of inertia J_{xy}:

$$J_{xy} = \frac{J_{x_c} - J_{y_c}}{2} \sin 2\alpha_0 + J_{x_c y_c} \cos 2\alpha_0 = \frac{3062.02 - 5070.43}{2} \sin 54.62° +$$
$$+1414.12 \cos 54.62° = -818.76 + 818.77 = 0.01\ \text{cm}^4$$

Relative error

$$\frac{0.01}{818.765} 100\% = 0.001\% < 2\%$$

Condition (2) is also satisfied: $J_{xy} \approx 0$. It means that the calculation has been carried out correctly.

Individual task

(calculation)

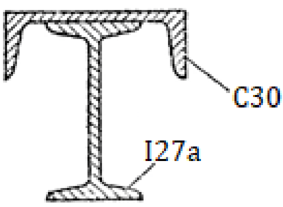
Task 2.4. Determine the moments of inertia (axial and centrifugal) and bending strength indices concerning the major central axes (see scheme for the task). The variant number is determined from the list.

Procedure:

1. Determine the coordinates of the centre of gravity of the entire section.
2. Determine the axial moments and centrifugal moments concerning the central axes.
3. Determine the position of the major central axes.
4. Determine the values of the major central moments of inertia of the section.
5. Determine the bending strength indices relative to the major central axes of the section.

Schemes for the Task 2.4

1



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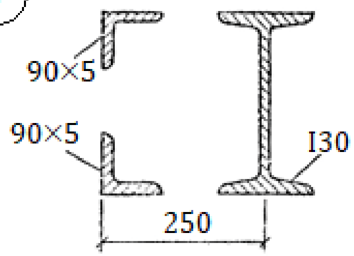

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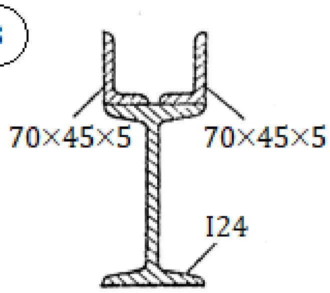

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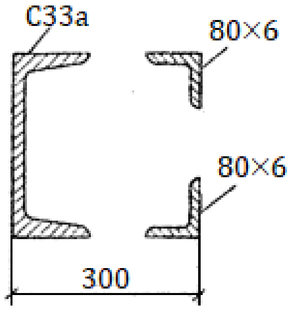

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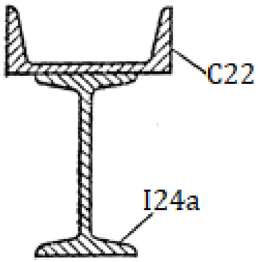

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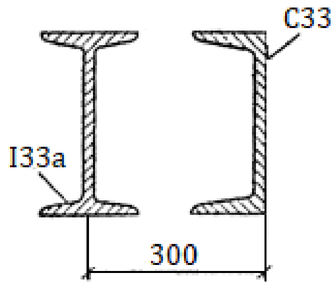

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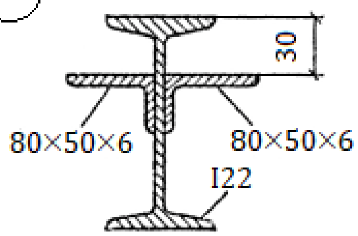

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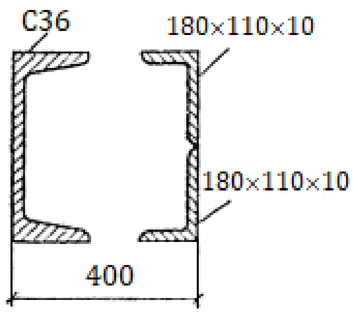

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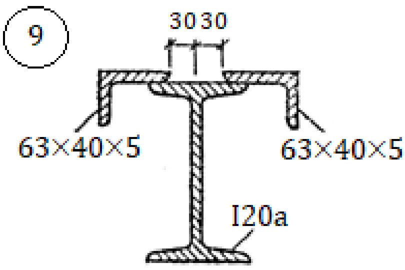

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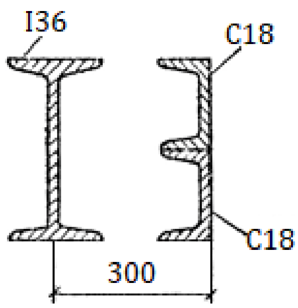

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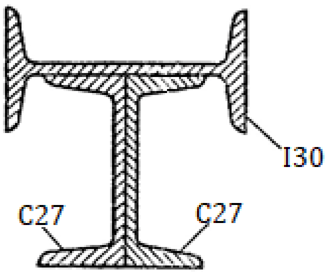

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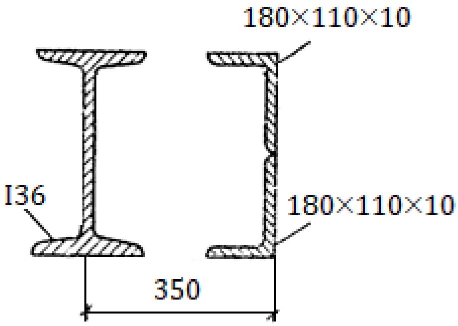

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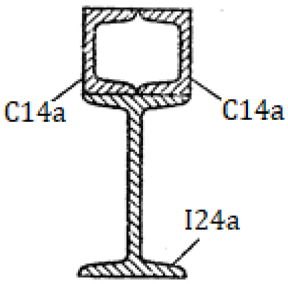

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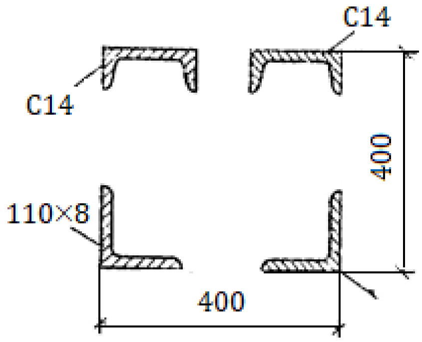

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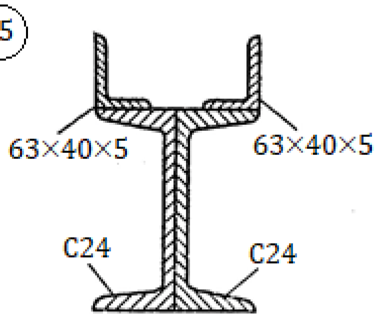

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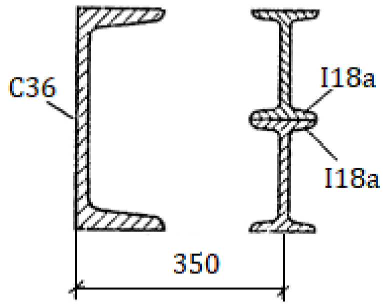

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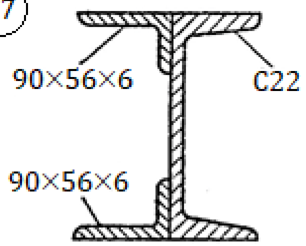

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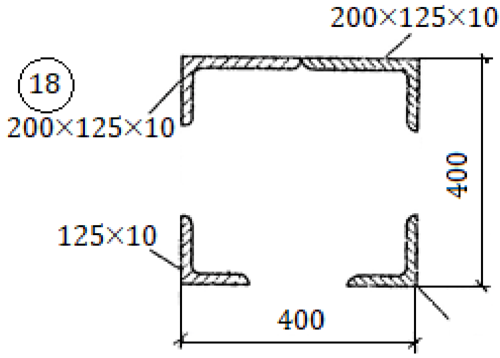

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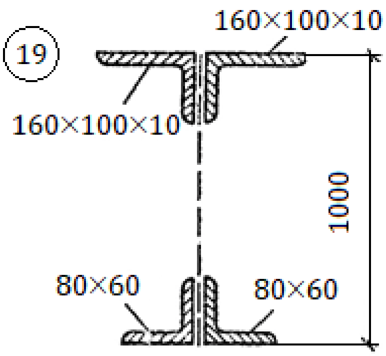

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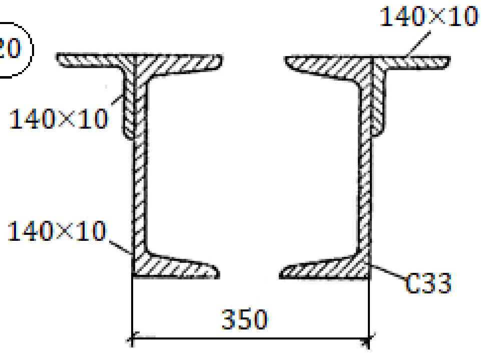

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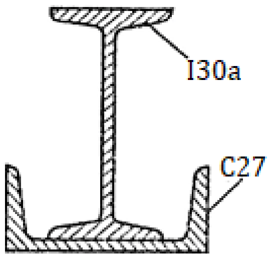

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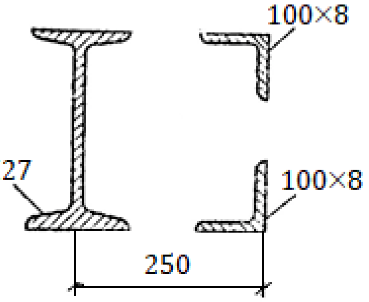

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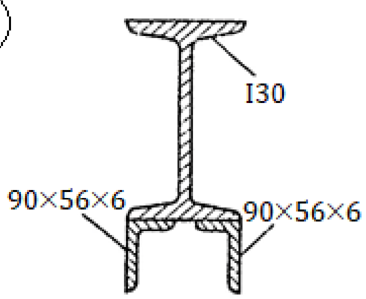

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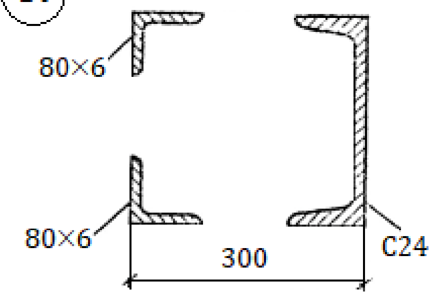

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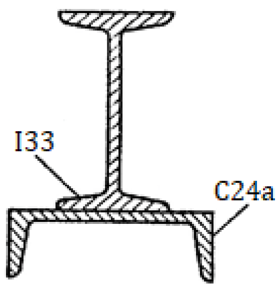

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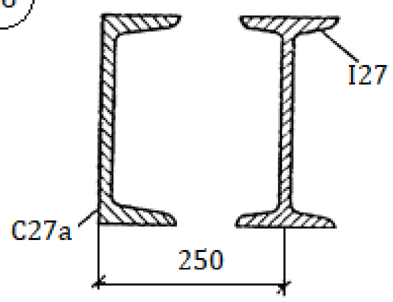

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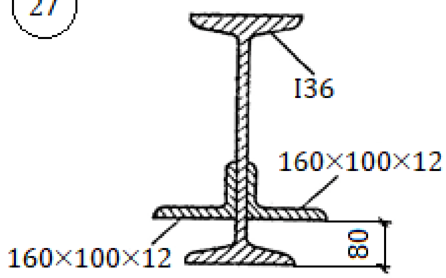

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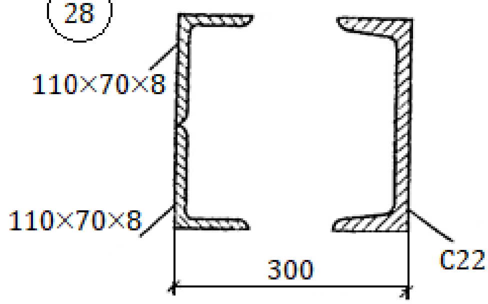

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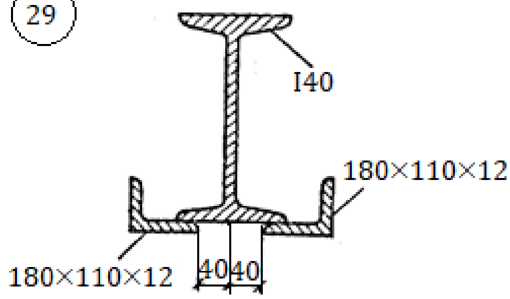

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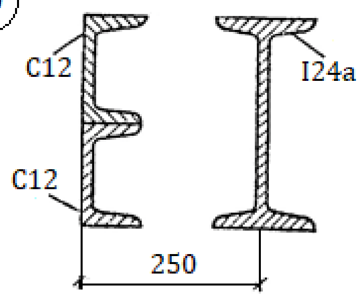

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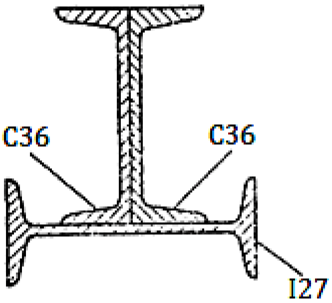

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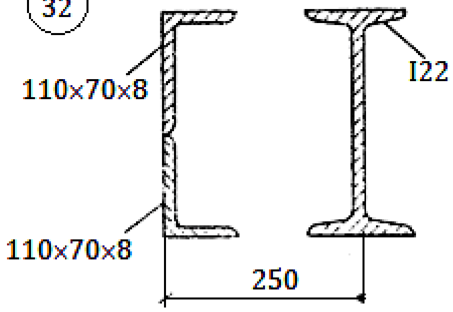

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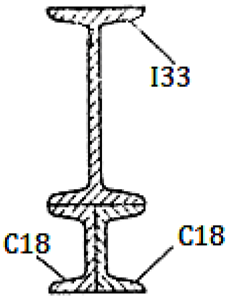

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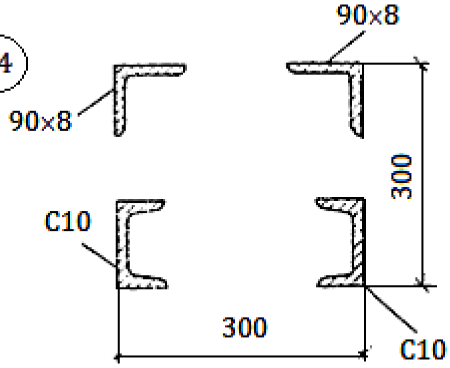

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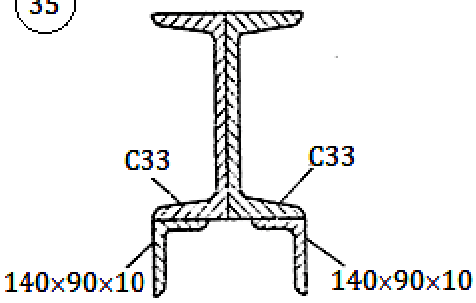

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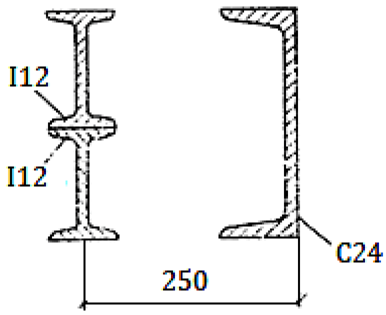


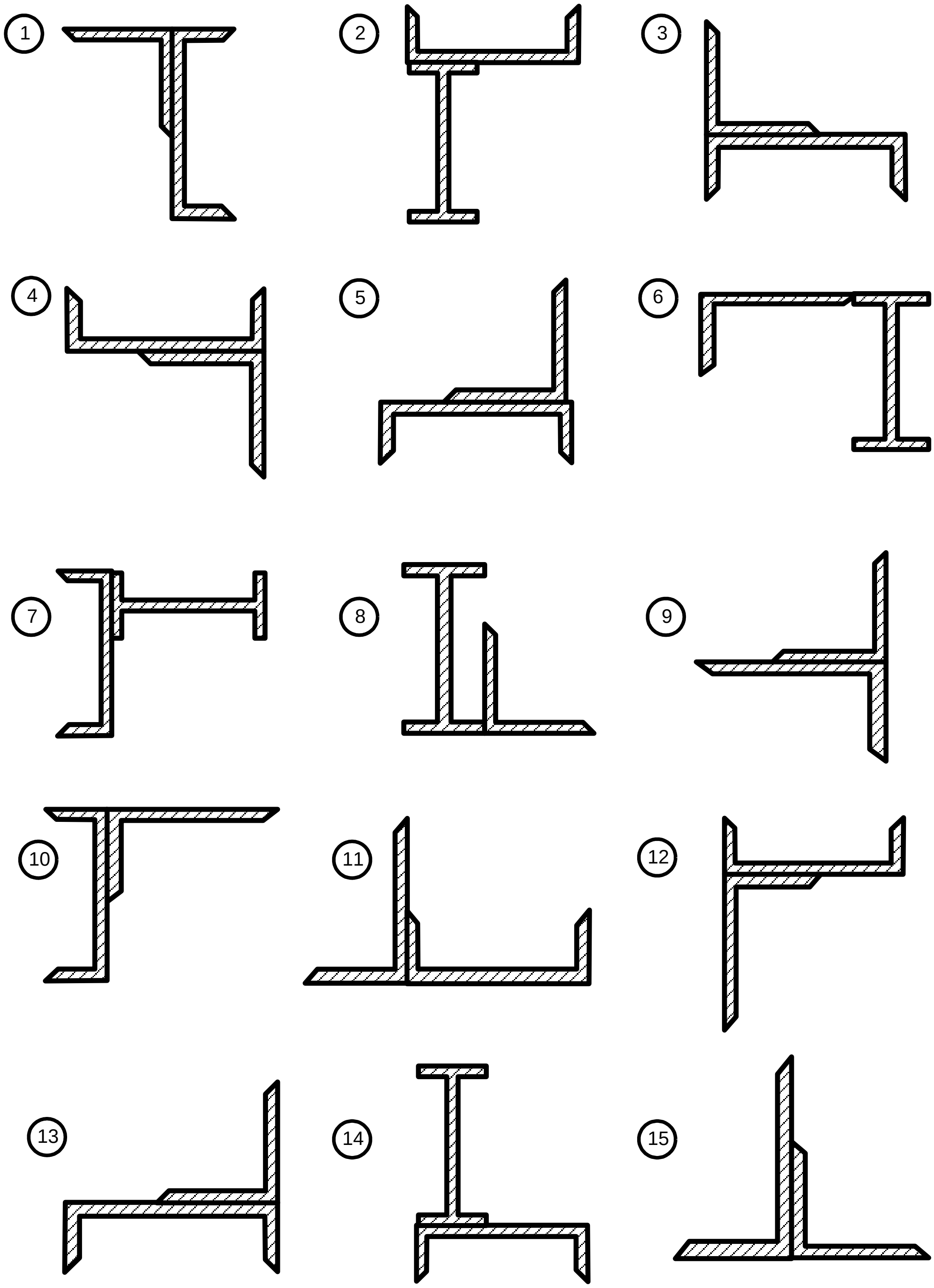
Task 2.5. Determine the position of the principal central axes, the values of the principal central moments of inertia and the bending strength indices (see diagram for the task).

Procedure:

1. Determine the coordinates of the centre of gravity of the entire section.
2. Determine the axial moments of inertia and the centrifugal moment of inertia of the section concerning the central axes.
3. Determine the position of the principal central axes.
4. Determine the values of the principal central moments of inertia of the section.
5. Determine the bending strength ratios concerning the major central axes of the section.

Initial data:
the cross-section contains: C-bar C20, I-beam I20, isosceles angle $100 \times 100 \times 10$, and non-isosceles angle $80 \times 50 \times 6$.

Schemes to the Task 2.5



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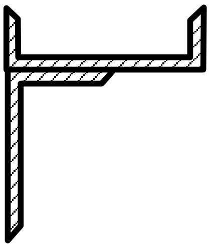
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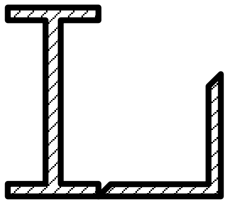
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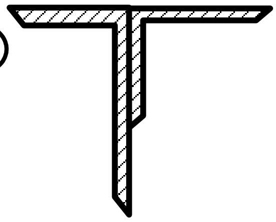
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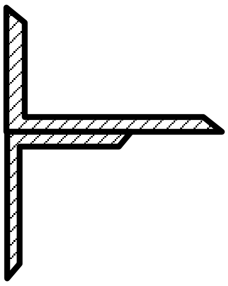
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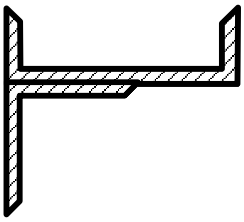
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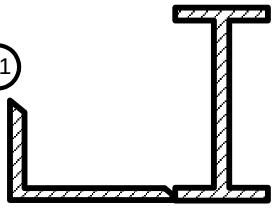
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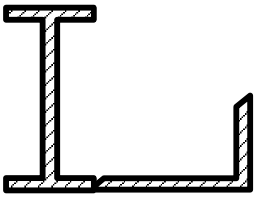
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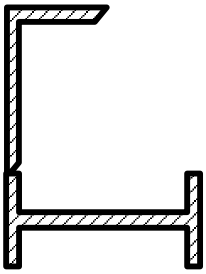
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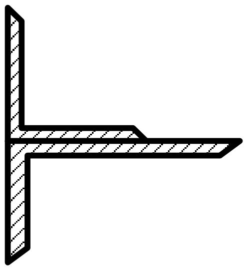
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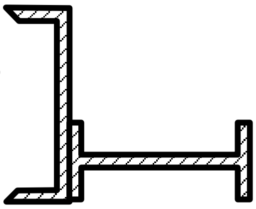
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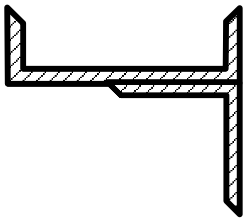
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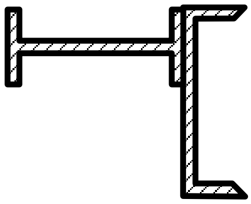
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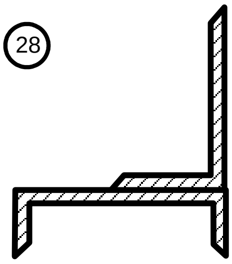
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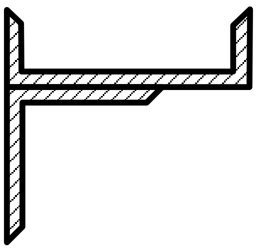
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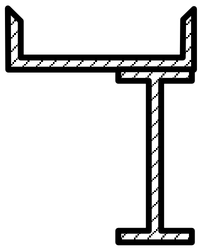

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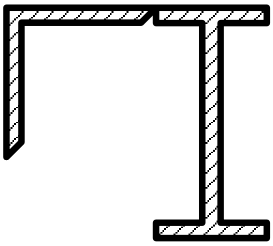

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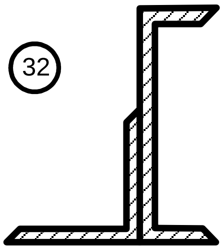

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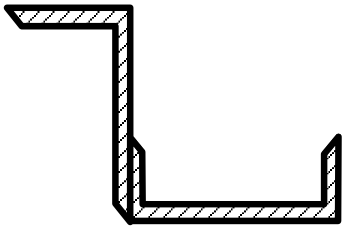

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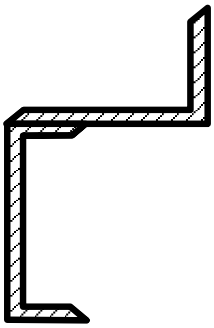

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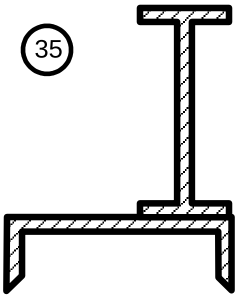

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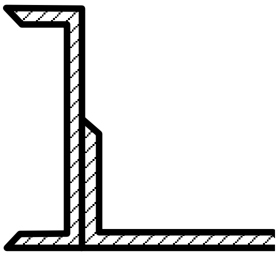

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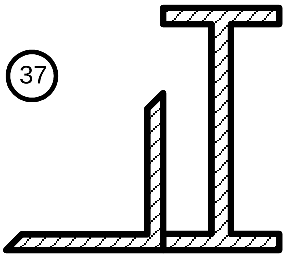

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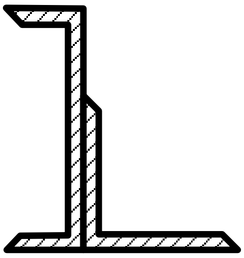

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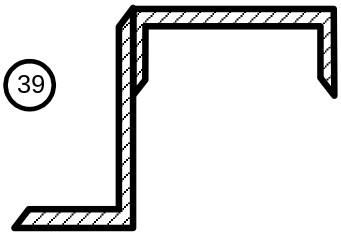

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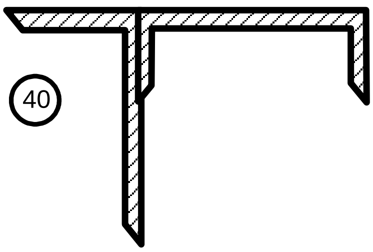

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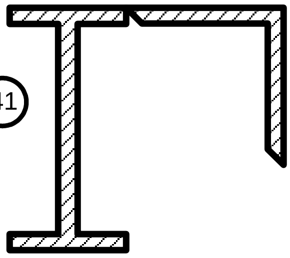

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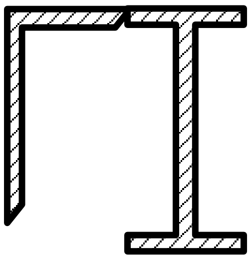

2.4. Torsion

General information

Torsional deformation occurs when a moment (pair of forces) acts on the rod in a plane perpendicular to the bar axis.

The internal moment M_s in any cross-section of the bar is equal to the algebraic sum of the torques applied on one side of the cross-section $M_s = \sum M_s$.

Principles for determining the sign of M_s

Concerning the cross-section, the acting moment is *positive* when it induces a clockwise rotation and *negative* when it induces a counter-clockwise rotation.

In calculations, the relationship between the torque M_s (Nm), the power P (W), the angular velocity ω (s^{-1}) or the number of revolutions per minute n (rpm):

$$M_s = \frac{P}{\omega} \text{ Nm}$$

It is known from a course in theoretical mechanics that angular velocity:

$$\omega = \frac{\pi n}{30} \ (s^{-1}),$$

then: $M_s = \frac{30P}{\pi n}$ Nm or $M_s = 9551\frac{P}{n}$ Nm

In the cross sections, only shear stresses [τ], arise with torsional deformation, which for the shafts are:

$$\tau = \frac{M_s}{W_o} \text{ MPa}$$

where W_o – is the torsional strength index, which is: $W_o = \frac{J_o}{r}$;

r – radius,

J_o – polar moment of inertia;

For the full section:

$$W_o = \frac{\pi d^3}{16} = 0.2d^3 \text{ cm}^2$$

For the ring section:

$$W_o = \frac{\pi D_z^3(1-\alpha^4)}{16} = 0{,}2D_z^3\ (1-\alpha^4), \alpha = \frac{d_w}{D_z}$$

D_z – outer diameter , d_w – inner diameter.

The angle of twist of the section is determined from the formula:

$$\varphi = \theta \cdot l = \frac{M_s \cdot l}{G \cdot J_o}$$

where Θ – relative torsion angle;

l – section length;

G – transverse modulus of elasticity of the material (MPa);

GJ_o – characterises the stiffness of the section under torsion.

Strength condition:

$$\tau_{max} = \frac{M_s}{W_o} \leq k_s$$

where τ_{max} – maximum shear stress occurring in the rod section;

M_s – torque;

W_o – torsional strength factor;

k_s – allowable torsional stress (k_s =0.5 ÷ 0.6 k).

Stiffness condition:

$$\theta_{max} = \frac{\varphi}{l} = \frac{M_s}{GJ_o} \leq [\theta] \frac{\text{rad}}{\text{m}}$$

where θ_{max} – the maximum relative torsion angle that occurs in the rod section;

M_s – torque in the section;

$[\theta]$ – permissible relative torsion angle.

Three types of problem

1. Checking the strength of a structural element (verifying calculations)

$\tau_{max} = \frac{M_s}{W_o} \leq k_s$- strength condition.

2. Section selection of a structural element (design calculations)

– in terms of strength:

for full section $d \geq \sqrt[3]{\frac{16M_s}{\pi k_s}}$

for ring section $D \geq \sqrt[3]{\frac{16M_s}{\pi k_s(1-\alpha^4)}}$, where $\alpha = \frac{d_w}{D_z}$

– in terms of stiffness:

for full section $d \geq \sqrt[4]{\frac{32M_s}{\pi G[\theta]}}$

for ring section $D \geq \sqrt[4]{\frac{32M_s}{\pi G[\theta](1-\alpha^4)}}$, where $\alpha = \frac{d_w}{D_z}$

$$[\theta] = (0.44 \div 1.75) \cdot 10^{-2} \frac{\text{rad}}{\text{m}}$$

3. Verification of stability (operational):

$$M_s \leq W_o k_s$$

Examples of calculation

Example 2.7. Pairs of forces act on a steel beam through three pulleys M_1, M_2, M_3 (Fig. 2.12, *a*). Make diagrams of M_s moments and rotation angles φ of the sections, verify the strength and stiffness of the beam if diameter d=70 mm, shear modulus G = 8·10^4 MPa, permissible relative rotation angle [θ] = 1.5 deg/m.

Solution

1. Drawing up torque diagrams.

Calculate the value of the torque for each segment:

$$M_{s_1} = M_1 = 2\text{ kNm}$$

$$M_{s_2} = M_1 + M_2 = 2 + 1 = 3\text{ kNm}$$

$$M_{s_3} = M_1 + M_2 + M_3 = 2 + 1 - 7 = -4\text{ kNm}$$

Plotting a diagram of the M_s torques (Fig. 2.12, b).

2. Plotting a diagram of the torsion angles.

The torsion angle of a segment is calculated according to the formula:

$$\varphi = \theta \cdot l = \frac{M_s \cdot l}{G \cdot J_o}$$

Polar moment of inertia: $J_o = \frac{\pi d^4}{32}$. The stiffness GJ_o along the whole shaft is constant, so the deformation sections are consistent with the load sections. Then:

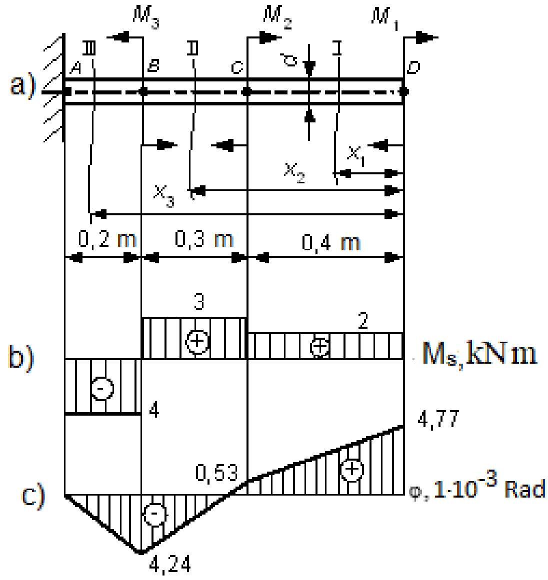

Fig. 2.12. Rod diagram to Example 2.7: *a* - load diagram; b- diagram of torsional moments; c - diagram of torsion angles of beam sections

$$GJ_o = G \cdot \frac{\pi d^4}{32} = 8 \cdot 10^4 \cdot 10^6 \cdot \frac{3.14 \cdot (72 \cdot 10^{-3})^4}{32} = 188574\text{ Nm}$$

Calculate the angles of rotation on each section:

$$\varphi_1 = \frac{M_{s_1} \cdot J_1}{GJ_\rho} = \frac{2 \cdot 10^3 \cdot 0.4}{188574} = 4.24 \cdot 10^{-3}\text{rad}$$

$$\varphi_2 = \frac{M_{s_2} \cdot J_2}{GJ_\rho} = \frac{3 \cdot 10^3 \cdot 0.3}{188574} = 4.77 \cdot 10^{-3}\text{rad}$$

$$\varphi_3 = \frac{M_{s_3} \cdot J_3}{GJ_\rho} = \frac{-4 \cdot 10^3 \cdot 0.2}{188574} = -4.24 \cdot 10^{-3}\text{rad}$$

We begin to plot the torsion angles φ relative to the fixed cross-section A:

$$\varphi_A = 0;$$

$$\varphi_B = \varphi_A + \varphi_3 = 0 + (-4.24 \cdot 10^{-3}) = -4.24 \cdot 10^3\text{rad}$$

$$\varphi_C = \varphi_A + \varphi_3 + \varphi_2 = 0 + (-4.24 \cdot 10^{-3}) + 4.77 \cdot 10^{-3} = 0.53 \cdot 10^{-3}\text{rad}$$

$$\varphi_D = \varphi_A + \varphi_3 + \varphi_2 + \varphi_1 = 0 + (-4.24 \cdot 10^{-3}) + 4.77 \cdot 10^{-3} + 4.2 \cdot 10^{-3} = 4.73 \cdot 10^{-3}\text{rad}$$

Based on the obtained data, we create a graph of the twist angles φ (Fig. 2.12, *c*).

3. Check the strength conditions of the shaft:

$$\tau_{max} \leq k_s$$

$$\tau_{max} = \frac{|M_{s_{max}}|}{W_o} = \frac{|M_{s_{max}}|}{\pi d^3/_{16}} = \frac{16 \cdot 4 \cdot 10^3}{3.14(70 \cdot 10^{-3})^3} = 59.4 \cdot 10^6 \text{ N/m}^2 = 59.4 \text{ MPa}$$

59.4 MPa ≤ 70 MPa – the strength condition is satisfied.

4. Check the shaft stiffness condition:

$$\theta_{max} \leq [\theta]$$

$$\theta_{max} = \frac{|M_{s_{max}}|}{GJ_o} = \frac{4 \cdot 10^3}{188574} = 2.12 \cdot 10^{-2}\text{m}^{-1} \cdot \frac{180°}{3.14} = 1.22 \text{ deg/m}$$

1,22 deg/m ≤ 1,5 deg/m – the stiffness condition is satisfied.

Example 2.8. For the shaft (Fig. 2.13, *a*), loaded as shown in the figure, determine the diameters and twist angles of individual sections, given [θ] = 0.03 rad/m, material: S215 steel, safety factor n = 1.5; shear modulus $G = 8 \cdot 10^4$ MPa, shaft angular velocity ω = 80 rpm, N_1 = 30 kW, N_2 = 15 kW, N_3 = 22 kW.

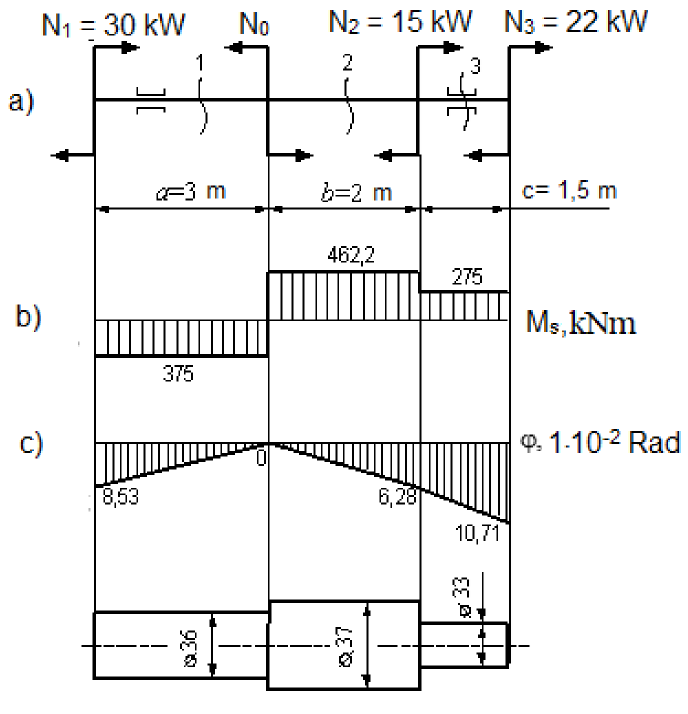

Fig. 2.13. Shaft load diagram:
a - shaft load; *b* - torques diagram; *c* - torsional angle diagram

Solution

1. Determine the torques transmitted by the pulleys using the formula:

$$M = \frac{N}{\omega}$$

$$M_1 = \frac{30 \cdot 10^3}{80} = 375 \text{ Nm}$$

$$M_2 = \frac{15 \cdot 10^3}{80} = 187.5 \text{ Nm}$$

$$M_3 = \frac{22 \cdot 10^3}{80} = 275 \text{ Nm}$$

The torque (M_0) is calculated based on the equilibrium condition of the shaft:

$$\sum M_i = M_1 - M_0 + M_2 + M_3 = 0; M_0 = 375 + 187.5 + 275 = 83.75 \text{ Nm}$$

2. Calculate the torque on each segment of the shaft using the sectioning method:

$$M_{s_1} = -M_1 = -375 \text{ Nm}$$

$$M_{s_2} = -M_1 + M_0 = -375 + 837.5 = 462.2 \text{ Nm}$$

$$M_{s_3} = -M_1 + M_0 - M_2 = -375 + 837.5 - 187.5 = 275 \text{ Nm}$$

Make a plot of torques (Fig. 2.13, b).

3. Determine the diameters of the shaft for each segment using the torsional strength criteria:

$$\tau_{max} = \frac{M_s}{W_o} \le k_s, \text{ where } k_s = (0.5 \div 0.6) \cdot k, k = \frac{R_e}{n}$$

For S215 steel, we have $k_s = \frac{240}{1.5} \cdot 0.5 = 80 \text{ MPa}$.

$$W_o = \frac{\pi d^3}{16}, \text{than } d_i \ge \sqrt[3]{\frac{16 \cdot M_s}{\pi k_s}}$$

$$d_1 = \sqrt[3]{\frac{16 \cdot 375}{3.14 \cdot 80 \cdot 10^6}} = 2.98 \cdot 10^{-2} \text{ m}, \ d_1 = 30 \text{ mm}$$

$$d_2 = \sqrt[3]{\frac{16 \cdot 462{,}5}{3.14 \cdot 80 \cdot 10^6}} = 3.08 \cdot 10^{-2} \text{ m}, d_2 = 31 \text{ mm}$$

$$d_3 = \sqrt[3]{\frac{16 \cdot 275}{3.14 \cdot 80 \cdot 10^6}} = 2.59 \cdot 10^{-2} \text{ m}, d_3 = 26 \text{ mm}$$

4. Determine the diameters of the shaft for each segment based on the condition of sufficient torsional rigidity:

$$\theta = \frac{M_s}{GJ_o} \le [\theta], \text{then } J_o = \frac{\pi \cdot d^4}{32}$$

$$\text{then } d_i \geq \sqrt[4]{\frac{32 \cdot M_s}{\pi \cdot G[\theta]}}$$

$$d_1 = \sqrt[4]{\frac{32 \cdot 375}{3.14 \cdot 8 \cdot 10^{10} \cdot 0.03}} = 3.59 \cdot 10^{-2} \text{ m}; \; d_1 = 36 \text{ mm}$$

$$d_2 = \sqrt[4]{\frac{32 \cdot 462{,}5}{3.14 \cdot 8 \cdot 10^{10} \cdot 0.03}} = 3.7 \cdot 10^{-2} \text{ m}; \; d_2 = 37 \text{ mm}$$

$$d_3 = \sqrt[4]{\frac{32 \cdot 275}{3.14 \cdot 8 \cdot 10^{10} \cdot 0.03}} = 3.29 \cdot 10^{-2} \text{ m}; \; d_3 = 33 \text{ mm}$$

Ultimately, we adopt:

$$d_1 = 36 \text{ mm}; \; d_2 = 37 \text{ mm}; \; d_3 = 33 \text{ mm}$$

With these diameters, both the torsional rigidity and the strength criteria are satisfied. Then sketch of the shaft (Fig. 2.13).

5. To construct the twist angle diagram, the twist angle of the shaft along its segments must be determined using the following formula:

$$\varphi_i = \frac{M_s l_i}{G \cdot J_{o_i}}$$

$$\varphi_1 = \frac{M_{s1} a}{G \cdot J_{o_1}} = \frac{-375 \cdot 3 \cdot 32}{8 \cdot 10^{10} \cdot 3.14(3.6 \cdot 10^{-2})^4} = -8.53 \cdot 10^{-2} \text{ rad}$$

$$\varphi_2 = \frac{M_{s2} b}{G \cdot J_{o2}} = \frac{462.5 \cdot 2 \cdot 32}{8 \cdot 10^{10} \cdot 3.14(3.7 \cdot 10^{-2})^4} = 6.28 \cdot 10^{-2} \text{ rad}$$

$$\varphi_3 = \frac{M_{s3} c}{G \cdot J_{o_3}} = \frac{275 \cdot 1.5 \cdot 32}{8 \cdot 10^{10} \cdot 3{,}14(3.3 \cdot 10^{-2})^4} = 4.43 \cdot 10^{-2} \text{ rad}$$

As a fixed reference, we conditionally consider the segment of the shaft where the zero-angle pulley is located. Concerning this reference, we plot the twist angles of the segments of the shaft, thereby constructing the twist angle diagram (Fig. 2.13, *c*).

The first segment: $\varphi_1 = -8.53 \cdot 10^{-2}$ rad.

The second segment: $\varphi_2 = 6.28 \cdot 10^{-2}$ rad.

The third segment: $\varphi_2 + \varphi_3 = (6.28 + 4.43) \cdot 10^{-2} = 10.71 \cdot 10^{-2}$ rad.

Individual task

(calculation)

Task 2.6. For a given shaft (see schematics for the task), on which pulleys are mounted and where forces act (table below), determine the diameters and twist angles of its segments, given $[\theta] = 0.03$ rad/m, material: S215 steel, safety factor $n = 1.5$; shear modulus $G = 8 \cdot 10^4$ MPa, shaft angular velocity $\omega = 80$ rpm.

Procedure:

1. Determine the torques.
2. Determine the torques on the shaft segments.
3. From the strength condition, determine the diameters of the shaft for each segment.
4. From the torsional rigidity condition, determine the diameters of the shaft for each segment.
5. Determine the twist angle of the shaft for each segment and plot the twist angle diagrams.

Table 2.4. Initial data for Task 2.6

Var. no	N_1	N_2	N_3	N_4	a	b	c	d	e
	kW				m				
1	10	30	20	20	1	2	1	2	1
2	20	15	10	25	2	1	2	2	1
3	40	15	25	30	2	1	3	2	1
4	5	8.5	10	3.5	2	4	1	2	2
5	8	8	10	6	1	2	2	2	1
6	6	16	20	2	1	2	1	1	1
7	4	16	10	10	2	1	2	1	1
8	20	10	18	12	1	3	1	2	1
9	24	6	10	8	3	1	1	3	1
10	4	12	10	6	1	2	2	1	1

Schemes for Task 2.6

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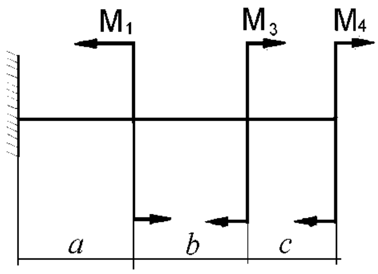

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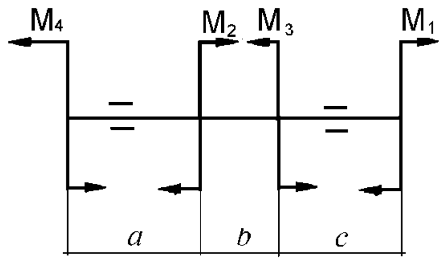

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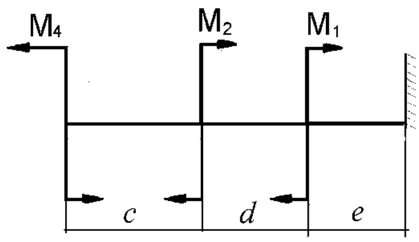

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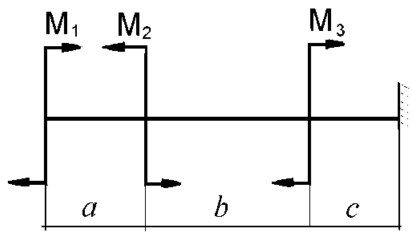

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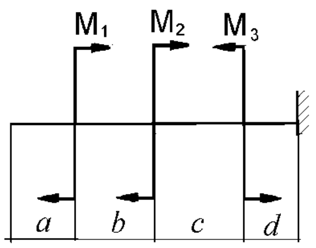

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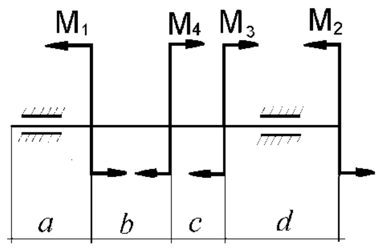

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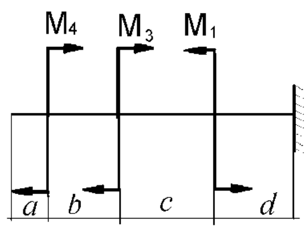

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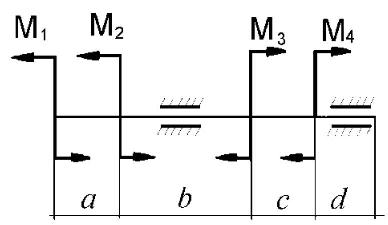

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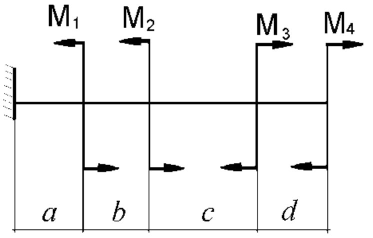

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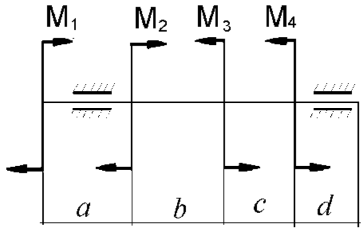

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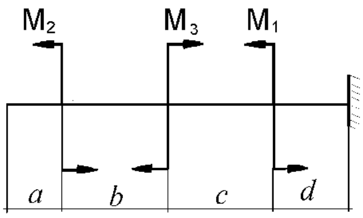

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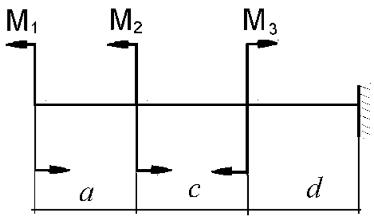

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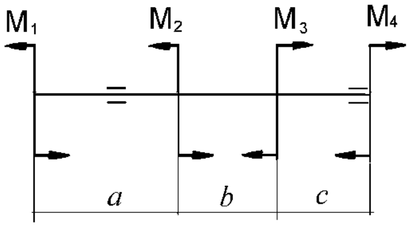

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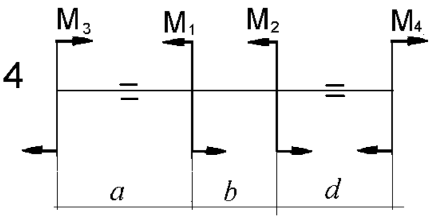

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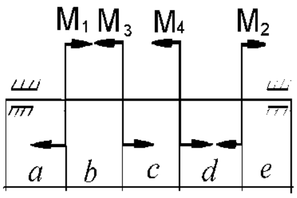

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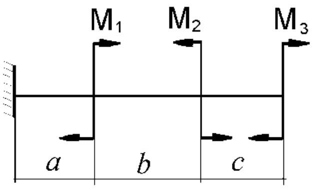

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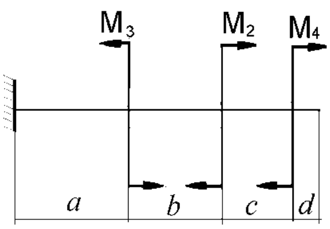

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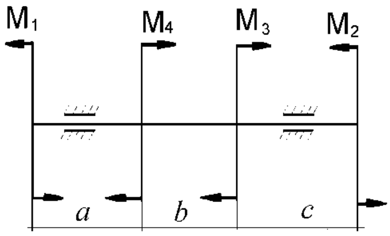

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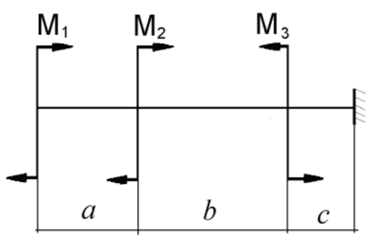

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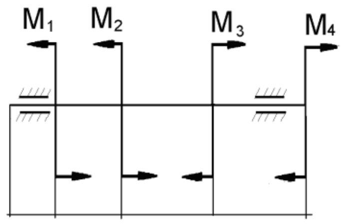

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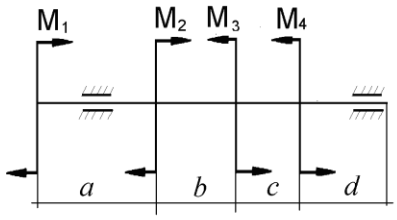

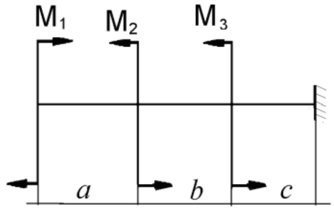

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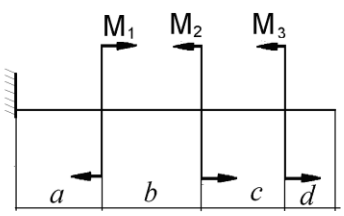

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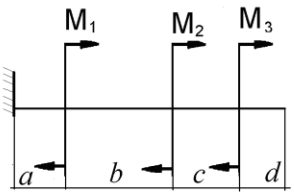

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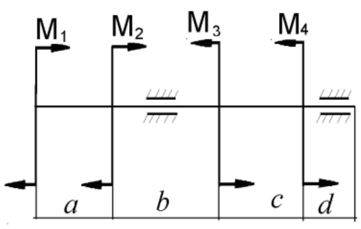

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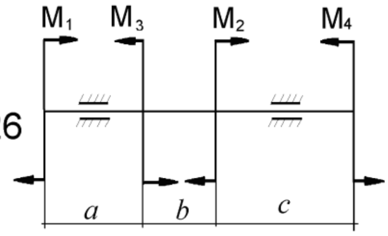

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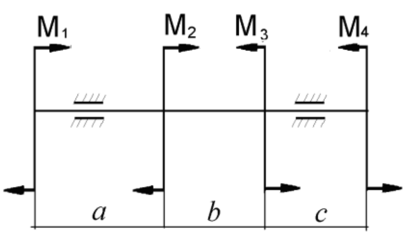

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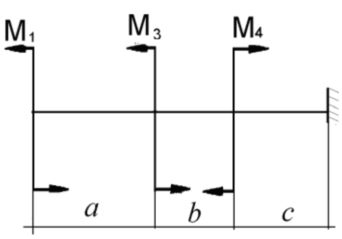

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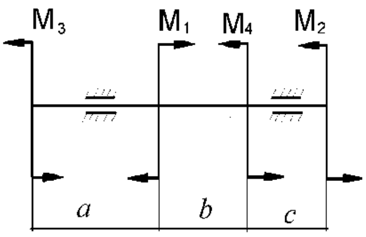

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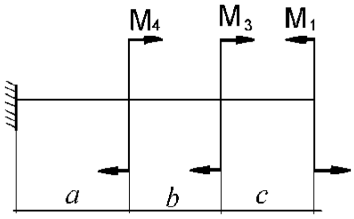

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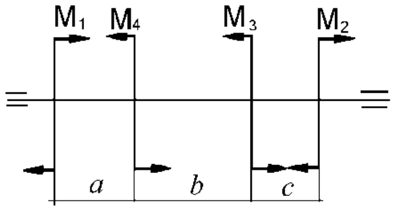

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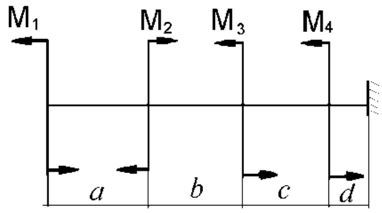

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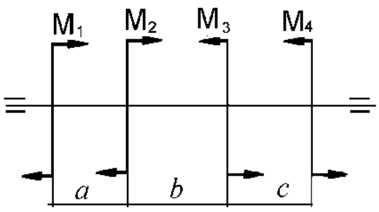

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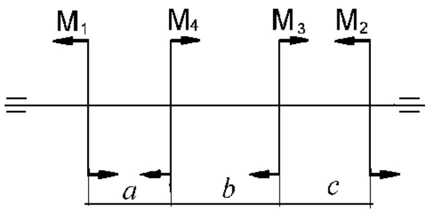

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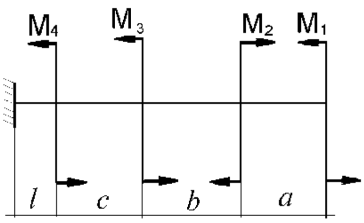

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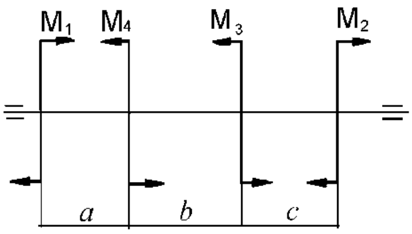

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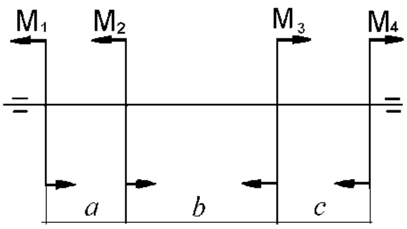

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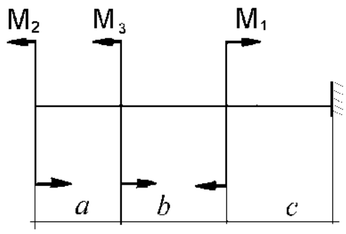

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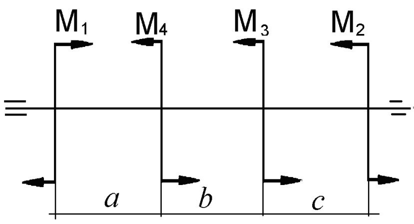

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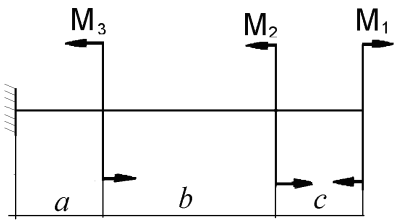


2.5. Bending

General information

Bending, where only the bending moment is present, is called **pure bending**.

Bending, where both the bending moment and shear force are present, is called **transverse bending.**

The planes in which the principal central axes of inertia lie are called **the principal planes of the beam.**

If the plane of action of the force coincides with one of the principal planes of the beam, i.e., the bending axis of the beam lies in the plane of the force, the bending is called **planar** or **simple**.

If the plane of the force does not coincide with any of the principal planes of the beam, i.e., the bending axis of the beam does not lie in the plane of the force, such bending is called **skew bending**.

Shear force and bending moment

In simple transverse bending, the cross-sections of the beam experience: shear force Q and bending moment M_g.

The shear force Q at the considered cross-section numerically equals the algebraic sum of the projections of the forces acting on one side of the section.

The bending moment M_g at the considered cross-section numerically equals the sum of the moments of all forces and couples of forces acting on one side of the section concerning the z-axis.

Sign convention

The shear force is **positive** if the external force tends to rotate the beam clockwise relative to the given section, and **negative** if in the opposite direction (Fig. 2.14).

The bending moment is positive if the beam bends with a sagging curvature (Fig. 2.14).

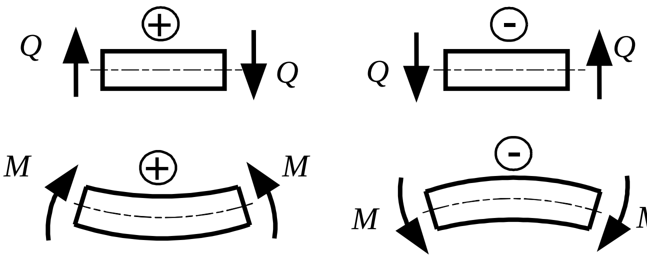


Fig. 2.14. Principle of determining forces and bending moments

Drawing diagrams of bending moments and tangential forces

Methods of verification Q i M_g diagrams

In the segment where continuous loading is present, the Q diagram takes the form of an inclined line, while the M_g diagram is a parabola.

In the segment where continuous loading is not present, the Q diagram takes the form of a line parallel to the beam axis, and the M_g diagram is an inclined line.

At the point where a concentrated force is applied, the Q diagram shows a jump in the absolute value equal to this force.

In the cross-section where a couple of forces are applied, the M_g diagram shows a jump in the absolute value equal to this moment.

In edge cross-sections, the bending moment is zero. The exception is cross-sections where a couple of forces (bending moment) are applied.

Calculating the strength of beams under normal stresses
The formula for the bending strength condition:

$$\sigma = \frac{M_{max}}{W_z} \leq k$$

where $W_z = \frac{J_z}{y_{\max}}$ – section modulus concerning the z-axis;

k – allowable stress of the material.

Three types of problems are solved using bending strength cond.

Verification of the beam strength (verification calculations).

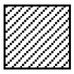
σ_{max} is calculated and compared with k.

$$\sigma = \frac{M_{\max}}{W_z} \leq k$$

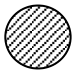
Selection of the beam cross-section (design calculations). The required cross-sectional dimension is calculated

$$W_z \geq \frac{M_{\max}}{k}$$

The values of W_z and W_y for I-beam, channel and angle profiles are selected from the catalogue tables. For square and circular cross-sections:



$$W_z = \frac{a^3}{6} \quad \Rightarrow \quad a = \sqrt[3]{6 \cdot W_z}$$



$$W_z = 0.1d^3 \quad \Rightarrow \quad d = \sqrt[3]{\frac{W_z}{0.1}}$$

Beam load-bearing capacity calculations. The maximum bending moment is calculated:

$$M_{max} \leq k \cdot W_z$$

Calculating the strength of beams under shear stresses

In simple transverse bending, both normal stresses σ, and shear stresses τ, occur in the beam's cross-sections, which are calculated according to the formula:

$$\tau_{max} = \frac{Q_{max} \cdot S_{z_{max}}}{J_z \cdot b},$$

where $Q_{\max}$ – shear force occurring in the cross-section;

$S_{z_{max}}$ – static moment concerning the neutral axis of the cross-section of a section that is on one side of a line drawn through the point under examination, parallel to the neutral axis;

J_z – moment of inertia of the entire cross-section concerning the neutral axis;

b – width of the cross-section (in the case of variable width, the value of b is taken at the level of the point of interest).

Calculation of the stiffness of a beam under bending

In bending a beam, the beam's axis deflects, causing points along this axis to shift. However, the distances are sufficiently small compared to the length of the beam, so their direction can be considered perpendicular to the beam's axis. These displacements are referred to as deflections.

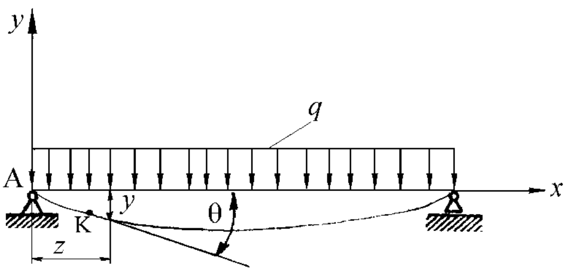


Fig. 2.15. Diagram of forces acting on the beam

The curve along which the original axis of the beam rotates under the action of external forces is called the deflected axis of the beam or the elastic line. Deflections at different cross-sections vary and depend on the distance z (Fig. 2.15) from the chosen coordinate system (in this case, 0 coincides with point A), that is, $y_z = f(z)$.

The angle formed by the tangent to any point k on the deflected axis with its initial position is denoted by θ (Fig. 2.15). The angle θ defines the rotational displacements of the cross-sectional plane of the beam under bending and is called the **angle of deflection** of the beam cross-section**.**

The formula for the stiffness condition

Linear or angular displacements of the cross-section should not exceed the permissible value:

$$f \leq [f]$$

where f – is the maximum deflection of the beam;

$[f]$ - is the permissible deflection of the beam.

The permissible deflection of the beam $[f]$ depends on the definitions and operating conditions of the designed structure. For example, for a manual crane $[f] = l/400$; for an electric crane $[f] = l/700$; for machine shafts used for cutting metals $[f] = 0.0005$ to $0.0010l$ (l – distance between beams).

$$\theta_{max} \leq [\theta]$$

The permissible angle of deflection is typically 0.001 rad.

To calculate the deflection of the beam, we use the universal elasticity equation.

$$f(z) = f_0 + \theta_0 z + \frac{M(z)^2}{2!\,EJ_x} + \frac{P(z)^3}{3!\,EJ_x} + \frac{q(z)^4}{4!\,EJ_x} + \sum \frac{M(z-2)^2}{2!\,EJ_x} + \sum \frac{P(z-b)^3}{3!\,EJ_x} + \sum \frac{q(z-c)^4}{4!\,EJ_x}$$

$$\theta(z) = \theta_0 + \frac{Mz}{EJ_x} + \frac{P(z)^2}{2!\,EJ_x} + \frac{q(z)^3}{3!\,EJ_x} + \sum \frac{M(z-a)}{EJ_x} + \sum \frac{P(z-b)^2}{2!\,EJ_x} + \sum \frac{q(z-c)^3}{3!\,EJ_x}$$

where “!” – denote factorial;

f_0, θ_0 – input parameters (deflection and angle of deflection of the left cross-section of the beam);

a, b, c – abscissa of sections where moments M, acting forces P, and the starting point of the load q occur;

EJ_x – stiffness of the beam cross-section;

Deflection f_0 and angle θ_0 are calculated based on the beam's fixation condition.

Examples of calculations

Example 2.9. For a beam loaded as shown in Fig. 2.16, plot the diagrams of Q and M_g.

Solution

1. Determination of the reactions at the supports of the beam. For symmetric loading:

$$R_A = R_B = \frac{P}{2} = \frac{10}{2} = 5\ \text{kN}$$

2. Plotting the shear force scheme Q

$$0 \le x_1 \le 2$$
$$Q(x_1) = R_A$$
$$Q(x_1 = 0) = 5\ \text{kN},$$
$$Q(x_1 = 2) = 5\ \text{kN}$$

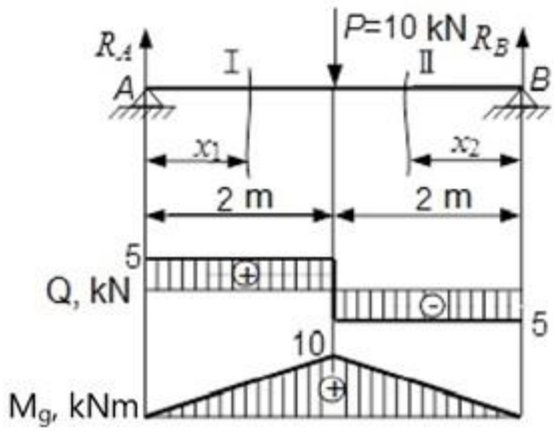


Fig. 2.16. Beam loading scheme

$$0 \le x_2 \le 20$$
$$Q(x_2) = R_B$$
$$Q(x_2 = 0) = -5\ \text{kN}, Q(x_2 = 2) = -5\ \text{kN}$$

3. Plotting the bending moment diagram M_g

$$0 \le x_1 \le 2$$
$$M(x_1) = R_A \cdot x_1;\ M(x_1 = 0) = R_A \cdot 0 = 0, M(x_1 = 2) = R_A \cdot 2 = 5 \cdot 2 = 10\ \text{kNm}$$
$$0 \le x_1 \le 2$$
$$M(x_2) = R_B \cdot x_2;\ M(x_2 = 0) = R_B \cdot 0 = 0, M(x_2 = 2) = R_B \cdot 2 = 5 \cdot 2 = 10\ \text{kNm}$$

Based on the obtained data, we create the diagrams $Q(x)$ and $M_g(x)$ (Fig. 2.16).

Example 2.10. For a beam loaded as shown in Fig. 2.17, plot the diagrams of Q and M_g.

Solution

1. Determination of the reactions at the supports of the beam.

$$R_A = R_B = \frac{q \cdot 6}{2}; R_A = R_B = \frac{4 \cdot 6}{2} = 12\ \text{kN}$$

2. Plotting the shear force diagram Q.

$$0 \le x \le 6$$
$$Q(x) = R_A - q \cdot x$$
$$Q(x = 0) = R_A - q \cdot 0 = 12\ \text{kN}$$

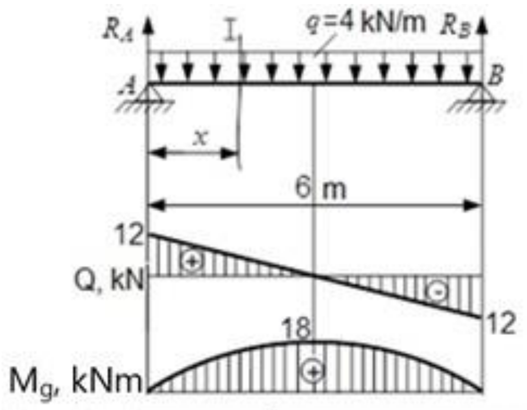


Fig. 2.17. Beam loading scheme

$$Q(x=6) = R_A - q \cdot 6 = 12 - 4 \cdot 6 = -12 \text{ kN}$$

4. Plotting the bending moment diagram M_g.

$$0 \le x \le 6$$

$$M(x) = R_A \cdot x - \frac{q \cdot x^2}{2};$$

$$M(x=0) = R_A \cdot 0 - \frac{q \cdot 0^2}{2} = 0 \text{ kNm}$$

$$M(x=6) = R_A \cdot 6 - \frac{q \cdot 6^2}{2} = 12 \cdot 6 - \frac{4 \cdot 6^2}{2} = 0 \text{ kNm}$$

In the section where $Q = 0$, the diagram M_g has the maximum bending moment. We calculate the maximum M_g. For this purpose, we set the transverse force equation to zero and determine the point where the extremum occurs:

$$Q(x) = R_A - q \cdot x = 0, \text{then } x = \frac{R_A}{q} = \frac{12}{4} = 3 \text{ kN}$$

$$M(x=3) = R_A \cdot 3 - \frac{q \cdot 3^2}{2} = 12 \cdot 3 - \frac{4 \cdot 3^2}{2} = 18 \text{ kNm}$$

Based on the obtained data, plot the diagrams $Q(x)$ and $M_g(x)$ (Fig. 2.17).

Example 2.11. For a beam loaded as shown in Fig. 2.18, plot the diagrams of Q and M_g.

Solution

1. Plotting the shear force diagram Q.

$$0 \le x \le 3$$

$$Q(x_1) = P_1$$

$$Q(x_1 = 0) = P_1 = 6 \text{ kN}$$

$$Q(x_1 = 3) = P_1 = 6 \text{ kN}$$

$$3 \le x \le 7$$

$$Q(x_2) = P_1 - P_2 + q(x_2 - 3)$$

$$Q(x_2 = 3) = P_1 - P_2 + q(3-3) = 6 - 14 + 5 \cdot 0 = -8 \text{ kN}$$

$$Q(x_2 = 7) = P_1 - P_2 + q(7-3) = 6 - 14 + 5 \cdot 4 = 12 \text{ kN}$$

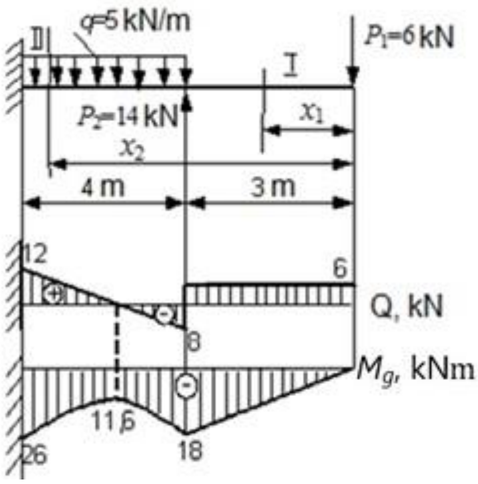


Fig. 2.18. Beam loading scheme

2. Plotting the bending moment diagram M_g.

$$0 \le x_1 \le 3$$

$$M(x_1 = 0) = -P_1 \cdot 0 = 0 \text{ kNm}$$

$$M(x_1 = 3) = -P_1 \cdot 3 = -6 \cdot 3 = -18 \text{ kNm}$$

$$3 \le x_2 \le 7$$

$$M(x_2) = -P_1 \cdot x_2 + P_2(x_2 - 3) - \frac{q(x_2-3)^2}{2}$$

$$M(x_2 = 3) = -P_1 \cdot 3 + P_2(3-3) - \frac{q(3-3)^2}{2} = -6 \cdot 3 + 14 \cdot 0 - \frac{5 \cdot 0^2}{2} = -18 \text{ kNm}$$

$$M(x_2 = 7) = -P_1 \cdot 7 + P_2(7-3) - \frac{q(7-3)^2}{2} = -6 \cdot 7 + 14 \cdot 4 - \frac{5 \cdot 4^2}{2} = -26 \text{ kNm}$$

We calculate the maximum value of $M_g(x_2)$ at the location where the shear force becomes zero in the second segment. We set the transverse force equation to zero:

$$Q(x_2) = P_1 - P_2 + q(x_2 - 3) = 0$$

$$P_1 - P_2 + q \cdot x_2 - x_2 \cdot 3 = 0 \quad \Rightarrow \quad x_2 = \frac{-P_1 + P_2 + q \cdot 3}{q} = \frac{-6 + 14 + 5 \cdot 3}{5} = 4.6 \text{ m}$$

$$M(x_2 = 4.6) = -6 \cdot 4.6 + 14 \cdot (4.6 - 3) - \frac{5 \cdot (4.6-3)^2}{2} = -11.6 \text{ kNm}$$

Based on the obtained data, we create the diagrams $Q(x)$ and $M_g(x)$ (Fig. 2.18).

Example 2.12. Based on the strength and stiffness conditions, determine the required size of the C-bar for the beam (Fig. 2.19), given that the permissible deflection $[f] = l/400$, the permissible stress k and the modulus of elasticity $E = 2 \cdot 10^5$ MPa.

Data:
$[f] = l/400$
k = 210 MPa
$E = 2 \cdot 10^5$ MPa

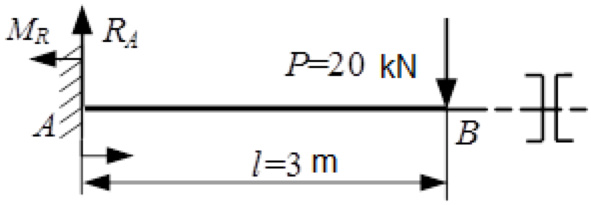


Fig. 2.19. Beam loading scheme

Searched for:
C-bar - ?

Solution

From the equilibrium conditions, determine the support reactions of the beam:

$$\sum M_A = 0; \quad M_R - P \cdot l = 0; \; M_R = P \cdot l = 30 \cdot 3 = 60 \text{ kNm}$$
$$\sum Y_i = 0; \quad R_A - P = 0; R_A = P = 20 \text{ kN}; \; R_A = P = 20 \text{ kN}$$

Determine the maximum bending moment:

$$M(x) = -P \cdot x$$
$$M(x = 0) = -P \cdot 0 = 0$$
$$M(x = 3) = -P \cdot 3 = -20 \cdot 3 = -60 \text{ kNm}$$

From the strength condition, select the cross-section:

$$W_x \geq \frac{M_{\max}}{k} = \frac{60 \cdot 10^3}{210 \cdot 10^6} = 0.286 \cdot 10^{-3} \text{ m}^3 = 286 \text{ cm}^3$$

The beam's cross-section consists of two C-bars so one C-bar

$$W_x = \frac{286}{2} = 143 \text{ cm}^3$$

From the Table D.58, C-bar C20 has a bending strength index W_x = 152 cm^3 and a moment of inertia J_x = 1520 cm^4. Therefore, for two C-bars:

$$W_x = 2 \cdot 152 = 304 \text{ cm}^3$$
$$J_x = 2 \cdot 1520 = 3040 \text{ cm}^4$$

Verify the stresses:

$$\sigma_{\max} = \frac{M_{\max}}{W_x} \le k$$

$$\sigma_{\max} = \frac{60 \cdot 10^3}{304 \cdot 10^{-6}} = 0.197 \cdot 10^9 \frac{N}{m^2} = 197 \text{ MPa} < 210 \text{ MPa}$$

The strength is ensured.

According to the stiffness condition, select the beam cross-section:

$$f \le [f]$$

$$[f] = \frac{l}{400} = \frac{3}{400} = 0.0074 \text{ m} = 7.4 \text{ mm}$$

The universal elasticity equation for a beam:

$$f(x) = f_0 + \theta_0 x - M_R \frac{x^2}{2EJ_x} + R_A \frac{x^3}{6EJ_x}$$

The deflection f_0 and the angle θ_0 from the beam's support condition are equal to zero; therefore, the maximum deflection for the given case is:

$$f(\text{x} = l = 3 \text{ m}) = -Pl \frac{l^2}{2EJ_x} + P \frac{l^3}{6EJ_x} = \frac{-3Pl^3 + Pl^3}{6EJ_x} = -\frac{Pl^3}{3EJ_x}$$

$$f_{max} = -\frac{20 \cdot 10^3 \cdot 3^3}{3 \cdot 2 \cdot 10^{11} \cdot 2 \cdot 1520 \cdot 10^{-8}} = -0.0296 \text{ m} = -29.6 \text{ mm}$$

The maximum deflection exceeds the permissible deflection value, so a different C-bar needs to be selected. We determine the required moment of inertia for the new C-bar:

$$f = \frac{-Pl^3}{3EJ_x} \le [f]$$

$$J_x \ge \frac{P \cdot l^3}{3E[f]} = \frac{20 \cdot 10^3 \cdot 3^3}{3 \cdot 2 \cdot 10^{11} \cdot 7.4 \cdot 10^{-3}} = 12.162 \cdot 10^{-5} \text{m}^4 = 12162 \text{ cm}^4$$

For one C-bar J_x = 12162 / 2 = 6081 cm^4.

From the Table D.58, we select C-bar C33 J_x = 7980 cm^4.

Finally, we adopt C-bar C33, thus, the stiffness condition is satisfied.

Example 2.13. Verify the serviceability of a cantilever wooden beam (Fig. 2.20), given that γf = 1.2 (load safety factor), service condition factor γ_c = 1.1; and the computational strength R = 15 MPa.

Data:	Searched for:
$\gamma f = 1.2$	The serviceability
$\gamma_c = 1.1$	of the beam - ?
$R = 15$ MPa	

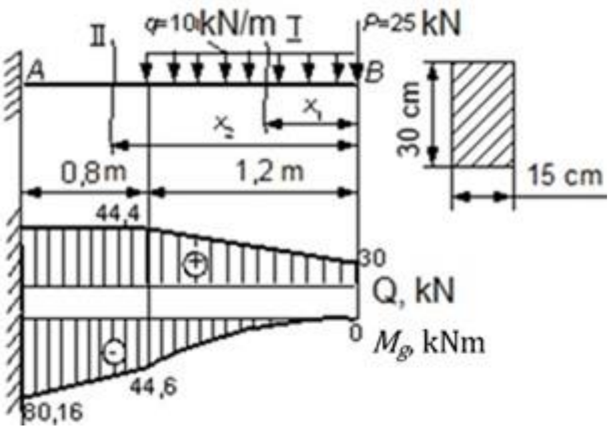


Fig. 2.20. Beam loading scheme

Solution

1. Determine the design load.

$$q_{calc} = q \cdot \gamma f = 10 \cdot 1.2 = 12 \text{ kN/m}$$
$$P_{calc} = P \cdot \gamma f = 25 \cdot 1.2 = 30 \text{ kN}$$

2. Plot the shear force diagram Q.

$$0 \leq x_1 \leq 1.2$$
$$Q(x) = P_{calc} + q_{calc} \cdot x_1$$
$$Q(x = 0) = P_{calc} + q_{calx} \cdot 30 \text{ kN}$$
$$Q(x = 1{,}2) = P_{calc} + q_{calc} \cdot 1.2 = 30 + 12 \cdot 1.2 = 44.4 \text{ kN}$$
$$1{,}2 \leq x_1 \leq 2$$
$$Q(x) = P_{calc} + q_{calc} \cdot 1.2$$
$$Q(x = 1{,}2) = P_{calc} + q_{calc} \cdot 1.2 = 30 + 12 \cdot 1.2 = 44.4 \text{ kN}$$
$$Q(x = 2) = P_{calc} + q_{calc} \cdot 1.2 = 30 + 12 \cdot 1.2 = 44.4 \text{ kN}$$

3. Make a scheme M_g.

$$0 \leq x_1 \leq 1.2$$
$$M(x_1) = -P_{calc} \cdot x - \frac{q_{ob} \cdot x^2}{2}$$
$$M(x_1 = 0) = -P_{calc} \cdot 0 - \frac{q_{calc} \cdot 0^2}{2} = 0 \text{ kNm}$$
$$M(x_1 = 1.2) = -P_{calc} \cdot 1.2 - \frac{q_{calc} \cdot 1.2^2}{2} = -30 \cdot 1.2 - \frac{12 \cdot 1.2^2}{2} = -44.6 \text{ kNm}$$
$$1.2 \leq x_1 \leq 2$$
$$M(x) = -P_{calc} \cdot x - q_{calc} \cdot 1.2 \cdot (x_2 - 0.6)$$
$$M(x_2 = 1{,}2) = -30 \cdot 1.2 - 12 \cdot 1.2 \cdot (1.2 - 0.6) = -44.6 \text{ kNm}$$
$$M(x_2 = 2) = -30 \cdot 2 - 12 \cdot 1.2 \cdot (2 - 0.6) = -80.16 \text{ kNm}$$

Based on the obtained data, plot the diagrams $Q(x)$ and $M_g(x)$ (Fig. 2.20).

Determine the bending strength index for a rectangular cross-section:

$$W_x = W_z = \frac{bh^2}{6} = \frac{15 \cdot 30^2}{6} = 2250 \text{ cm}^3$$

Check the serviceability of the beam. From the strength condition:

$$M_g = 80.16 \text{ kNm} \leq M_{max}$$
$$M_{max} \leq k \cdot W_x = \gamma_c \cdot R \cdot W_x = 1.1 \cdot 15 \cdot 10^6 \cdot 2250 \cdot 10^{-6} = 37125 \text{ Nm} = 37.3 \text{ kNm}$$

The actual bending moment M_g is greater than the design moment M_{max}.

Conclusion: The serviceability of the beam is not ensured; it is necessary to either reduce the load or increase the cross-sectional area of the rectangle.

Example 2.14. From the conditions of strength and stiffness, select the dimensions of the beam's cross-sectional area (Fig.2.21).

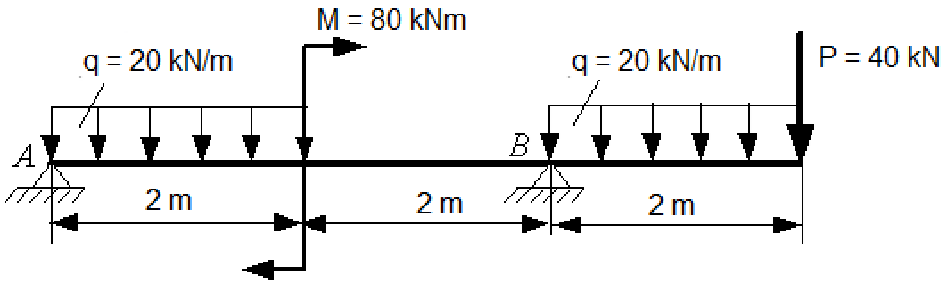


Fig. 2.21. Beam loading scheme

For the loaded beam:

1. Determine the values of shear forces Q_i and bending moments M_{g_i} and plot their diagrams;

2. From the strength condition, determine the required dimensions of the beam for three cross-sectional variants:

a) circular cross-section;

b) rectangular cross-section (where $h/b = 2$);

c) I-beam cross-section.

Choose the most rational cross-sectional shape among the three options and justify the choice.

3. For the selected rational cross-section, plot the normal and shear stress diagrams. Determine the equivalent stress for this beam.

4. Check the stiffness of the beam with the chosen cross-section using the strength condition, given that $[f] = 0{,}001l$; $E = 2 \cdot 10^5$ MPa; $k = 160$ MPa.

Solution

1. From the equilibrium conditions, calculate the support reactions of the beam.

$$\sum M_A = 0; -q \cdot 2 \cdot 1 - M + R_B - q \cdot 2 \cdot 5 - P \cdot 6 = 0$$

$$R_B = \frac{+q \cdot 2 \cdot 1 + M + q \cdot 2 \cdot 5 + P \cdot 5}{4} = \frac{20 \cdot 2 \cdot 1 + 80 + 20 \cdot 2 \cdot 5 + 40 \cdot 6}{4} = 140 \text{ kN}$$

$$\sum M_A = 0; R_A \cdot 4 + q \cdot 2 \cdot 3 - M - q \cdot 2 \cdot 1 - P \cdot 2 = 0$$

$$R_A = \frac{-q \cdot 2 \cdot 3 + M + q \cdot 2 \cdot 1 + P \cdot 2}{4} = \frac{-20 \cdot 2 \cdot 3 + 80 + 20 \cdot 2 \cdot 1 + 40 \cdot 2}{4} = 20 \text{ kN}$$

Verification:

$$\sum Y = 0 - R_A - q \cdot 4 - P + R_B = 0$$

$$-20 - 80 - 40 + 140 = 0$$

2. Determine the transverse forces Q

$$0 \le x_1 \le 2$$
$$Q(x_1) = -R_A - q \cdot x_1$$
$$Q(x_1 = 0) = -R_A - q \cdot 0 = -20 - 20 \cdot 0 = -20 \text{ kN}$$
$$Q(x_1 = 2) = -R_A - q \cdot 2 = -20 - 20 \cdot 2 = -60 \text{ kN}$$
$$2 \le x_2 \le 4$$
$$Q(x_2) = -R_A - q \cdot x_2$$
$$Q(x_1 = 2) = P + q \cdot = -20 - 20 \cdot 2 = -60 \text{ kN}$$
$$Q(x_1 = 4) = -R_A - q \cdot 2 = -20 - 20 \cdot 2 = -60 \text{ kN}$$
$$0 \le x_3 \le 2$$
$$Q(x_3) = P + q \cdot x_3$$
$$Q(x_3 = 0) = P + q \cdot 0 = 40 + 20 \cdot 0 = 40 \text{ kN}$$
$$Q(x_3 = 2) = P + q \cdot 2 = 40 + 20 \cdot 2 = 80 \text{ kN}$$

3. Determine the bending moment M_g.

$$0 \le x_1 \le 2$$
$$M(x_1) = -R_A \cdot x_1 - \frac{q \cdot {x_1}^2}{2}$$
$$M(x_1 = 0) = -R_A \cdot 0 - \frac{q \cdot 0^2}{2} = 0 \text{ kNm}$$
$$M(x_1 = 2) = -R_A \cdot 2 - \frac{q \cdot 2}{2} = -80 \text{ kNm}$$
$$2 \le x_2 \le 4$$
$$M(x_2) = -R_A \cdot x_2 - \mathrm{q} \cdot 2(x_2 - 1) + M$$
$$M(x_2 = 2) = -R_A \cdot 2 - \mathrm{q} \cdot 2(2 - 1) + M = 0 \text{ kNm}$$
$$M(x_2 = 4) = -R_A \cdot 4 - \mathrm{q} \cdot 2(4 - 1) + M = -120 \text{ kNm}$$
$$0 \le x_3 \le 2$$
$$M(x_3) = -\mathrm{P} \cdot x_3 - \frac{q \cdot {x_3}^2}{2}$$
$$M(x_3 = 0) = -\mathrm{P} \cdot 0 - \frac{q \cdot 0^2}{2} = 0 \text{ kNm}$$
$$M(x_3 = 2) = -\mathrm{P} \cdot 2 - \frac{q \cdot 2^2}{2} = -120 \text{ kNm}$$

4. Plot the diagrams for Q and M_g, considering that in two intervals there is a uniform load q. In this case, the M_g diagram is a parabola, with the bulge oriented in the direction opposite to the load. In the second interval, the M_g diagram is bounded by straight lines (Fig. 2.22). As shown in the diagram, the maximum moment M_g = 120 kNm occurs at the section where x = 4 m.

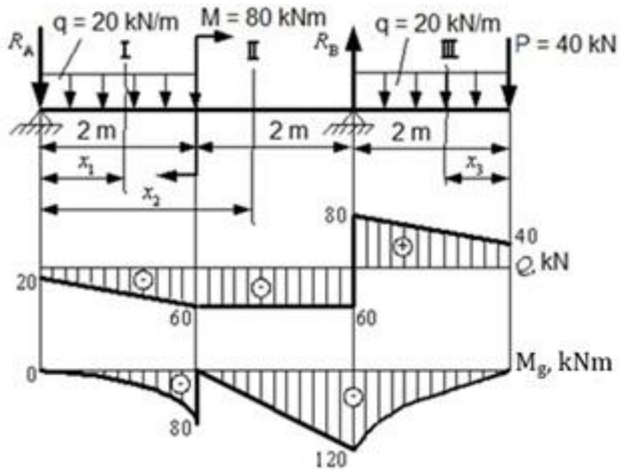


Fig. 2.22. Diagram of shear forces and bending moments for the beam of Example 2.14

5. From the normal stress strength condition, select the required cross-sectional dimensions of the beam:

$$\sigma_{\max} = \frac{M_{\max}}{W_x} \le k$$

Required bending strength of the beam cross-section:

$$W_x \ge \frac{M_{max}}{k}, \text{ so } W_x \ge \frac{120 \cdot 10^3}{160 \cdot 10^6} = 0{,}75 \cdot 10^{-3}\ \text{m}^3 = 750\ \text{cm}^3$$

For a circular cross-section of the beam, we have:

$$W_x = \frac{\pi d^3}{32}, \text{ then}$$

$$d \ge \sqrt[3]{\frac{32 W_x}{\pi}} \ge \sqrt[3]{\frac{32 \cdot 750 \cdot 10^{-6}}{3.14}} \ge \sqrt[3]{7.643 \cdot 10^{-3}} \ge 1.97 \cdot 10^{-3 \cdot \frac{1}{3}} \ge 1.97 \cdot 10^{-1}\ \text{m} = 19.7\ \text{cm}$$

We assume d = 20 cm, $A = \frac{\pi d^2}{4} = \frac{3.14 \cdot 20^2}{4} = 314\ \text{cm}^2$

The stresses acting in the beam will be:

$$\sigma_{max} = \frac{M_{max}}{W_x} = \frac{M_{max}}{\frac{\pi d^3}{32}} = \frac{120 \cdot 10^3}{3.14 \cdot 0.20^3} \cdot 32 = 153 \cdot 10^6\ \frac{\text{N}}{\text{m}^2} = 153\ \text{MPa}$$

The stresses in the beam are:

$$\frac{153 - 160}{160} \cdot 100\% = -4{,}4\% < [5\%]$$

For a rectangular cross-section of the beam:

$$W_x = \frac{bh^2}{6}$$

because $b = \frac{h}{2}$, then $W_x = \frac{\frac{h}{2} \cdot h^2}{6} = \frac{h^3}{12}$.

$$h = \sqrt[3]{12 W_x} = \sqrt[3]{12 \cdot 750 \cdot 10^{-6}} = \sqrt[3]{9.0 \cdot 10^{-3}} = 2.082 \cdot 10^{-3 \cdot \frac{1}{3}} = 2.082 \cdot 10^{-1}\ \text{m} = 20.82\ \text{cm}$$

We assume $h = 21$ cm, $b = \frac{h}{2} = 10.5$ cm, A = 220.5 cm^2.

For this cross-section, the acting stresses are:

$$\sigma_{max} = \frac{M_{max}}{W_x} = \frac{M_{max}}{\frac{bh^2}{6}} = \frac{120 \cdot 10^3}{0.105 \cdot 0.21^2} \cdot 6 = 15.5 \cdot 10^6\ \frac{\mathrm{N}}{\mathrm{m}^2} = 155.5\ \mathrm{MPa}$$

The stresses in the beam will be:

$$\frac{155{,}5 - 160}{160} \cdot 100\% = -2.8\% < [5\%]$$

For the I-beam: from Table D.59 the closest values for W_z = 750 cm^3 are for I-beam no. 36 with a bending strength indicator W_z = 743 cm^3.

The stresses acting in I-beam I36 are:

$$\sigma = \frac{M_{max}}{W_z} = \frac{120 \cdot 10^3}{743 \cdot 10^{-6}} = 0.161 \cdot 10^9 = 162\ \mathrm{MPa}$$

The overload on the beam is:

$$\frac{162-160}{160} \cdot 100\% = +1.25\% > [5\%]$$

Ultimately, we choose I-beam I36: the moment of inertia J_z = 13380 cm^4, and the cross-sectional area A = 61.9 cm^2.

Conclusion: The most rational choice among the three cross-sections is I-beam I 36.

6. Check the strength of the I-beam concerning the maximum shear stresses τ:

$$\tau_{max} = \frac{Q_{max} \cdot S_{x_{max}}}{J_x \cdot b} \leq k_s$$

where: $S_{x_{max}}$ - the static moment of the beam's cross-section about the neutral axis (from the tables in the appendix),

b – the width of the I-beam flange (Table D.59).

For materials if $k_s = (0.5 \div 0.6)k$, then $k_s = 0.56 \cdot 160 = 90$ MPa.

$$\tau_{max} = \frac{80 \cdot 10^3 \cdot 423 \cdot 10^{-6}}{7.5 \cdot 10^{-3} \cdot 13380 \cdot 10^{-8}} = 0.337 \cdot 108\ \mathrm{Pa} = 33.7\ \mathrm{MPa} < k_s = 90\ \mathrm{MPa}$$

7. Plot the diagrams of normal stresses σ and shear stresses τ for the I-beam.

Normal stresses: $\sigma = \frac{M_{max}}{W_x} = \frac{M_{max} \cdot y}{J_x}$,

where y – the distance from the neutral axis to the considered point.

$\sigma(1) = \frac{120 \cdot 10^3 \cdot 18 \cdot 10^{-2}}{13380 \cdot 10^{-8}} = 0.161 \cdot 10^9 \frac{\mathrm{N}}{\mathrm{m}^2} = 161\ \mathrm{MPa}$

$\sigma(2) = \frac{120 \cdot 10^3 \cdot (18-1{,}23) \cdot 10^{-2}}{13380 \cdot 10^{-8}} = 0.150 \cdot 10^9 \frac{\mathrm{N}}{\mathrm{mm}^2} = 150\ \mathrm{MPa}$

$\sigma(3) = \frac{120 \cdot 10^3 \cdot 0}{13380 \cdot 10^{-8}} = 0\ \mathrm{MPa}$

Shear stresses:

$$\tau(1) = \frac{Q_{max} \cdot S_{\left(\frac{h}{2}\right)}}{b \cdot J_x} = \frac{80 \cdot 10^3 \cdot 0}{145 \cdot 10^{-3} \cdot 13380 \cdot 10^{-8}} = 0$$

where $S_{\left(\frac{h}{2}\right)} = 0$ – the static moment of inertia about the neutral axis for the portion of the area that is cut off from the cross-section by a line (flange edge).;

$$\tau(1') = \frac{Q_{max} \cdot S_{flange}/_{1p}}{b \cdot J_x} = \frac{80 \cdot 10^3 \cdot 310 \cdot 10^{-6}}{145 \cdot 10^{-3} \cdot 13380 \cdot 10^{-8}} = 0.0128 \cdot 10^8 \frac{\text{N}}{\text{m}^2} = 1.28 \text{ MPa}$$

where $S_{\text{flange/1p}}$ – the static moment of the flange area about the neutral axis:

$$S_{flange}/_{1p} = b \cdot t\left(\frac{h}{2} - \frac{t}{2}\right) = 145 \cdot 12.3\left(\frac{360}{2} - \frac{12.3}{2}\right) = 310062 \text{ mm}^2 = 310 \text{ cm}^3$$

where b – the flange width of the I-beam,

t – the flange thickness of the I-beam (Table D.59).

$$\tau(2) = \frac{Q_{max} \cdot S_{flange/1p}}{d \cdot J_x} = \frac{80 \cdot 10^3 \cdot 310 \cdot 10^{-6}}{7.5 \cdot 10^{-3} \cdot 13380 \cdot 10^{-8}} = 0.247 \cdot 10^8 \frac{\text{N}}{\text{m}^2} = 24.7 \text{ MPa}$$

$$\tau(3) = \frac{Q_{max} \cdot S_{flange/2p}}{b \cdot J_x} = \frac{80 \cdot 10^3 \cdot 423 \cdot 10^{-6}}{7.5 \cdot 10^{-3} \cdot 13380 \cdot 10^{-8}} \frac{\text{N}}{\text{m}^2} = 33.7 \text{ MPa},$$

where $S_{\text{flange/2p}}$ the static moment of inertia of the half-section of the I-beam concerning the neutral axis (from the table in the appendix).

The scheme of normal stresses σ and shear stresses τ of the I-beam cross-section are shown in Fig. 2.23.

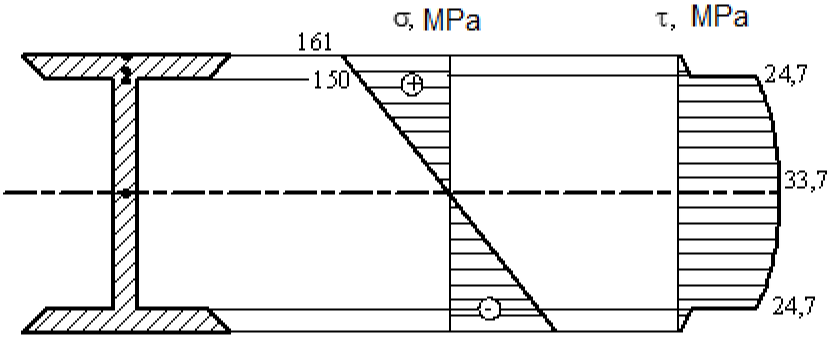


Fig. 2.23. Diagram of normal and tangential stresses in an I-beam

8. The equivalent stresses of the I-beam cross-section are determined according to the Huber-Mises yield criterion:

$$\sigma_{red}^{IV} = \sqrt{\sigma^2 + 3 \cdot \tau^2} \leq k$$

$$\sigma_{\text{red}}^{IV} = \sqrt{\sigma_2^2 + 3 \cdot \tau_2^2} = \sqrt{150^2 + 3 \cdot 24,.} = 156.13 \text{ MPa} < 160 \text{ MPa}$$

9. Check the selected the I-beam no. 36 using the stiffness criterion.

$$\textit{The stiffness criterion}:\ f \geq [f] = 0.001l = [0.006\text{ m}]$$

To determine the deflection of the I-beam, we place the origin of the coordinate system at the left support and write the universal equation of the elastic line for the last segment of the beam:

$$EJ_x f(x) = EJ_x f_0 + EJ_x \theta_0 \cdot x - R_A \frac{x^3}{6} - q\frac{x^4}{24} + M\frac{(x-2)^2}{2} +$$
$$+q\frac{(x-2)^4}{24} + R_B\frac{(x-4)^3}{6} - q\frac{(x-4)^4}{24}$$

From the beam's support condition, it follows:

$$f(x=0) = f_0 = 0, f(x=4) = 0$$

The angle θ_0 is calculated from the condition that for x = 4, the deflection of the beam f = 0.

$$EJ_x f(x=4) = EJ_x f_0 + EJ_x\theta_0 \cdot 4 - R_A\frac{4^3}{6} - q\frac{4^4}{24} + M\frac{(x-2)^2}{2} + q\frac{(x-2)^4}{24} = 0$$
$$EJ_x \cdot 0 + EJ_x\theta_0 \cdot 4 - 20\frac{4^3}{6} - 20\frac{4^4}{24} + 80\frac{(4-2)^2}{2} + 20\frac{(4-2)^4}{24} = 0$$

Thus, the rotation angle of the cross-section at the left support is $\theta_0 = \frac{63.3}{EJ_x}$.

Determine the deflection of the I-beam at critical locations:

$$EJ_x f(x=2) = EJ_x \cdot 0 + EJ_x\frac{63{,}3}{EJ_x} \cdot 2 - 20\frac{2^3}{6} - 20\frac{2^4}{24} = 86.7\text{ kNm}^3$$
$$f(x=2) = \frac{86{,}7 \cdot 10^3}{2 \cdot 10^{11} \cdot 13380 \cdot 10^{-8}} = 0.00324\text{ m} = 0.324\text{ cm}$$
$$EJ_x f(x=6) = EJ_x \cdot 0 + EJ_x\frac{63{,}3}{EJ_x} \cdot 6 - 20\frac{6^3}{6} - 20\frac{6^4}{24} + 80\frac{(6-2)^2}{2} +$$
$$+20\frac{(6-2)^4}{24} + 140\frac{(6-4)^3}{6} - 20\frac{(6-4)^4}{24} = -393\text{ kN}\cdot\text{m}^3$$
$$f(x=6) = \frac{-393 \cdot 10^3}{2 \cdot 10^{11} \cdot 13380 \cdot 10^{-8}} = -0.0148\text{ m} = -1.48\text{ cm}$$

The maximum deflection occurs at the end of the beam for x = 6 m and is f = –0.0148 m, which exceeds the permissible deflection

$[f]$ = 0.001l = 0.006 m = 0.6 cm.

Since the maximum deflection exceeds the permissible deflection, it is necessary to select a different I-beam. To do this, we determine the required moment of inertia for the new I-beam:

$$J_x \geq \frac{-393 \cdot 10^3}{2 \cdot 10^{11} \cdot 0.6 \cdot 10^{-2}} = 327.5 \cdot 10^{-6}\text{ m}^4 = 32750\text{ cm}^4$$

According to Table D.59, the I-beam I50 with J_z = 39727 cm^4 meets the stiffness requirements.

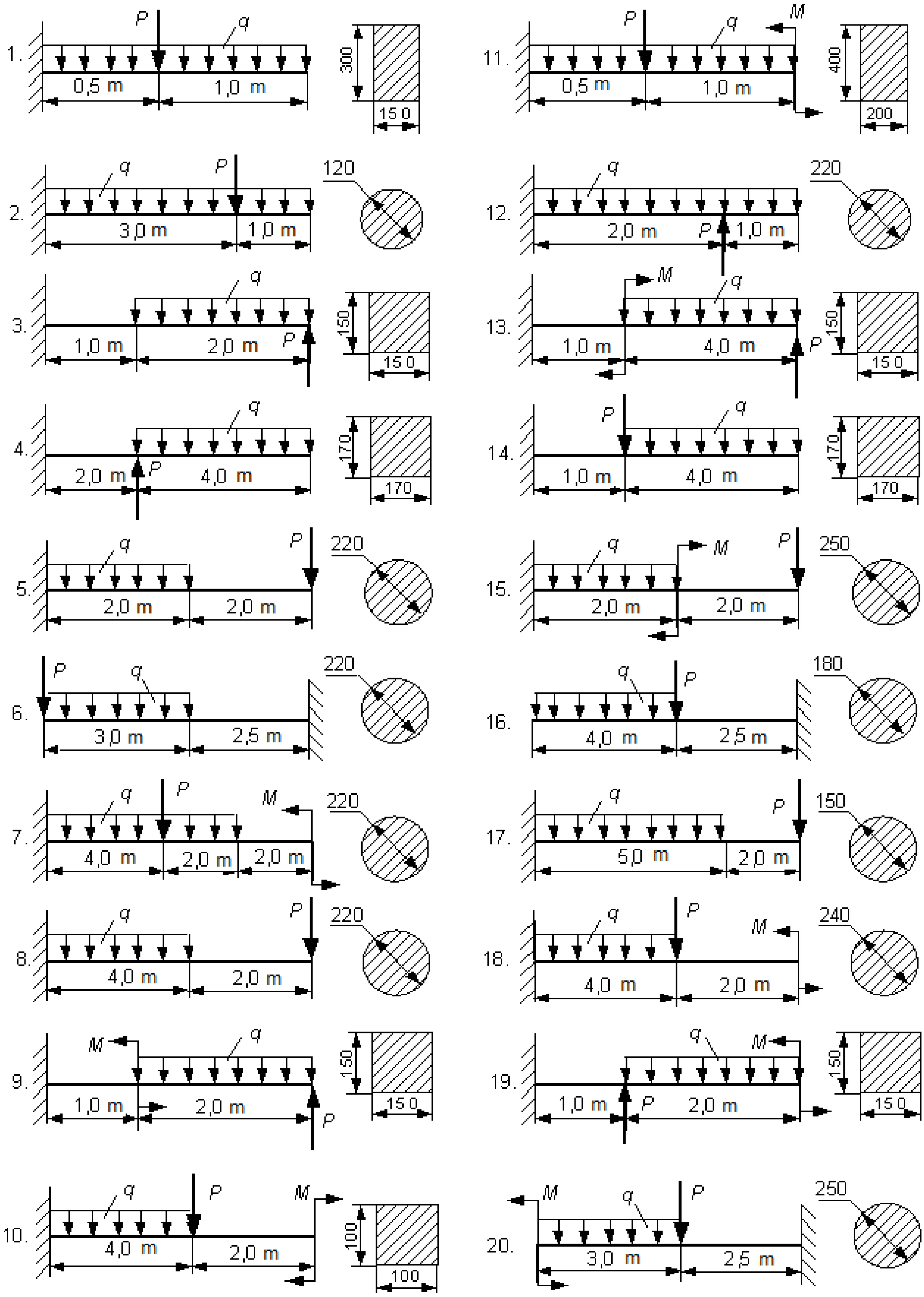
Individual tasks
(calculation)

Task 2.7. Check the serviceability of the beam (Table 2.5, calculation schemes).

Table 2.5. Initial data for Task 2.5

Var. no	Force *P*, kN	Torque *M*, kNm	Continuous load *q*, kN/m	Reliability factor γf	Service condition factor γ_c	Computational strength *R*, MPa
1	25	30	18	1.2	1.1	20
2	40	40	12	1.3	1.2	15
3	15	35	10	1.1	1.1	18
4	20	46	15	1.2	1.2	22
5	35	30	14	1.0	1.1	16
6	25	25	18	1.2	1.2	20
7	22	28	10	1.3	1.1	16
8	34	18	8	1.1	1.2	18
9	28	34	15	1.3	1.1	20
10	18	20	16	1.2	1.2	15

Schemes to Task 2.7



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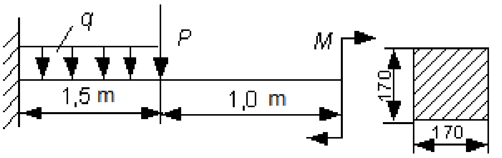

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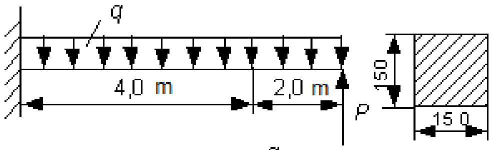

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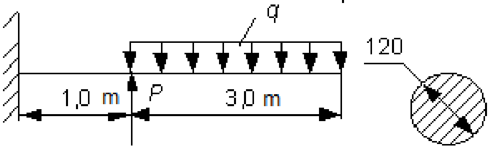

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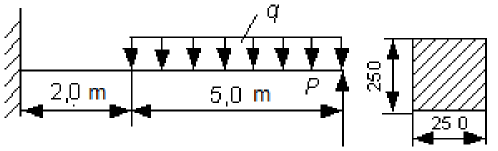

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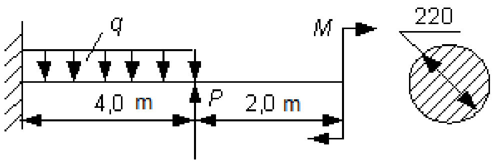

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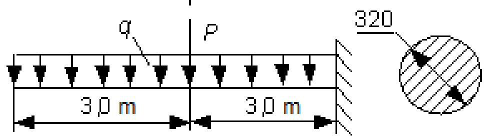

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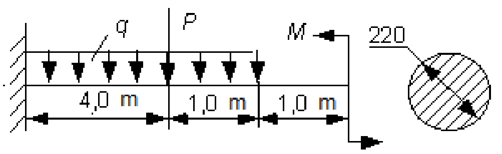

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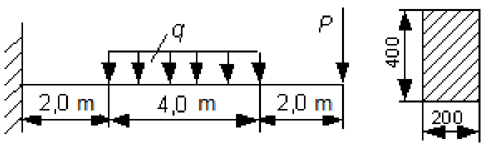

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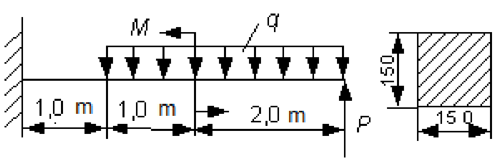

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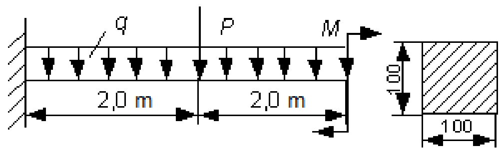

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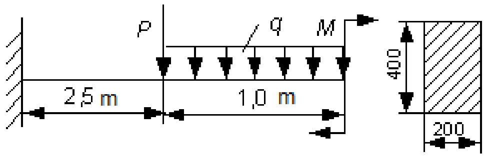

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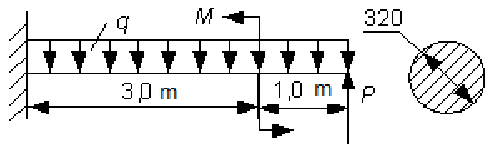

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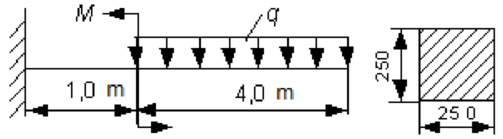

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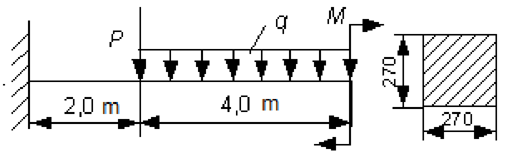

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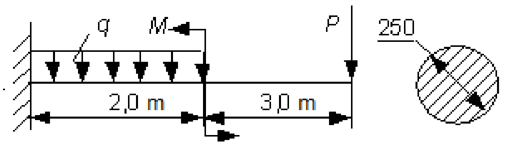

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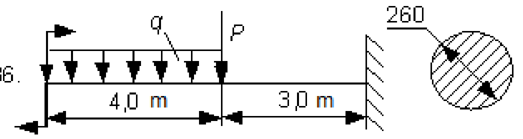

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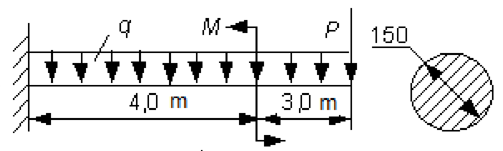

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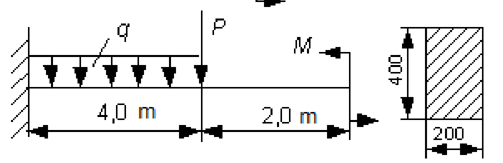

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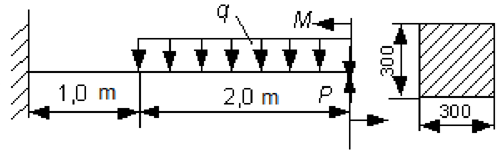

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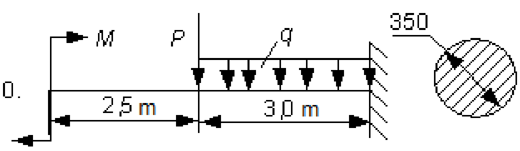


Task 2.8. Based on the strength and stiffness criteria, select the cross-section of the beam (calculation schemes).

For the loaded beam:

- determine the values of shear forces Q, bending moments M_g and plot their diagrams;

- from the strength criteria, determine the required dimensions of the beam cross-section in three variants:

a) circular cross-section;

b) rectangular cross-section (h/b = 2);

c) I-beam cross-section.

Choose the most rational cross-section shape from the three variants and justify your choice.

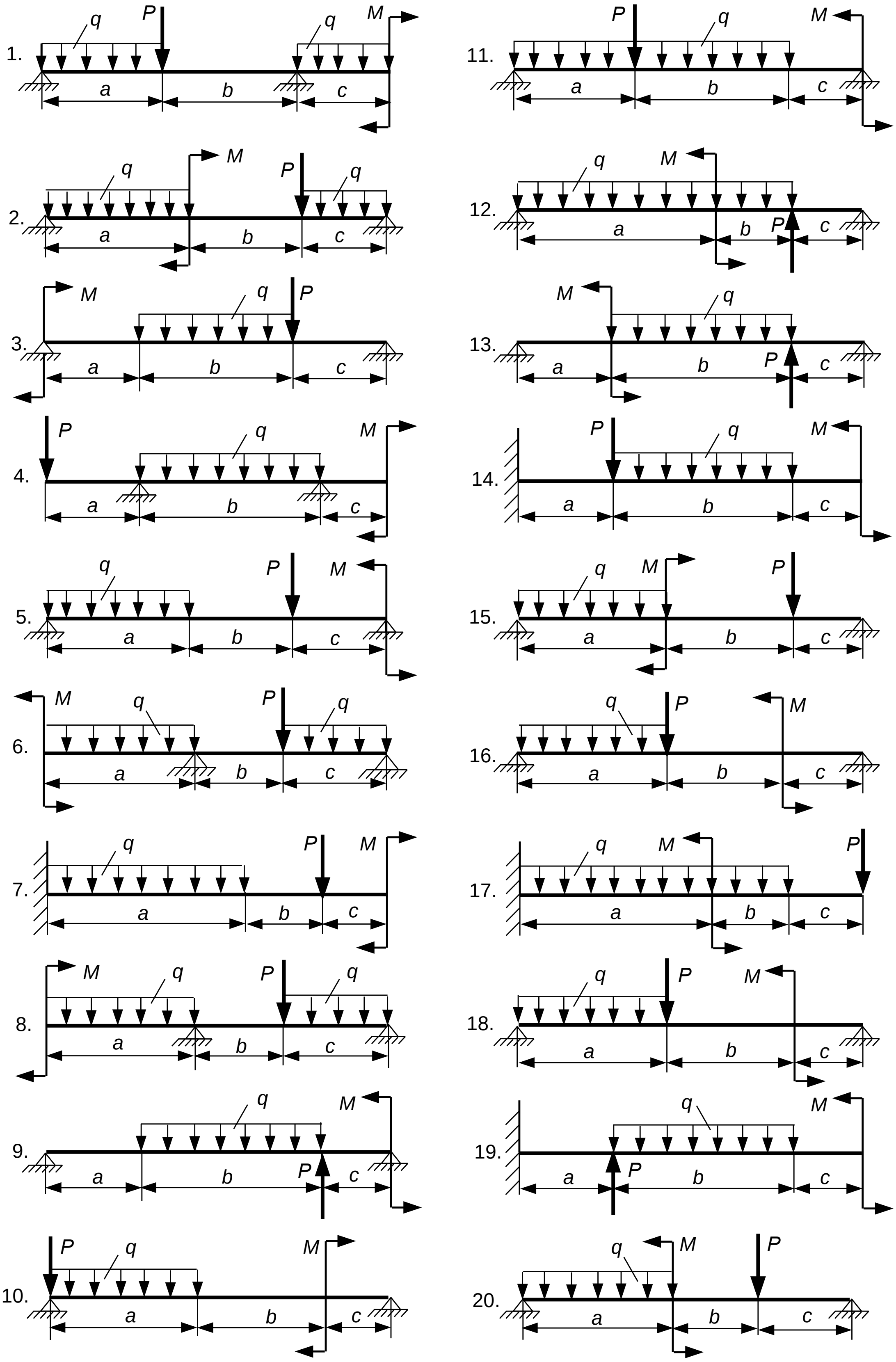
- For the selected cross-section, draw the diagrams of normal and shear stresses. Using the appropriate failure hypothesis, determine the reduced stresses for the given beam.

- Using the strength criterion, verify the stiffness of the beam with the selected cross-section, given $[f] = 0{,}001l$; $E = 2{\cdot}10^5$ MPa.

Table 2.6. Initial data for Task 2.8

Var. no	Force P, kN	Moment of a couple of forces M, kNm	Continuous load q, kN/m	Permissible stresses k, MPa	a, m	b, m	c, m
1	15	30	8	200	2	3	1
2	14	20	6	150	3	2	2
3	15	15	5	180	1	4	1
4	20	16	10	220	2	4	1
5	15	20	7	160	3	2	2
6	25	25	8	200	3	1	2
7	20	15	10	160	4	1	1
8	14	18	8	180	3	1	2
9	20	30	5	200	2	4	1
10	18	20	6	150	3	2	1

Schemes to Task 2.8



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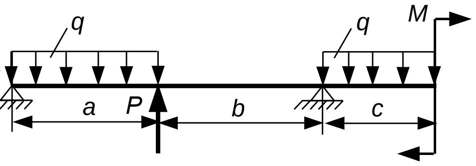

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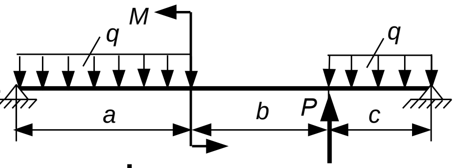

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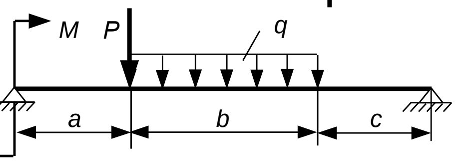

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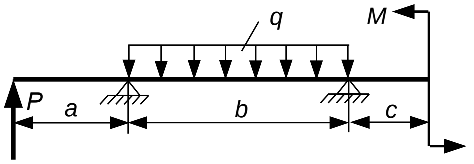

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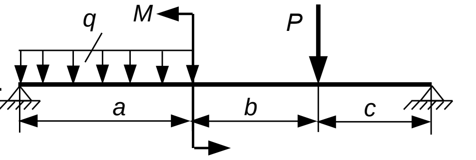

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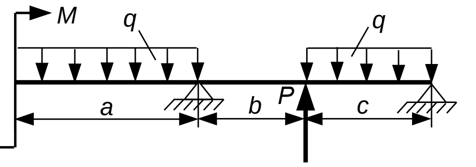

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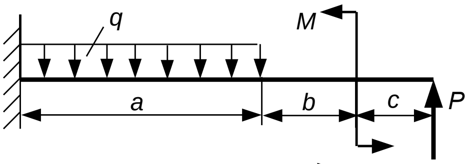

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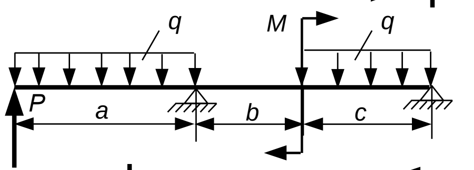

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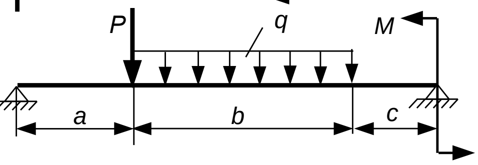

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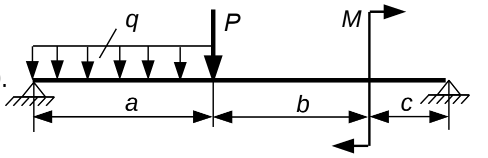

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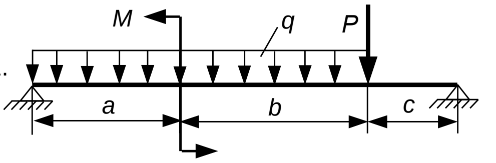

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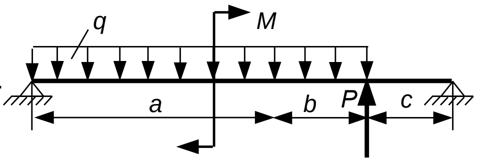

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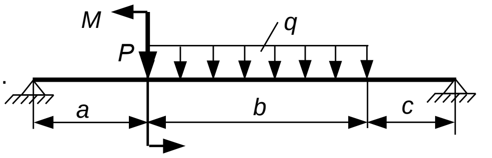

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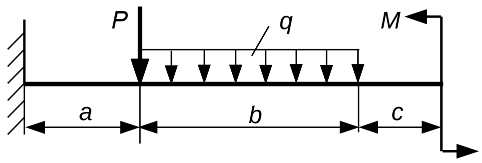

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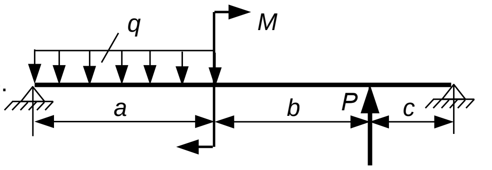

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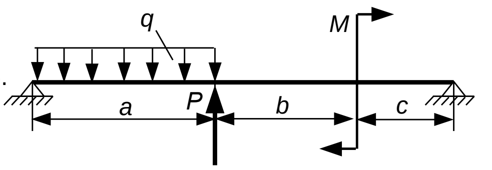

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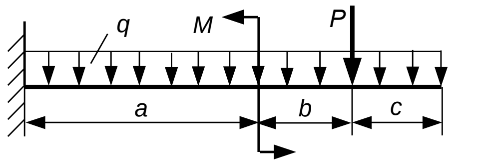

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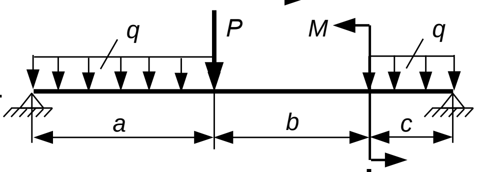

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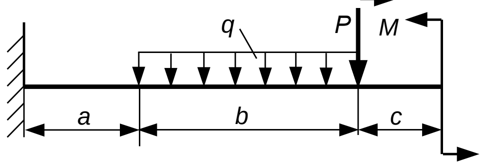

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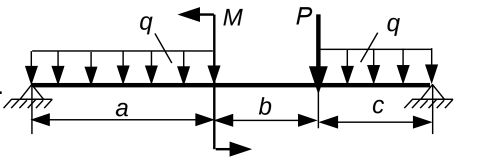


2.6. Bending with torsion of round rods

General information

Shafts of various machines typically operate under the influence of both bending and torsional stresses. When torque is transmitted to the shaft via a pulley and belt drive, a pair of forces is generated: torsional force and bending force. a similar scenario is observed in gear transmissions. In most cases, shafts are bent in two planes rather than one.

If the belt drive is set at an angle (Fig. 2.24), the shaft is bent in the horizontal plane by the projection of the belt tension forces onto the horizontal axis, and in the vertical plane by the weight of the pulley and the projection of the belt tension forces onto the vertical axis.

Bearings supporting the shaft are considered in calculations as spatial hinge supports, i.e., connections that prevent linear movements but do not interfere with the rotation of fixed sections of the shaft.

In the case of simultaneous bending and twisting, the bending moment M_g and the torque M_s are taken into account for the cross-section of the shaft.

Shafts are usually made of medium-carbon structural steel. Construction calculations are based on failure hypotheses.

Examples of calculation

Example 2.15. Three pulleys are mounted on a shaft. The pulley with a diameter $D_1 = 0.6$ m and an inclination angle $\alpha_1 = 45°$ rotates at $n = 500$ rpm and transmits a power of $N = 75$ kW. The other two pulleys have the same diameter $D_2 = 0.4$ m and the same inclination angle $\alpha_2 = 45°$, each transmitting a power of $N/2$ (Fig. 2.24).

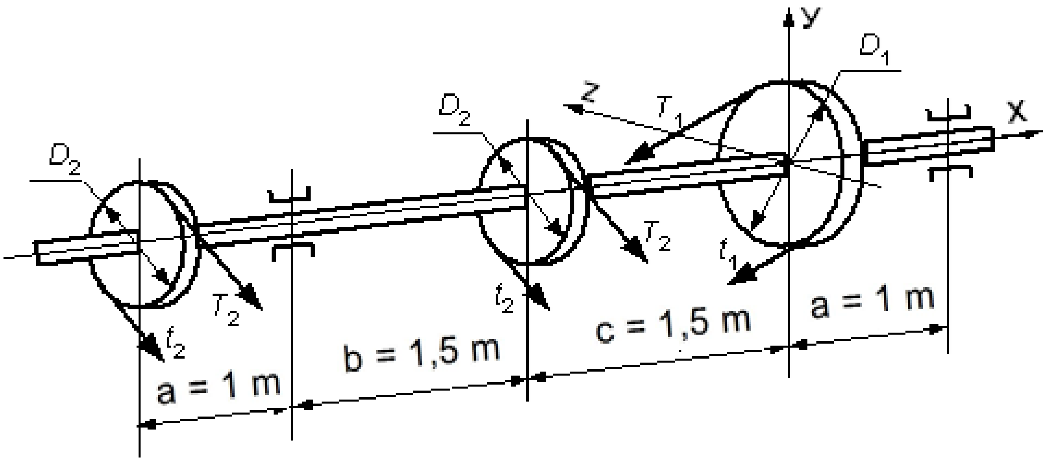


Fig. 2.24. Shaft diagram to Example 2.15

Procedure:

- determine the moments applied to the pulleys based on the given values of N (kW) and n (rpm);
- plot the torque diagrams M_s;
- given the moments and the specified pulley diameters D_1 and D_2 determine the tension forces t_1 and t_2, acting on the pulleys;
- calculate the shaft loads, assuming the forces are equal to the three peripheral forces;
- determine the bending forces acting in the horizontal and vertical planes (without considering the weights of the pulleys and shaft);
- plot diagrams of the horizontal bending moments M_g^h and vertical bending moments M_g^v;
- plot the diagrams of the total bending moments M_g;
- using the M_s and M_g diagrams, identify the critical cross-section and calculate the maximum computional moment M_{comp} (according to the appropriate failure hypothesis);
- select the shaft diameter d for $k = 70$ MPa.

Initial data: a = 1 m; b = 1.5 m; c = 1.5 m.

Solution

Wheel 1 – driving wheel, wheels 2 and 3 (of the same diameter) – driven wheels. T_1, T_2 – tension of the belt on the driving wheel, $t_1 = t_2 = T/2$ – tension of the belt in the driven part (Fig. 2.25).

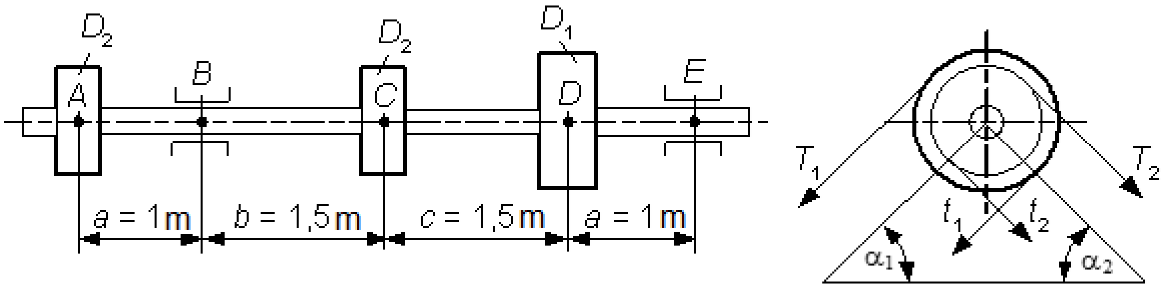


Fig. 2.25. Schematic of the drive shaft

1. Calculate the torques acting on the wheels using the given values of N and n according to the formulas:

$$M_1 = \frac{N}{\omega} = \frac{N}{n\frac{\pi}{30}} = \frac{75 \cdot 10^3}{500\frac{3.14}{30}} = 1433.12 \text{ Nm}$$

$$M_2 = \frac{N/2}{\omega} = \frac{N/2}{n\frac{\pi}{30}} = \frac{75 \cdot 10^3/2}{500\frac{3.14}{30}} = 716.56 \text{ Nm}$$

Plot the diagram of the computed torques on the shaft (Fig. 2.26, a).

2. Determine the values of the torques along the segments of the shaft as the sum of the moments on one side of the examined segment in the computed shaft diagram (Fig. 2.26, a):

$$M_s^{\mathrm{AC}} = M_2 = 716{,}56 \text{ Nm}, M_s^{CD} = M_2 + M_2 = M_1 = 1433.12 \text{ Nm}$$

Based on the obtained data, plot the torque diagram (Fig. 2.26, b*)*.

3. Calculate the belt tensions t_1 and t_2 acting on the wheels:

For wheel 1:

The torque of the belt drive is equal to the product of the difference in tension forces and half of the pulley diameter:

$$M_1 = (T_1 - t_1)\frac{D_1}{2} = (2t_1 - t_1)\frac{D_1}{2} = \frac{t_1 D_1}{2}$$

then,

$$t_1 = \frac{2M_1}{D_1} = \frac{2 \cdot 1433.12}{0.6} = 4777 \text{ N}$$

For wheel 2:

$$M_2 = (T_2 - t_2)\frac{D_2}{2} = \frac{t_2 D_2}{2}$$

then,

$$t_2 = \frac{2M_2}{D_2} = \frac{2 \cdot 716.56}{0.4} = 3583 \text{ N}$$

4. Determine the belt contact forces on the shaft:

$$P_A = T_2 + t_2 = 3t_2 = 10.75 \text{ kN}$$
$$P_C = T_2 + t_2 = 3t_2 = 10.75 \text{ kN}$$
$$P_D = T_1 + t_1 = 3t_1 = 14.37 \text{ kN}$$

5. Calculate the bending forces on the shaft in the horizontal plane (neglecting the mass of the wheels and the shaft):

$$P_A^H = P_A \cdot cos\,\alpha_2 = 10.75 \cdot cos\,45° = 7.6 \text{ kN}$$
$$P_C^H = P_C \cdot cos\,\alpha_2 = 10.75 \cdot cos\,45° = 7.6 \text{ kN}$$
$$P_D^H = -P_D \cdot cos\,\alpha_1 = 14.33 \cdot cos\,45° = -10.13 \text{ kN}$$

Plot the diagram of forces in the horizontal plane (Fig. 2.26 c).

Determine the support reactions:

$$\sum M_B = 0 - P_A^H \cdot a + P_C^H \cdot b - P_D^H \cdot (b + c) + R_E^H \cdot (b + c + a) = 0$$
$$R_E^H = \frac{P_A^H \cdot a - P_C^H \cdot b + P_D^H(\text{в} + \text{c})}{b + c + a} = \frac{7.6 \cdot 1 - 7.6 \cdot 1.5 + 10.13(1.5 + 1.5)}{1.5 + 1.5 + 1} = 6{,}65 \text{ kN}$$
$$\sum M_E = 0 - P_A^H(a + b + c + a) + R_B^H(b + c + a) - P_C^H(c + a) + P_D^H \cdot a = 0$$
$$R_B^H = \frac{P_A^H(a + b + c + a) + P_C^H(c + a) - P_D^H \cdot a}{b + c + a} =$$
$$= \frac{7.6 \cdot (1 + 1.5 + 1.5 + 1) - 7.6(1.5 + 1) - 10.13 \cdot 1}{1.5 + 1.5 + 1} = 11.72 \text{ kN}$$

Verification:

$$\sum y = 0 + P_A^H - R_B^H + P_C^H - P_D^H + R_E^H = 0 + 7.6 - 11.72 + 7.6 - 10.13 + 6.65 = 0$$

6. Calculate the values of the bending moments M_g^H from the horizontal forces:

$$M_{\text{g}_A}^H = P_A^H \cdot 0 = 0 \text{ Nm}$$
$$M_{g_\text{B}}^H = P_\text{A}^H \cdot \text{a} = 7.6 \cdot 1 = 7.6 \text{ Nm}$$
$$M_{g_\text{C}}^H = P_\text{A}^H(a + b) - R_B^\Gamma \cdot b = 7{,}6(1 + 1.5) - 11.72 \cdot 1.5 = 1.42 \text{ Nm}$$
$$M_{g_D}^H = P_\text{A}^H(a + b + c) - R_B^H(b + c) + P_C^H \cdot \text{c} =$$
$$=7.6(1 + 1.5 + 1.5) - 11.72(1.5 + 1.5) + 7.6 \cdot 1.5 = 6.65 \text{ Nm}$$
$$M_{g_D}^H = P_E^H \cdot d = 6.65 \cdot 1 = 6.65 \text{ Nm}$$
$$M_{g_E}^H = P_E^H \cdot 0 = 0 \text{ Nm}$$

Based on the obtained results, plot the bending moment diagram in the horizontal plane (Fig. 2.26, d).

7. Determine the bending forces on the shaft in the vertical plane (neglecting the mass of the wheels and the shaft):

$$P_A^V = -P_A \cdot \sin\alpha_2 = -10.75 \cdot \sin 45° = -7.6 \text{ kN}$$
$$P_C^V = -P_C \cdot \sin\alpha_2 = -10.75 \cdot \sin 45° = -7.6 \text{ kN}$$
$$P_D^V = -P_D \cdot \sin\alpha_1 = -14.33 \cdot \sin 45^\circ = -10.13 \text{ kN}$$

Plot the diagram of forces in the vertical plane (Fig. 2.26, e).

Determine the support reactions:

$$\sum M_B = 0 = P_A^V \cdot a - P_C^V \cdot b - P_D^V \cdot (b+c) + R_E^V \cdot (b+c+a) = 0$$

$$R_E^V = \frac{-P_A^V \cdot a + P_C^V \cdot b + P_D^V(\text{в} + \text{c})}{b+c+a} = \frac{-7.6 \cdot 1 + 7.6 \cdot 1.5 + 10.13(1.5+1.5)}{1.5+1.5+1} = 8.55 \text{ kN}$$

$$\sum M_E = 0;\ P_A^V(a+b+c+a) - R_B^V(b+c+a) + P_C^V(c+a) + P_D^V \cdot a = 0$$

$$R_B^V = \frac{P_A^V(a+b+c+a) + P_C^V(c+a) + P_D^V \cdot a}{b+c+a} =$$

$$= \frac{7.6 \cdot (1+1.5+1.5+1) + 7.6(1.5+1) + 10.13 \cdot 1}{1.5+1.5+1} = 16.78 \text{ kN}$$

Verification:

$$\sum y = 0 - P_A^V + R_B^V - P_C^V - P_D^V + R_E^V = 0$$

$$-7.6 + 16.78 - 7.6 - 10.13 + 8.55 = 0$$

8. Calculate the values of the bending moments M_{g}^V from the vertical forces:

$$M_{\text{g}_A}^V = -P_{\text{A}}^V \cdot 0 = 0 \text{ kNm}$$

$$\text{M}_{\text{g}_\text{B}}^V = -P_{\text{A}}^V \cdot \text{a} = -7.6 \cdot 1 = -7.6 \text{ kNm}$$

$$\text{M}_{\text{g}_\text{C}}^V = -P_{\text{A}}^V(a+b) + R_B^V \cdot b = -7.6(1+1.5) + 16.78 \cdot 1.5 = 6.125 \text{ kNm}$$

$$\text{M}_{\text{g}_D}^V = +P_E^V \cdot \text{a} = +8.55 \cdot 1 = 8.55 \text{ kNm}$$

$$M_{\text{g}_\text{E}}^V = -\text{P}_{\text{E}}^V \cdot 0 = 0 \text{ kNm}$$

Based on the obtained results, plot the bending moment diagram for the vertical section (Fig. 2.26, f).

9. Calculation and preparation of the combined bending moment diagram M_{g}.

We determine the total bending moments M_{g}^H and M_{g}^V in the shaft sections based on the diagrams:

$$M_{\text{g}} = \sqrt{\left(M_{\text{g}}^H\right)^2 + \left(M_{\text{g}}^V\right)^2} \text{ kNm}$$

$$M_{\text{g}}^A = 0 \text{ kNm};\ \text{M}_{\text{g}}^B = \sqrt{(-7.6)^2 + (-7.6)^2} = 10.75 \text{ kNm}$$

$$M_{\text{g}}^C = \sqrt{(-1.42)^2 + (6.125)^2} = 6.29 \text{ kNm}$$

$$\text{M}_{\text{g}}^D = \sqrt{(-6.65)^2 + (8.55)^2} = 10.83 \text{ kNm}$$

$$M_{\text{g}}^E = 0 \text{ kNm}$$

The diagram of the combined bending moments is shown in Fig. 2.26, *g*.

10. Identify the critical cross-section from the diagrams M_{s} and M_{g} and calculate the value of the maximum calculated moment according to the appropriate strength hypothesis (Fig. 2.26, *h*).

The critical cross-section for the shaft is section *D*, where $M_{\text{s}} = 1.433$ kNm and $M_{\text{g}} = 10.83$ kNm.

According to the Coulomb-Tresca failure criterion:

$$M_{\text{red}} = \sqrt{M_g^2 + M_s^2} = \sqrt{10.83^2 + 1.433^2} = 10.92 \text{ kNm}$$

11. We determine the shaft diameter based on the strength criterion:

$$\sigma_{\text{red}} = \frac{M_{\text{red}}}{W} \leq k$$

where $W = \frac{\pi d^3}{32}$ – the bending strength index for a shaft with a continuous cross-section is, therefore:

$$d = \sqrt[3]{\frac{32 \cdot M_{calc}}{\pi \cdot k}} = \sqrt[3]{\frac{32 \cdot 10.92 \cdot 10^3}{3.14 \cdot 70 \cdot 10^6}} = 0.1167 \text{ m}$$

From Table D.43, we adopt a shaft diameter d = 120 mm.

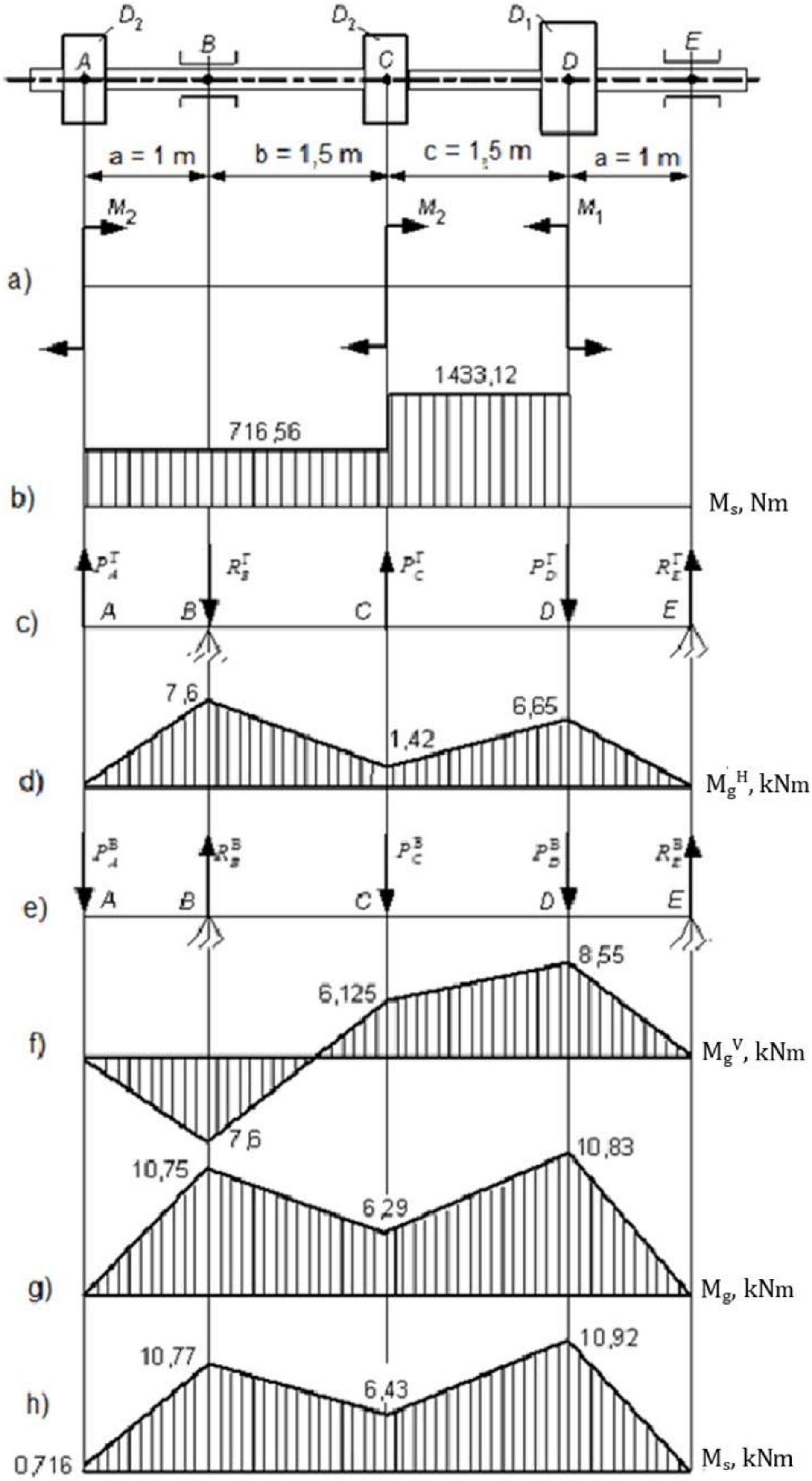


Fig. 2.26 Shaft load charts

Individual tasks
(calculation)

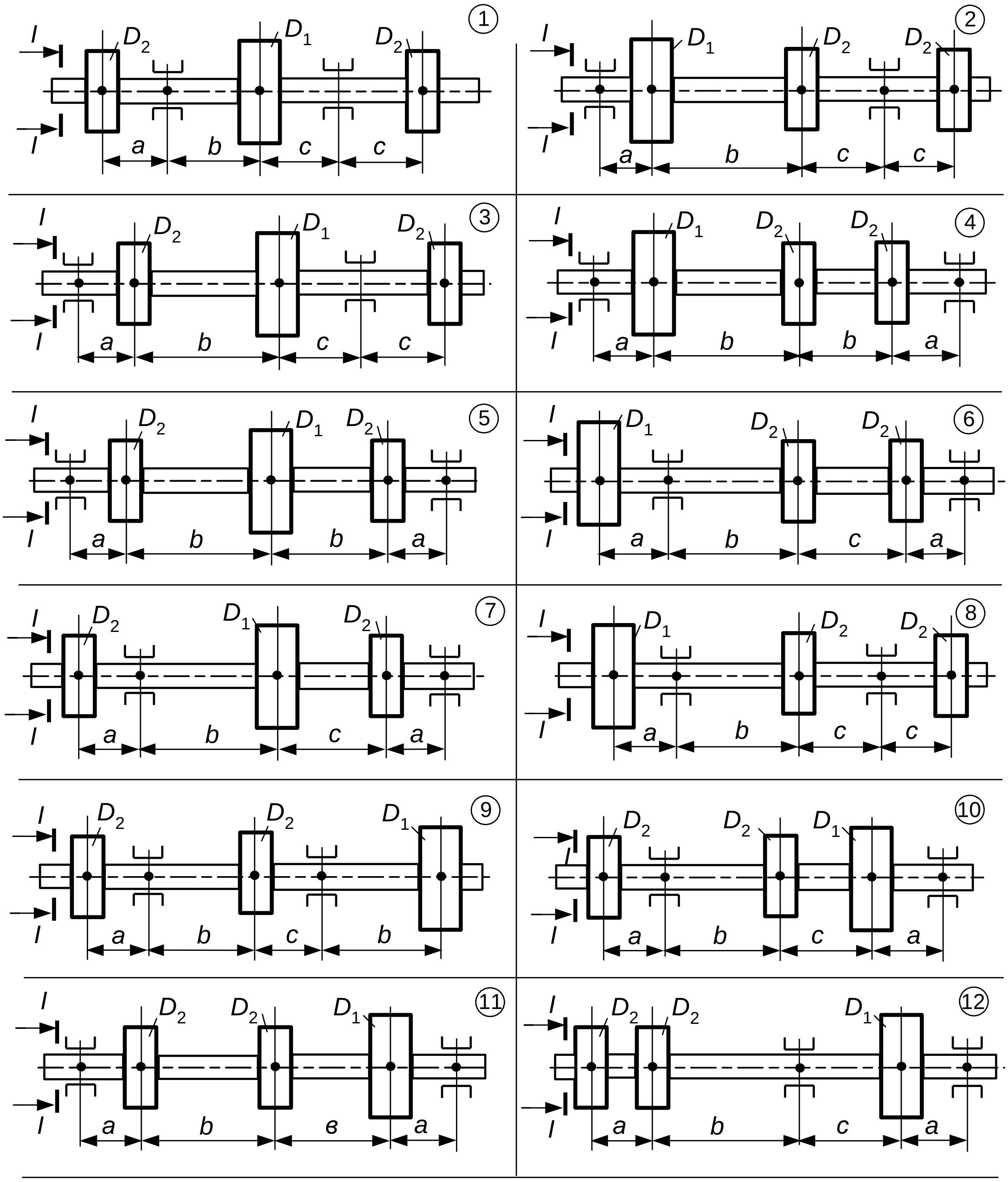
Task 2.9. Determine the shaft diameter based on bending and torsional strength conditions (Table 2.7, schemes below).

Table 2.7. Initial data for Task 2.9

Var. no	Scheme no	*N*, kW	*n*, rpm	*a*, m	*b*, m	*c*, m	D_1, mm	D_2, mm	α_1, deg	α_2, deg
1	7	10	100	0.5	1	1.5	600	400	30	30
2	3	75	600	0.5	2	1	800	650	45	45
3	5	80	800	1	2	1	1000	600	30	45
4	12	100	1000	1	2	0.5	1200	800	45	30
5	9	80	900	1	2.5	1.5	1400	700	45	60
6	2	70	700	0.5	2.5	1	1200	800	60	30
7	1	85	900	0.5	2	1.5	800	500	30	45
8	4	95	1000	1	2.5	2	1000	800	30	30
9	6	75	800	0.5	1.5	1	800	600	60	45
10	7	40	300	1	2	1.5	800	500	45	45
11	10	60	600	0.5	1.5	1	850	650	30	45
12	8	70	900	1	2	1.5	1200	900	60	45
13	7	80	1000	0.5	2	1	850	550	30	60
14	3	90	800	0.5	1.5	1	800	450	30	30
15	5	70	600	0.5	1.5	1	1200	800	60	30
16	2	80	900	1	2	1.5	1300	900	30	45
17	9	100	1000	0.5	1	1.5	1000	600	30	30
18	12	75	900	1	2	1	1000	700	45	30
19	1	40	500	0.5	2	1.5	1000	800	30	45
20	8	50	400	0.5	1	2	600	300	30	60
21	11	80	900	0.5	1	1.5	600	400	30	45
22	10	65	700	1	3	2	1200	900	30	45
23	7	75	800	1	2	1.5	1000	700	45	30
24	6	50	500	0.5	2	1.5	1200	800	30	60
25	11	100	400	0.5	1	1.5	1000	700	45	60
26	5	50	700	1	3	2	1200	400	30	45
27	2	60	800	0.5	2	2.5	1300	900	45	30
28	9	75	750	0.5	2	1.5	900	600	60	60
29	12	85	850	1	3	2	800	500	45	30
30	1	90	900	1	2	1.5	900	600	30	30
31	8	120	1200	1	3	2	1200	800	45	30
32	7	140	1400	0.5	2	1	1400	900	30	30

Var. no	Scheme no	N, kW	n, rpm	a, m	b, m	c, m	D_1, mm	D_2, mm	α_1, deg	α_2, deg
33	5	150	500	0.5	1	1	1000	800	45	60
34	6	65	600	0.5	1.5	1	800	500	30	45
35	4	55	450	0.5	1	1.5	800	500	45	45
36	2	90	900	0.5	2	1	900	500	30	60
37	12	120	1200	0.5	2.5	1	1200	900	45	45
38	1	100	1000	0.5	1.5	1	1000	800	60	45
39	11	90	900	1	3	2	900	600	60	45
40	8	80	900	0.5	1	1.5	600	400	30	45
41	3	100	400	0.5	1	1.5	1000	700	45	60
42	4	50	400	0.5	1	2	600	300	30	60
44	6	65	560	0.5	1.5	1	700	500	45	45
45	7	70	700	0.5	1.5	1	700	500	60	45
46	9	90	1000	1	2	1	1000	800	45	45
47	2	120	1200	0.5	1.5	1	1200	600	45	45

Schemes to Task 2.9



2.7. The strength of compressed rods

General information

Constructions and their elements can be destroyed due to the loss of the original elastic equilibrium state – loss of stability. The length of a compressed rod significantly impacts the nature of the damage. So-called "stubby" rods fail due to loss of strength; the only deformation type is compression.

As the length of the compressed rod increases, a loss of stability occurs, manifesting as a transition from a straight form of equilibrium to a curved one. Due to the curvature of the axis in the rod, both compressive and bending deformations occur. This happens suddenly when the load on the rod increases slightly, and the stress level is low enough that the strength has not yet been exceeded.

The stress at which a centrally compressed straight rod loses its stability can be much less than the tensile strength of the material from which it is made. When the rod loses its straight shape, additional bending stresses appear in its cross-sections, leading to its failure. Therefore, after calculating the strength, the compressed rod must be checked for stability, and if necessary, assessed for stability.

Bending caused by an axial force is referred to as **longitudinal bending**.

The compressive force at which a rod may lose stability is known as the **critical force** (F_{cr}).

Equilibrium can be stable, unstable, or neutral (Fig. 2.27).

The value of the critical force (F_{cr}) for a compressed rod of length l is calculated using *Euler's formula***:**

$$F_{cr} = \frac{\pi^2 E \cdot J_{\min}}{(\mu \cdot l)^2}$$

where E – longitudinal modulus of elasticity of the rod;

J_{min} – minimum axial moment of inertia of the rod's cross-section;

μ – length reduction factor (depends on the method of the rod's end conditions).

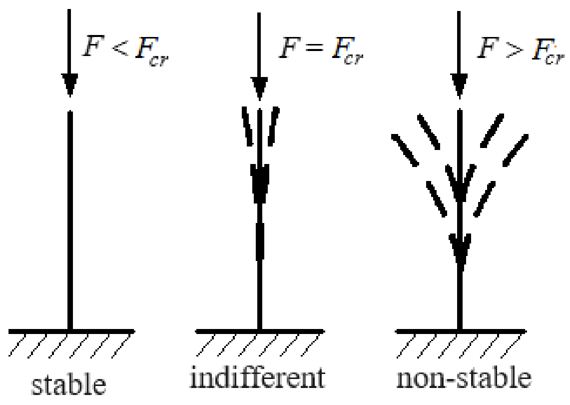


Fig. 2.27. Types of bar equilibrium

The values of the coefficient μ are selected from the relevant tables. Some cases are shown in Fig. 2.28.

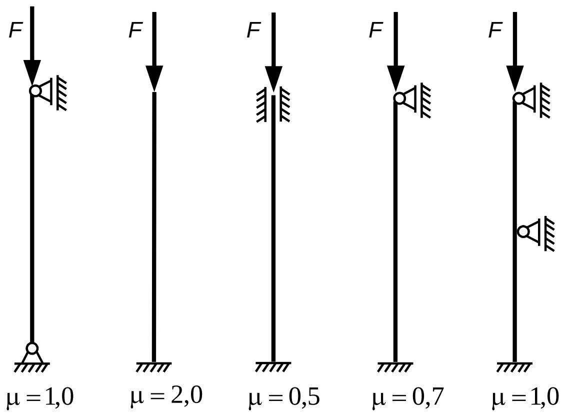


Fig. 2.28. Values of μ for selected cases

To ensure the stability of a compressed rod with a certain safety margin, the following conditions must be met.

$$F < [F], \text{ if } [F] = \frac{F_{cr}}{n},$$

where n – safety factor.

Critical stresses. Slenderness of the rod

Stability condition in terms of stresses:

$$\sigma_{cr} = \frac{F_{cr}}{A}$$

The critical stresses σ_{cr} are calculated using the formula:

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}, \qquad \text{if} \qquad \lambda = \frac{\mu l}{i_{min}},$$

λ - slenderness of the rod (characterizes the stiffness of the cross-section);

$i = \sqrt{\frac{J_{\min}}{A}}$ – the smaller of the radius of inertia of the rod's cross-section (geometric characteristic of the cross-section);

A – cross-sectional area of the rod.

Euler's formula can be applied provided that the critical stress does not exceed the material's proportional limit:

$$\sigma_{cr} \leq \sigma_H \text{ namely } \sigma_{\mathrm{cr}} = \frac{F_{\mathrm{cr}}}{A} = \frac{\pi^2 E}{\lambda^2} \leq R_m$$

Typically, the condition for applying Euler's formula is expressed by the inequality:

$$\lambda \geq \lambda_{lim} = \sqrt{\frac{\pi^2 E}{R_m}},$$

where λ_{lim} – limiting slenderness of a rod made of a material.

In contrast to the slenderness of the rod λ, which is a geometric characteristic, the limiting slenderness λ_{lim} depends solely on the physical and mechanical properties of the rod material and does not depend on its dimensions.

For a rod made of steel S215 (E = 2,1 · 10^5 MPa, σ_H = 200 MPa):

$$\lambda_{\mathrm{lim}} = \sqrt{\frac{3.14^2 \cdot 2 \cdot 10^5}{200}} \cong 100$$

This means that if a rod made of steel S215 has a slenderness of 100, applying Euler's formula to calculate F_{cr} and σ_{cr} will result in an incorrect outcome.

From practical experience, stable equilibrium phenomena can also occur at stresses exceeding the proportional limit.

Jasinski F.S. conducted experimental studies on the stability of rods beyond the proportional limit and derived an empirical formula for critical stresses dependent on the slenderness of the rod:

$$\sigma_{cr} = a - b\lambda + c\lambda,$$

where a, b, c – empirical coefficients that have the dimension of stress.

For deformable materials, it is most often assumed that c = 0, simplifying the formula to:

$$\sigma_{cr} = a - b\lambda$$

These formulas apply to rods whose slenderness falls within the limits:

$$\lambda_0 < \lambda < \lambda_{lim},$$

where λ_0 – slenderness at which σ_{cr} equals the ultimate stress σ_{lim}.

For a deformable material, σ_{lim} is equal to the yield strength R_e, and for an undeformable material, it is equal to the compressive strength R_c. For slenderness $\lambda < \lambda_0$ $\sigma_{cr} = \sigma_{lim}$, and in this case, the rod's strength should not be checked.

Values of empirical coefficients and slenderness λ_0 and λ_{lim} for some materials are presented in Table 2.8.

Table 2.8. Empirical coefficient values and slenderness for selected materials

Material	a, MPa	b, MPa	c, MPa	λ_0	λ_{lim}
Steel S215	310	1.14	0	61	100
Steel S275	350	1.15	0	57	90
Duraluminum A7	406	2.83	0	30	53
Cast iron	776	12	0.053	10	80
Pin	29.3	0.194	0	–	70

The critical force is determined by the stresses σ_{cr} as for axial compression of the rod:

$$F_{cr} = \sigma_{cr} \cdot A$$

Cross-sectional radius of inertia

In calculations of stability, it is sometimes convenient to use the radius of inertia ρ:

$$\rho = \frac{i_{\min}}{\sqrt{A}} = \frac{\sqrt{J_{\min}}}{A}$$

The radius of inertia characterizes the shape of the cross-section and does not depend on its dimensions. The larger the ρ, the greater the load-bearing capacity of a compressed rod with the same cross-sectional area. Table 2.9 presents the values of ρ for some cross-sections.

Table 2.9. Values of ρ for certain sections

Cross-section	ρ
Tubular $\left(c = \frac{d}{D} = 0.95 \div 0{,}8\right)$	1.246 ÷ 0.602
Tubular (c = 0,7 ÷ 0,5)	0.482 ÷ 0.364
Angle bracket	0.5 ÷ 0.3
I-beam	0.41 ÷ 0.27
C-bar	0.41 ÷ 0.29
Square	0.289
Circular	0.283
Rectangle	0.204

Calculations of Stability Using the Stress Reduction Factor

There is a relationship between the allowable compressive stress k and the allowable stability stress k_{st}:

$$k_{st} = \varphi k,$$

where φ - the reduction factor for the allowable stress in a compressed rod.

The values of the reduction factor φ have been calculated for rods made of different materials, depending on their slenderness. Table 2.10 presents these values.

Table 2.10. Values of φ for selected materials in relation to slenderness

Slenderness	Steels S215 and S235		Steel S275		Cast iron		Wood		Steels 17Mn4, 15GA, 13Mn6, S355J2	
λ	φ	$\lambda/\sqrt{\varphi}$	φ	$\lambda/\sqrt{\varphi}$	φ	$\lambda/\sqrt{\varphi}$	φ	$\lambda/\sqrt{\varphi}$	φ	$\lambda/\sqrt{\varphi}$
0	1.00	0	1.00	0	1.00	0	1.00	0	1.00	0
10	0.99	10.05	0.98	10.10	0.97	10.15	0.99	10.05	0.98	10.10
20	0.97	20.31	0.96	20.41	0.91	20.97	0.97	20.31	0.95	20.52
30	0.95	30.78	0.93	31.11	0.81	33.33	0.93	31.11	0.92	31.28
40	0.92	41.70	0.89	42.40	0.69	48.15	0.87	42.88	0.89	42.40
50	0.89	53.00	0.85	54.23	0.57	66.23	0.80	55.90	0.84	54.56
60	0.86	64.70	0.80	67.08	0.44	90.45	0.71	71.21	0.78	67.93
70	0.81	77.78	0.74	81.37	0.34	120.15	0.60	90.37	0.71	83.08
80	0.75	92.38	0.67	97.74	0.26	156.9	0.48	115.5	0.63	100.8
90	0.69	108.4	0.59	117.2	0.20	201.3	0.38	146.0	0.54	122.5
100	0.60	129.1	0.50	141.4	0.16	250.0	0.31	179.6	0.46	147.4
110	0.52	152.5	0.43	167.8			0.25	220.0	0.39	176.1
120	0.45	178.9	0.37	197.3			0.22	255.8	0.33	208.9
130	0.40	205.6	0.32	229.8			0.18	306.4	0.29	241.4
140	0.36	233.3	0.28	264.6			0.16	350.0	0.25	280.0
150	0.32	265.2	0.25	300.0			0.14	400.9	0.23	312.8
160	0.29	297.1	0.23	333.6			0.12	461.9	0.21	349.1
170	0.26	333.4	0.21	371.0			0.11	512.6	0.19	390.0
180	0.23	375.3	0.19	413.0			0.10	569.2	0.17	436.6
190	0.21	414.6	0.17	460.8			0.09	633.3	0.15	490.6
200	0.19	458.8	0.15	516.4			0.08	707.1	0.13	555.1
210	0.17	509.3	0.14	561.3						
220	0.16	550.0	0.13	610.2						

Considering the coefficient φ the strength condition takes the form:

$$\sigma = \frac{F}{A} \leq \varphi \cdot\ k$$

The stability condition allows for two types of calculations for compressed members - verification and design calculations. Additionally, it enables the calculation of the permissible load on the member.

Verification calculations for compressed rods

The stresses in the rod are calculated using the formula:

$$\sigma = \frac{F}{A}$$

Based on the known dimensions and shape of the cross-section, the smallest axial moment of inertia is determined, and the minimum radius of gyration is calculated:

$$i = \sqrt{\frac{J_{\min}}{A}}$$

Slenderness of the rod: $\lambda = \frac{\mu l}{i_{min}}$

Determine the value of the factor of stress reduction φ from Table 2.10. Calculate the permissible stress to ensure stability:

$$k_{st} = \varphi \cdot k$$

Check the condition:

$$\sigma = \frac{F}{A} \leq k_{st} = \varphi \cdot \mathrm{k}$$

Design calculations for the stability of compressed rods using the coefficient φ

In design calculations, the task is to select the shape of the cross-section, the area, and the rod's material based on the rod's known load and length. This condition contains two unknown values φ and A. Therefore, rods are calculated using an iterative approximation method.

1. For the first approximation, the value of the coefficient φ is assumed to be:

$$\varphi_1 = 0.5 \div 0.6$$

2. Calculate the cross-sectional area of the rod:

$$A \geq \frac{F}{\varphi_1 k}$$

3. Assume the cross-sectional shape and calculate the minimum radius of inertia for the known area:

$$i_{min} = \sqrt{\frac{J_{min}}{A}}$$

4. Calculate the slenderness based on the known end restraints of the rod:

$$\lambda = \frac{\mu l}{i_{min}}$$

5. Knowing the effective slenderness λ from Table 2.10, select the value of the coefficient φ_1'.

6. Calculate the permissible stability stresses:

$$\sigma_{\text{st}} = \varphi_1' \cdot k$$

7. Compare the calculated stresses in the rod $\sigma_{\text{calc}} = \frac{F}{A}$ and the permissible stability stresses k_{st}:

$$\frac{\sigma_{\text{calc}} - k_{st}}{k_{st}} \leq \eta = 0{,}05$$

where η - the accuracy of the calculations (usually the accuracy η = ± 0.05 or ± 5 %).

8. If the condition from step 7 is satisfied, the design calculation task is completed. If the condition is not met, repeat the entire calculation for a new value $\varphi_2 = \frac{\varphi_1 + \varphi}{2}$ until the condition from step 7 is met. Typically, three to four iterations are required to satisfy the condition η = ±0.05.

Examples of calculations

Example 2.16. Check the stability of a rod with a length of l = 2.5 m, an outer diameter D = 76 mm, and an inner diameter d = 64 mm. The rod is supported on one end and fixed on the other end. The compressive force F = 150 kN, the material is chrome-molybdenum steel (k_r = 540 MPa, $E = 2.15 \cdot 10^5$ MPa), and the stability safety factor k_{st} = 3.5.

Solution

Determine the critical slenderness of the given material:

$$\lambda_{lim} = \sqrt{\frac{\pi^2 E}{R_m}} = \sqrt{\frac{3.14^2 \cdot 2.15 \cdot 10^{11}}{540 \cdot 10^6}} = 63$$

To determine the slenderness of the rod (λ) calculate the axial moment of inertia of its cross-section:

$$J_{min} = J = \frac{\pi}{64}(D^4 - d^4) = \frac{3{,}14}{4}(76^4 - 64^4) = 81.4 \cdot 10^4 \text{mm}^4 = 81.4 \cdot 10^{-8}\ \text{mm}^4$$

Cross-sectional area:

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{3.14}{4}(76^2 - 64^2) = 1319\ \text{mm}^2$$

The radius of inertia of the cross-section:

$$i_{min} = i = \sqrt{\frac{J_{min}}{A}} = \sqrt{\frac{81.4 \cdot 10^4}{1319}} = 24.8\ \text{mm}$$

Calculate the slenderness of the rod, assuming the coefficient μ = 0.7:

$$\lambda = \frac{\mu l}{i} = \frac{0.7 \cdot 2.5 \cdot 10^3}{24.8} = 70.7$$

Since the slenderness of the rod is greater than the critical slenderness ($\lambda > \lambda_{\text{lim}}$) the critical force is determined using Euler's formula:

$$F_{\text{cr}} = \frac{\pi^2 E \cdot J_{min}}{(\mu \cdot l)^2} = \frac{3.14^2 \cdot 2.15 \cdot 10^{11} \cdot 81.4 \cdot 10^{-8}}{(0.7 \cdot 2.5)^2} = 564 \cdot 10^3\ \text{N} = 564\ \text{kN}$$

Determine the stability safety factor and compare it with the specified value [k_{st}]:

$$k_{st} = \frac{F_{st}}{F} = \frac{564}{150} = 3.76 > 3.5$$

Conclusion: The stability of the rod is ensured.

Example 2.17. Check the stability of a steel column loaded as shown in Fig. 2.29, with a section of an I-beam I20, a height of 1.5 m, and fixed at one end, if: F = 160 kN, k_{st} = 1.5, and the material of the column with S215 z σ_e = 240 MPa, n = 1,045.

Solution

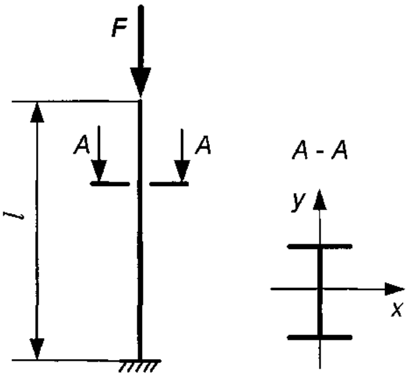


Fig. 2.29. Diagram of column to Example 2.16

From Table D.59 for the I-beam I 20, take the geometric characteristics required for the calculations:

$$A = 26.8 \cdot 10^{-2}\ \text{m}^2$$
$$J_y = J_{min} = 115 \cdot 10^{-8}\text{m}^4$$
$$i_{min} = i_y = 2.07 \cdot 10^{-2}\text{m}$$

Calculate the slenderness of the column, assuming the coefficient μ = 2 (Fig. 2.28).

$$\lambda = \frac{\mu l}{i_{min}} = \frac{2 \cdot 1.5}{2.07 \cdot 10^{-2}} = 145$$

Calculate the values of the coefficient φ using the linear interpolation formula:

$$\varphi = \varphi_1 - \frac{\varphi_1 - \varphi_2}{\lambda_2 - \lambda_1}(\lambda - \lambda_1)$$

Since λ = 145 select the value from Table 2.4 (material of the column – S215)

$$\lambda_1 = 140 \qquad \varphi_1 = 0.36$$
$$\lambda_2 = 150 \qquad \varphi_2 = 0.32$$

$$\varphi_{145} = 0.36 - \frac{0.36 - 0.32}{150 - 140}(145 - 140) = 0.34$$

Calculate the permissible stresses for steel S215:

$$k = \frac{R_{0.2}}{n} = \frac{210}{1.045} = 230\ \text{MPa}$$

If the safety factor n is not available, use the values from Table D.2.

Calculate the permissible stresses:

$$k_{st} = \varphi \cdot k = 0.34 \cdot 230 = 78.2\ \text{MPa}$$

Calculate the applied stresses in the steel:

$$\sigma = \frac{F}{A} = \frac{160 \cdot 10^3}{26.8 \cdot 10^{-4}} = 59.7 \cdot 106\ \text{Pa}\ =\ 59.7\ \text{MPa}$$

Since the stresses in the column are less than the permissible stresses

$$59.7\ \text{MPa} < k = 78.2\ \text{MPa}$$

the stability of the column is ensured.

Calculate the critical force for the given column. Since the slenderness of column $\lambda = 145 > \lambda_{lim} = 100$ use Euler's formula to determine σ_{cr}:

$$F_{cr} = \sigma_{cr} \cdot A = \frac{\pi^2 E}{\lambda^2} \cdot A = \frac{3.14^2 \cdot 2.1 \cdot 10^{11}}{145^2} 26.8 \cdot 10^{-4} = 2.64 \cdot 10^5 = 264\ \text{kN}$$

The safety factor of the column:

$$k_{\text{st}} = \frac{F_{\text{cr}}}{F} = \frac{264}{160} = 1.65$$

Since $k_{\text{st}} = 1.65 \geq 1.5$, the stability of the column is ensured.

Conclusion: The stability of the column is ensured.

Example 2.18. Determine the permissible load [*F*] on a rod (Fig. 2.30) consisting of two angles (110 × 70 × 8) with $l = 3.4$ m; $k = 190$ MPa; $\delta = 12$ mm; material – S275 steel.

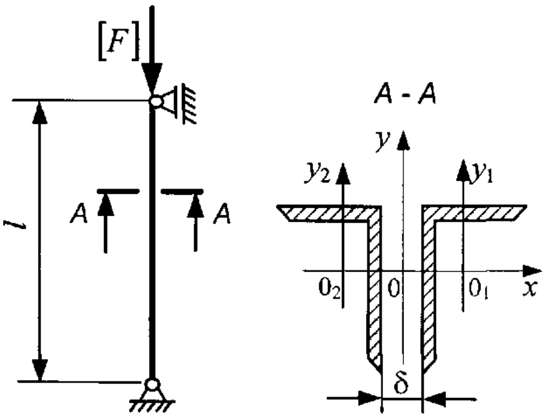


Fig. 2.30. Beam bar to Example 2.17

Solution

Using the stability condition for the permissible load [*F*], we have:

$$[F] = \varphi \cdot k \cdot A$$

To calculate φ, it is necessary to determine the slenderness λ, which in turn requires calculating the minimum radius of inertia for the given cross-section of the rod.

Using data from Table D.61 for a single angle 110 × 70 × 8 (mm):

$$J_{x_1} = 172\ \text{cm}^4;\ J_{y_1} = 54.6\ \text{cm}^4$$

$$A_1 = 13.9\ \text{cm}^2, x_0 = 1.64\ \text{cm}$$

Concerning the principal central axes x_0y *of* the cross-section, we have:

$$J_x = 2J_{x_1} = 2 \cdot 172 = 344\ \text{cm}^4$$

$$J_{\text{y}} = 2 \cdot \left[J_{\text{y}1} + \left(\text{x}_0 + \frac{\delta}{2}\right)\text{A}_1\right] = 2 \cdot [54.6 + (1.64 + 0.6) \cdot 2 \cdot 13.9] = 249\ \text{cm}^4$$

Since $J_y < J_x$, the minimum radius of inertia is:

$$i_{min} = i_y = \sqrt{\frac{J_{\text{y}}}{2A_1}} = \sqrt{\frac{249}{2 \cdot 13.9}} = 2.99\ \text{cm} = 2.99 \cdot 10^{-2}\text{m}$$

Slenderness of the rod:

$$\lambda = \frac{\mu l}{i_{min}} = \frac{1 \cdot 3.4}{2.99 \cdot 10^{-2}} = 114$$

Given that λ = 114 assigns the value from Table 2.10 (material of the column – steel S275).

$$\lambda_1 = 110;\ \varphi_1 = 0.43$$

$$\lambda_2 = 120;\ \varphi_2 = 0.37$$

$$\varphi_{114} = 0{,}43 - \frac{0.43 - 0.37}{120 - 110}(114 - 110) = 0.406$$

Permissible value of the force:

$$[F] = 0.406 \cdot 190 \cdot 10^6 \cdot 2 \cdot 13.9 \cdot 10^{-4} = 2144.5 \cdot 10^2 \text{ N} = 214.45 \text{ kN}$$

Example 2.19. For a column created of two channel sections (Fig. 2.31) select their dimensions and the distance δ using the stability condition of the column concerning the principal axes of the cross-section with:
F = 400 kN, l = 6.6 m, for material S235 R_e = 260 MPa, n = 1,18.

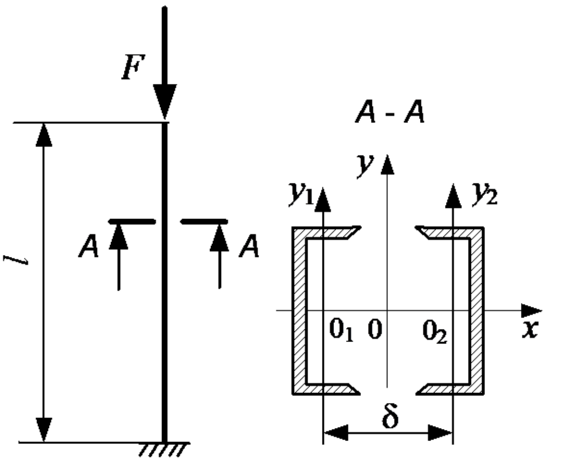


Fig. 2.31. Column scheme to Example 2.18

Solution

A peculiarity of the designed column is that concerning the principal axis x, the moment of inertia of the section depends only on the dimensions of the channel sections, whereas concerning the axis y, the moment of inertia depends on both the dimensions of the channel sections and the distance δ between them, i.e.

$$J_x = 2J_{x_1}, \quad J_y = 2\left(J_{y_1} + a^2 A\right),$$

where A – cross-sectional area of a single C-bar;

a = δ/2 – centroid coordinates for the given axis.

Then, from the stability condition of the column in the two principal planes, we formulate the equation: $Jx = Jy$ or $2J_{x_1} = 2\left(J_{y_1} + a^2 A\right)$, from which

$$a = \sqrt{\frac{J_{x_1} - J_{y_1}}{\mathrm{A}}}$$

Determine the permissible stress for S235 steel

$$k = \frac{R_e}{n} = \frac{210}{1.18} = 220 \text{ MPa}$$

If the safety factor n is not available, use the values from Table D.2.

Select the dimensions of the channel sections using an iterative approximation method. Assume an initial approximation of φ_1 = 0.5 and calculate the cross-sectional area of the column based on the stability condition in the y_0z plane, where the moment of inertia depends only on the type of C-bar:

$$A \geq \frac{F}{\varphi_1 \cdot k} \geq \frac{400 \cdot 10^3}{0.5 \cdot 220 \cdot 10^6} \geq 36.36 \cdot 10^{-4} \text{m}^2 = 36.36 \text{ cm}^2$$

Since the column consists of two C-bars, the calculated cross-sectional area of a single channel section should satisfy the condition:

$$A_1 \geq \frac{36.36}{2} \geq 18.18 \approx 18.2 \text{ cm}^2$$

From Table D.58, select the C-bar C 16 and list its characteristics:

$$A = 18.1 \text{ cm}^2, i_{min} = 6.42 \text{ cm}$$

Additionally, Comment that for a column consisting of two channel sections, the radius of inertia concerning the x-axis will be equal to the radius of inertia of a single channel section because:

$$i_x = \sqrt{\frac{2J_{x_1}}{2A}} = \sqrt{\frac{J_{x_1}}{A}} = 6.42 \text{ cm}$$

Calculate the slenderness of the column in the plane y_0z:

$$\lambda = \frac{\mu \cdot l}{i_x} = \frac{0{,}7 \cdot 6.6}{6.42 \cdot 10^{-2}} = 71.96 \approx 72$$

From Table 2.10, find the corrected value of the coefficient φ_1' (material steel S235):

$$\lambda = 70, \varphi = 0.81,$$
$$\lambda = 80, \varphi = 0.75$$

Then, for $\lambda = 72$ we have:

$$\varphi_1' = 0.81 - \frac{0.81 - 0.75}{10}(72 - 70) = 0.798$$

Since φ_1 and φ_1' differ significantly, perform calculations in the second approximation by assuming:

$$\varphi_2 = \frac{0.5 + 0.798}{2} = 0.65$$

Calculate the cross-sectional area of the column and the cross-sectional area of a single channel section:

$$A \geq \frac{400 \cdot 10^3}{0.65 \cdot 220 \cdot 10^6} \geq 27.97 \cdot 10^{-4} \text{ m}^2 = 28 \text{ cm}^2, \quad A_1 \geq \frac{28}{2} = 14 \text{ cm}^2$$

From Table D.58, select C-bar C 14 and take its characteristics:

$$A = 15.6 \text{ cm}^2, i_{min} = 5.6 \text{ cm}$$

Calculate the slenderness of the column:

$$\lambda = \frac{\mu \cdot l}{i_x} = \frac{0.7 \cdot 6.6}{5.6 \cdot 10^{-2}} = 82.5$$

Calculate the corrected values of the coefficient φ_2' (material - S235 steel):

$$\lambda = 80, \varphi = 0.75,$$
$$\lambda = 90, \varphi = 0.69$$

Then, for $\lambda = 82.5$ we have:

$$\varphi_2' = 0.75 - \frac{0.75 - 0.69}{10}(82.5 - 80) = 0.78$$

Since $\varphi_2' >> \varphi_2$, proceed to the third approximation:

$$\varphi_3 = \frac{0.6 + 0.78}{2} = 0.715$$

$$A \geq \frac{400 \cdot 10^3}{0.715 \cdot 220 \cdot 10^6} \geq 25.43 \cdot 10^{-4}\text{m}^2 = 25.43\ \text{cm}^2$$

$$A_1 \geq \frac{25.43}{2} = 12.7\text{cm}^2$$

From Table D.58, select C-bar C12, for which:

$$A = 13.3\ \text{cm}^2, i_{min} = 4.78\ \text{cm}$$

Calculate the slenderness of the column:

$$\lambda = \frac{\mu \cdot l}{i_x} = \frac{0.7 \cdot 6.6}{4.78 \cdot 10^{-2}} = 96.55$$

Calculate the corrected values of the coefficient φ_3' (material - steelS235):

$$\lambda = 90, \varphi = 0.69$$

$$\lambda = 100, \varphi = 0.60$$

Then, for λ = 96.55 we have:

$$\varphi_2' = 0.69 - \frac{0.69 - 0.60}{10}(96.55 - 90) = 0.68$$

Check the fulfilment of the strength condition in the third approximation. To do this, calculate the stresses in the rod and the permissible stresses:

$$k_{st}''' = \varphi_3' \cdot k = 0.68 \cdot 220 = 149\ \text{MPa}$$

$$\sigma = \frac{F}{2 \cdot A} = \frac{400 \cdot 10^3}{2 \cdot 13.3 \cdot 10^{-4}} = 150.4 \cdot 10^6 = 150.4\ \text{MPa}$$

By comparing σ and k_{st}''' we determine that the excess in the column is:

$$\eta = \frac{150.4 - 149.6}{149.6} \cdot 100\% = +0.53\% < [5\%]$$

It follows that the strength condition of the column in the y_0z plane will be satisfied if it consists of two C-bars C12.

After determining the dimensions of the channel sections in one plane, calculate the size of the spacing δ in the x_0z plane. From the Table D.58, select the necessary additional data:

$$J_{x_1} = 304\ \text{cm}^4; \qquad J_{y_1} = 31.2\ \text{cm}^4$$

We calculate the distance δ between channel sections of the column:

$$\delta = 2a = 2 \cdot \sqrt{\frac{J_{x_1} - J_{y_1}}{A}} = 2 \cdot \sqrt{\frac{304 - 31.2}{13.3}} = 9.06\ \text{cm}$$

Answer: According to the condition for equal column strength in two planes, the column should be constructed from two C-bars C12 spaced at a distance of δ = 9.06 cm.

Individual tasks
(calculations)

Task 2.10. Determine the dimensions of the compressed steel rod section and the distance δ based on strength requirements, by calculating the factor φ (Fig. 2.32, Table 2.11).

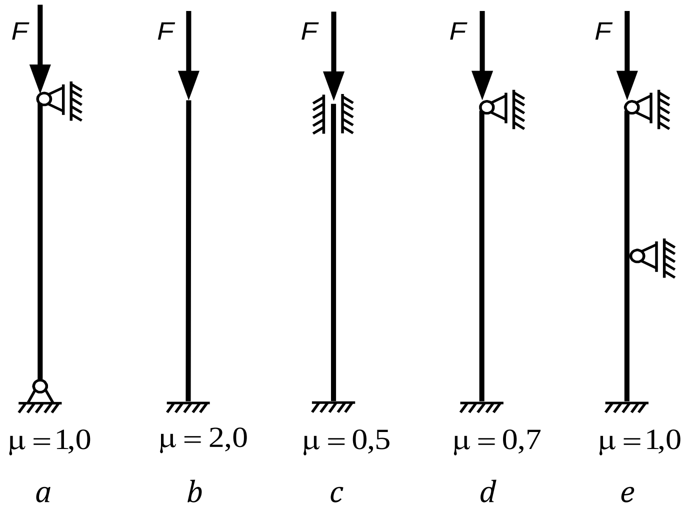


Fig. 2.32. Beam schemes to Task 2.10

Table 2.11. Initial data for Task 2.10

Var. no	Scheme	Fixing scheme μ (Fig. 2.32)	Compressive force F, kN	Length of the rod l, m	Steel
1	2	*a*	280	3.0	S215
2	8	*b*	300	4.8	S275
3	1	*c*	450	5.0	S215
4	3	*b*	350	5.5	S235
5	5	*c*	350	3.6	S235
6	9	*a*	400	5.0	S215
7	10	*d*	200	2.8	S275
8	4	*b*	350	3.0	S235
9	6	*c*	260	3.6	S215
10	11	*e*	400	6.5	S275
11	15	*b*	500	4.8	S215
12	7	*a*	360	2.8	S215
13	4	*c*	420	4.8	S275
14	15	*e*	280	3.0	S215
15	6	*d*	340	2.8	S235
16	13	*a*	380	4.8	S235
17	12	*b*	280	2.5	S215

Var. no	Scheme	Fixing scheme μ (Fig. 2.32)	Compressive force F, kN	Length of the rod l, m	Steel
18	9	*c*	450	2.8	S275
19	7	*d*	380	5.8	S215
20	1	*e*	360	4.0	S275
21	5	*d*	300	3.0	S235
22	15	*c*	550	3.8	S215
23	2	*s*	380	3.5	S235
24	5	*e*	380	2.8	S275
25	10	*b*	280	2.6	S215
26	3	*d*	350	3.5	S235
27	8	*a*	400	4.0	S215
28	14	*e*	340	4.8	S275
29	9	*c*	420	3.0	S235
30	1	*b*	400	3.6	S215
31	13	*d*	370	5.0	S235
32	8	*b*	430	2.8	S275
33	7	*e*	340	2.8	S275
34	12	*c*	360	3.5	S235
35	14	*a*	300	3.6	S215
36	3	*b*	440	3.0	S275
37	4	*e*	650	6.0	S215
38	15	*d*	380	2.5	S215
39	9	*a*	420	5.0	S235
40	5	*c*	360	3.8	S275

Schemes to Task 2.10

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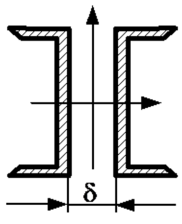

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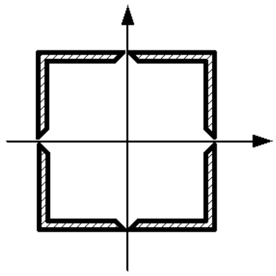

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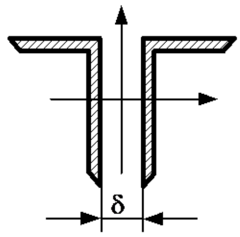

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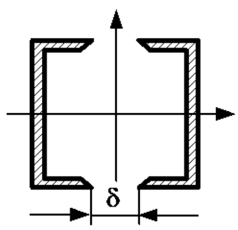

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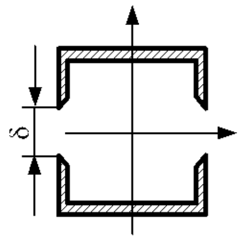

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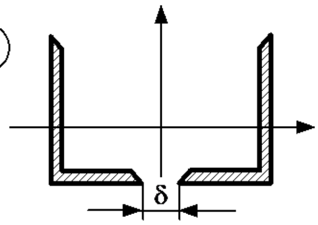

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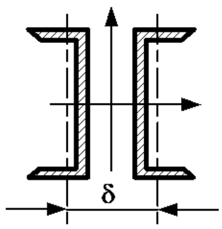

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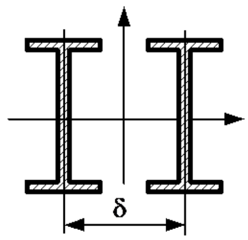

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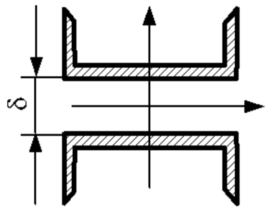

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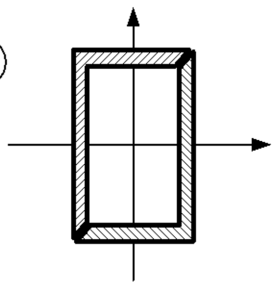

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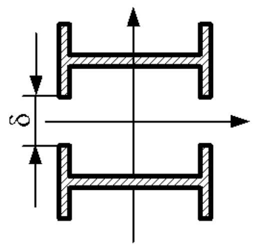

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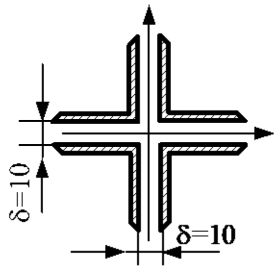

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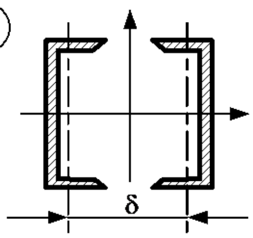

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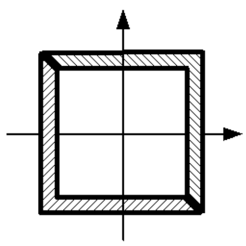

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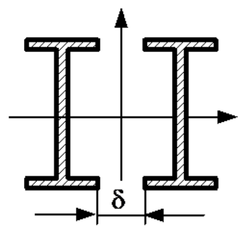


CHAPTER III
MACHINE PARTS

3.1. Calculation of welded joints

General information

Welded joints are permanent connections.

Welding - the process of obtaining a permanent joint through intermolecular interaction forces, as a result of either general or local heating or pressure.

Of the many different types of welding, electric arc welding, resistance welding, and gas welding are the most commonly used in mechanical engineering.

Types of welded joint

Welded joints - joints formed by welding.

Welded joints are **strong** and **tight**.

Depending on the construction (relative position of the parts to be joined), a distinction is made between **butt**, **corner**, **lap** and **overlap** joints.

The basic types of welded joints made by arc welding are standardised. Welded joints made by manual arc welding are governed by EN ISO 15614, butt and fillet welds can be used in welded joints.

Basic calculation formulae

The main criterion for weld performance is **strength**. The calculation of strength is based on the assumption that the stresses in the weld are distributed uniformly along the length as well as the cross-section. In general form, the strength condition for butt and fillet welds can be written as:

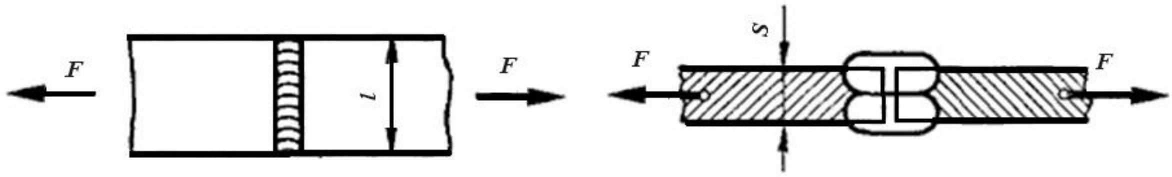
$$\sigma \leq k'_{r(c)}$$

$$\tau \leq k'_t$$

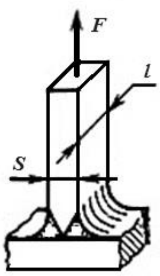
where $k'_{r(c)}$ – allowable tensile stress in the butt joint, MPa;

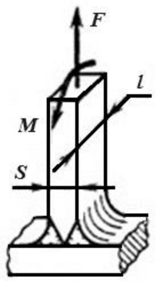
k'_t– allowable shear stress of fillet weld, MPa.

The butt welds (Fig. 3.1, a) are calculated in the cross-section of the parts to be joined without taking into account the weld thickness.



a





b *c*

Fig. 3.1 For the calculation of welds:
a - there are tensile forces in the butt welds;
b - in fillet welds with machined
tensile forces occur at the edges;
c - there are tensile forces and a bending moment in fillet welds

Bond tensile strength condition (compression) (Fig. 3.1, *a*, b)

$$\sigma = \frac{F}{sl} \leq k'_{r(c)},$$

where F – load, N;

s – thickness of welded elements, mm;

l – bond length, mm;

$k'_{r(c)}$ – permissible stresses for the butt weld in tension (compression), MPa.

Strength condition of a weld loaded simultaneously with a tensile force and a bending moment (Fig. 3.1, c)

$$\sigma = \frac{F}{sl} + \frac{M_u}{W_S} = \frac{F}{sl} + \frac{6M_u}{sl^2} \leq k_r{}'$$

Fillet welds are calculated in shear. The failure of fillet welds takes place at or near the smallest cross-section passing through the bisector of the right angle (Fig. 3.2).

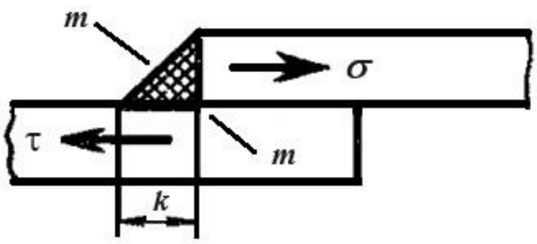
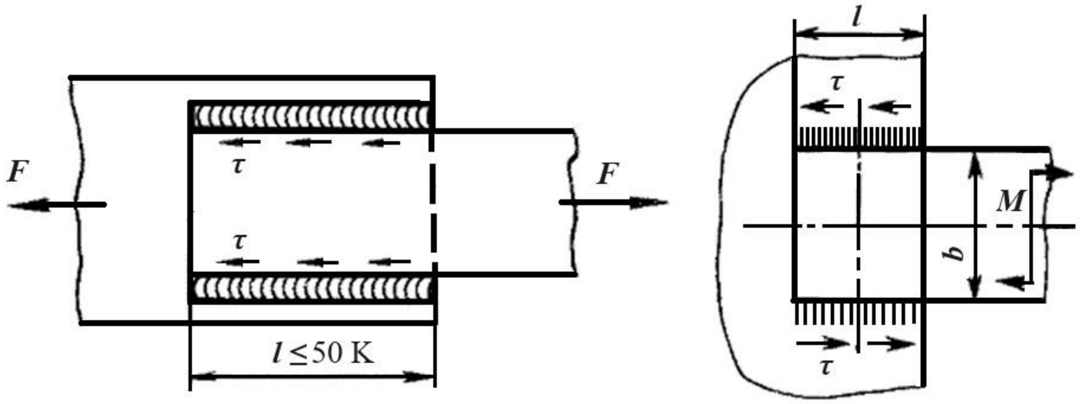


Fig. 3.2. Diagram of a fillet weld and the forces involved

Calculation of flange welds

a) loaded by axial force (Fig. 3.3, *a*)



a *b*

Fig. 3.3. Forces occurring in a flange weld:
a - loaded with axial force; *b* - loaded with axial force and bending moment

Strength condition

$$\tau = \frac{F}{2l\beta k} \le k_t'$$

or the permissible stress on the weld

$$F \le 2l\beta k k_t'$$

where β is a coefficient that characterises the depth of remelting;

k - weld root, mm. The weld fossa (k) is taken as the smaller fossa of the triangle inscribed in the weld cross-section. If *the thickness* of the parts to be welded is the same, the weld bead is **equal to the thickness of the parts to be joined**, $k = \delta$. When *the thickness* of the parts to be welded is *different*, the weld bead is equal to **the smallest thickness of the part**. For technological reasons, **the minimum value of the weld bead is 3 mm, the maximum is 20 mm**;

l - weld length, mm ($l_k \le 50 \div 60k$);

k_t' - allowable tangential stresses of fillet weld, MPa;

βkl - design weld *cross-section*, mm^2.

For multi-run automatic and semi-automatic welding and manual welding β = 0.7, for two- and three-leg semi-automatic welding β = 0.8, for automatic welding with the same parameters β = 0.9 and for single-leg automatic welding β = 1.1;

b) loaded with a bending moment (Fig. 3.3, *b*)

For relatively short welds ($l < b$), the strength condition,

$$\tau = \frac{M}{\beta klb} \le k_t',$$

where *b* – the width of the plate, mm.

Calculation of overlap welds

a) axially loaded (Fig. 3.4, a)

Strength condition

$$\tau = \frac{F}{\beta kl} \le k_t',$$

where *l* is the length of the weld, mm, if the weld is made from one side and *2l* if the weld is made from two sides.

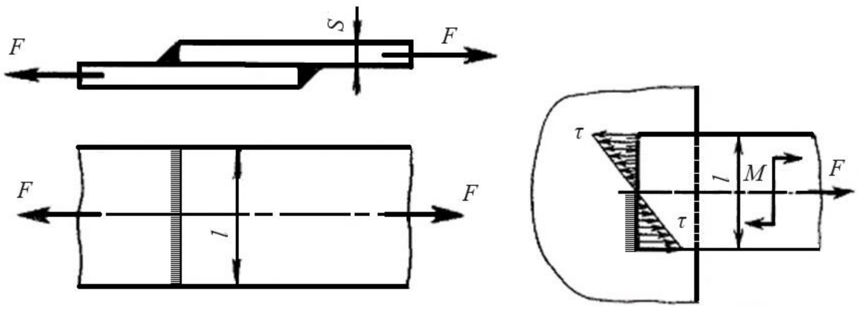


Fig. 3.4 Diagram of forces and moments in an overlap weld:
a - loaded with axial force; *b* - loaded with bending moment

b) loaded by a bending moment (Fig. 3.4, b - without *F*)

Strength condition

$$\tau = \frac{\mathrm{M}}{W_s} = \frac{6M}{\beta kl^2} \le k_t'$$

where $W_{oc} = \frac{\beta kh^2}{6}$ – section strength index of the seam.

c) loaded with axial force and bending moment (Fig. 3.4, b)

Strength condition

$$\tau = \frac{6M}{\beta kl^2} + \frac{F}{\beta kl} \le k_t'$$

Calculation of combination welds

Combination welds are used when a simple angle weld (butt, fillet, flange) does not provide the required weld strength.

Combination welds are calculated based on the principle of load distribution in proportion to the load capacity of the individual joints.

a) Calculation of the combined weld under axial force (Fig. 3.5, *a*)

Strength condition

$$\tau = \frac{F}{\beta k(2l_k + l_{cz})} \leq k_t'$$

b) calculation of the combined moment weld (Fig. 3.5, b – without *F*)

Strength condition

$$\tau = \frac{\mathrm{M}}{(\beta k l_k l_{cz} + \beta k l_k^2/6)} \leq k_t'$$

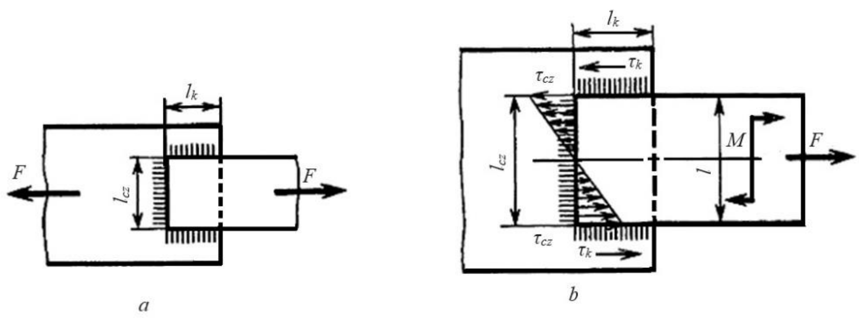


Fig. 3.5. Distribution of forces in combined welds:
a - loaded by an axial force;
b - loaded by bending moment

c) calculation of combined welds loaded by axial force and bending moment (Fig. 3.5, b)

Strength condition

$$\tau = \tau_M + \tau_F \leq k_t'$$

$$\tau_M = \frac{M}{(\beta k l_k l_{cz} + \beta k l_{cz}^2/6)}$$

$$\tau_F = \frac{F}{\beta k(2l_k + l_{cz})},$$

where l_k, l_{cz} – joint and butt joint lengths, mm.

When loading *non-symmetrical profiles*, for example, an angle iron (Fig. 3.6), the load passes through the centre of mass of the profile. When the welds are uniformly loaded, their length is inversely proportional to the distance of the weld from the line of incidence of the load.

So

$$\frac{l_1}{l_2} = \frac{a}{b}$$

Since the total length of the welds $L = l_1 + l_2$, then

$$l_1 = l\frac{a}{a+b};\ l_2 = l\frac{b}{a+b}$$

From the equation of statics, the load on the hinges follows:

$$F_1 = F\frac{a}{a+b};\ \ F_2 = F\frac{b}{a+b}$$

For an equilateral angular profile, it can be roughly assumed that F_1 = 0,7F and F_2 =0.3F, then from the strength condition the length of the welds is:

$$l_1 = \frac{F_1}{\beta k k_t'};\ l_2 = \frac{F_2}{\beta k k_t'}$$

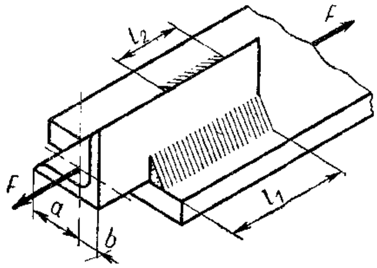


Fig. 3.6. Flange welds in asymmetric member joints loaded with axial force

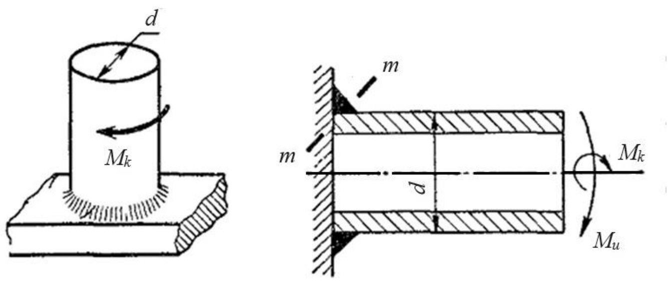
When the fillet weld is loaded with a torque (welded gears, pulleys, sprockets, couplings, drums, shafts, etc.), the strength condition will take the form of the following. (Fig. 3.7, *a*), the strength condition will take the form:

$$\tau = \frac{2T}{\beta k \pi d^2} \le k_s'$$

When the fillet weld is loaded with a torque (Fig. 3.7, *b*), and bending, the strength condition will take the form:

$$\tau = \sqrt{\tau_s^2 + \tau_g^2} \le k_t',$$

where $\tau_s = \frac{2T}{\beta k \pi d^2}$; $\tau_g = \frac{M_u}{W_p} \approx \frac{4M_4}{\beta k \pi d^2}$.



a *b*

Fig. 3.7. Calculation scheme of a fillet weld:
a - load of the flange weld in torque;
b - load of the flange weld in torsion and bending moment

The strength condition for spot welded joints made by arc welding (Fig. 3.8) will take the form:

$$\tau = \frac{4F}{\pi d^2 zi} \leq k_t'$$

where z is the number of welding points; and i is the number of shear planes.

For the structure in Fig. 3.8, a – $z = 4$, $i = 1$; in Fig. 3.8, b – $z = 2$, $i = 2$.

Welded point diameter:

$d = 1.2s + 4$ mm at $s < 3$ mm;

$d = 1.5s + 5$ mm at $s > 3$ mm.

The distance between the edges of t_1 and t_2 is normalized taking into account technological and energy factors. They usually take

$$t = 3d;\ t_1 = 2d;\ t_2 = 1.5d$$

A point connection is characterised by a high-stress concentration. Therefore, it does not perform very well under varying loads. Stress concentrations arise not only at the weld points but also in the parts themselves in the weld zone.

Spot-welded joints are often used not as working joints that carry the main load, but as bonding joints.

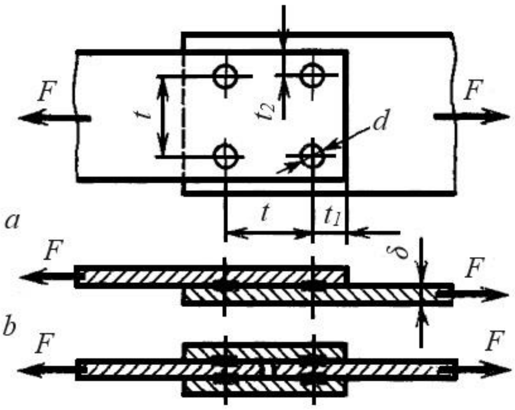


Fig. 3.8. Diagram of spot welding calculations:
a - overlapping;
b - with translation

For *continuous contact welding* (Fig. 3.9), the strength condition will take the form:

$$\tau = \frac{F}{bl} \leq k_t'$$

where b – width of welded joint, mm.

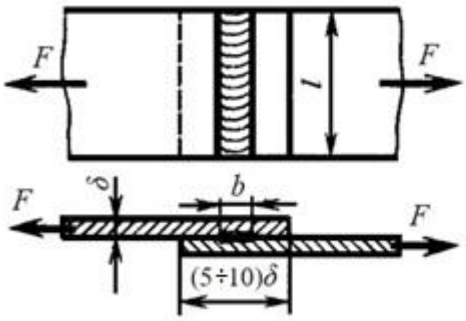


Fig. 3.9. Calculation scheme of continuous contact welding

The permissible stresses depend on the type of welding, type of electrode, type of weld, the material of the workpieces and the nature of the load. The permissible stresses under static loading are selected from Table D.3.

Examples of calculations

Example 3.1. Check the strength of a butt weld (Fig. 3.1, a) made with an electrode type E34, E42, E42A, with a constant tensile force acting on it F = 65 kN, width of strands b =100 mm, thickness s =5, of strands S215 steel with k_r=160 MPa.

Data:	*Searched for:*
Electrode type E34, E42, E42A F = 65 kN b = 100 mm s = 5 mm material – S215 steel constant loading	σ - ?

Solution

1. Determine the permissible stresses for the weld, taking into account that the load acting on the joint induces tensile stresses in the weld, from Table D.3 for S215 steel and the given electrode types:

$$E34:\ k_r' = 0{,}75k_r = 0.75 \cdot 160 = 120 \text{ MPa};$$
$$E42: k_r' = 0.9k_r = 0.9 \cdot 160 = 144 \text{ MPa};$$
$$E42A:\ k_r' = k_r = 160 \text{ MPa}.$$

2. Check the strength condition

We assume b = l = 100 mm.

$$\sigma = \frac{F}{s \cdot l} = \frac{65 \cdot 10^3}{5 \cdot 100} = 130 \frac{\text{N}}{\text{mm}^2} = 130 \text{ MPa}$$

The calculated values are compared with the permissible:

E34: 130 MPa > k_r' = 120 MPa – condition is not met;

E42: 130 MPa < k_r'= 144 MPa – condition is met;

E42A: 130 MPa < k_r'= 160 MPa – condition is met.

Conclusion: The strength condition is fulfilled for connections formed via electrode types E42 and E42A.

Example 3.2. Check the strength of a lap butt weld (Fig. 3.10), made by manual arc welding with an E50 electrode. The axial force F = 40 kN, the weld was made on one side, plate thickness s_1 = 5 mm; s_2 =10 mm; material of plates - steel S215 with k_r = 160 MPa; plate width b_1 = 100 mm; b_2 = 400 mm.

Data:
Electrode type E50
F = 40 kN
b_1 = 100 mm
b_2 = 400 mm
s_1 = 5 mm
s_2 = 10 mm
material–S215 steel
k_r = 160 MPa
constant loading

Searched for:
τ –?

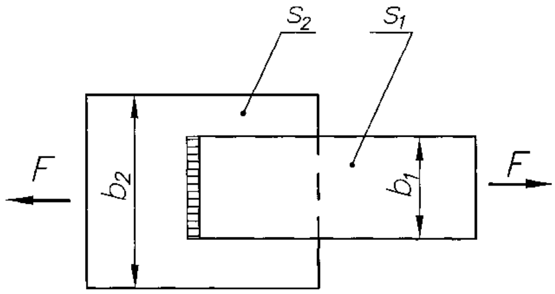


Fig. 3.10. Diagram for the calculation of the butt weld to Example 3.2

Solution

1. Determine the permissible stresses for the weld.

Taking into account that the load acting in the weld induces tensile stresses, from Table D.2 for S215 steel we derive the following.

$$k_t' = 0{,}6k_c = 0.6 \cdot 160 = 96 \text{ MPa}$$

2. Check the weld strength condition

Take the weld bead equal to the smallest thickness of the plate $k = s_1 = 5$ mm; we take the weld length $l = b_1$ =100 mm; for manual hatch welding, the remelting factor β = 0.7.

$$\tau = \frac{F}{\beta kl} = \frac{40 \cdot 10^3}{0.7 \cdot 5 \cdot 100} = 114 \ \frac{\text{N}}{\text{mm}^2} = 114 \text{ MPa} > k_t' = 96 \text{ MPa}$$

the condition is not met.

Conclusion: the weld will not ensure the strength of the welded joint.

Example 3.3. Check the strength of the butt weld (Fig. 3.7, *b*), made without edge treatment, performed by manual arc welding with E42 electrodes. The joint is loaded with torque T = 1500 Nm, the load is constant, pipe diameter d = 273 mm, wall thickness s = 7 mm and pipe material C10 steel.

Data:
Electrode type E42
T = 1500 Nm
d = 273 mm
s = 7 mm
material – C10 steel

Searched for:
τ –?

R_e = 220 MPa
constant loading

Solution

1. Determine the permissible stress for the weld.

In the case of a butt weld made without edge treatment, shear stresses occur under torque.

For C10 steel, according to Table D.2, we take σ_r = 210 MPa and determine the allowable stress. Taking n = 1.5 (Comment to Table D.3), we calculate the

$$k_r = \frac{R_e}{n} = \frac{210}{1.5} = 140 \text{ MPa}$$

From Table D.3 $k_t^{'} = 0.6k_r = 0.6 \cdot 140 = 84 \text{ MPa}$.

2. Check the strength of the weld.

Take the angular length of the weld equal to the thickness of the pipe wall.

$$k = s = 7 \text{ mm}$$

Take the angular length of the weld equal to the thickness of the pipe wall β = 0.7;

Moment in Nm is converted to Nmm

$$T = 1500 \text{ Nm} = 1500 \cdot 10^3 \text{ Nmm}$$

$$\tau = \frac{2T}{\beta k \pi d^2} = \frac{2 \cdot 1500 \cdot 10^3}{0.7 \cdot 7 \cdot 3.14 \cdot 273^2} = \frac{\text{N}}{\text{mm}^2} = 2.6 \text{ MPa} < k_t^{'} = 84 \text{ MPa}$$

the condition is met.

Conclusion: the weld will ensure the strength of the welded joint.

Example 3.4. Determine the permissible load that the lap joint can withstand (Fig. 3.4, *a*), made by manual arc welding with E42 electrode, S215 steel strip material with k_r = 160 MPa, weld made on both sides, plate width b = 100 mm, weld made on both sides, plate width s_1 = 6 mm, s_2 = 8 mm. Continuous load.

Data:	*Searched for:*
Electrode type E42	[F] - ?
b = 100 mm	
plate thickness	
s_1 = 6 mm	
s_2 = 8 mm	

k_r =160 MPa
material - S215 steel

Solution

1. Determine the allowable stress for the weld.

Taking into account that the acting load in the weld causes shear stresses using Table D.3 for S215 steel we determine:

$$k_t' = 0{,}6k_r = 0.6 \cdot 160 = 96 \text{ MPa}$$

2. Determine the permissible stresses.

Determine the permissible stresses: $k = s_1 = 6$ mm; the length of the weld is given by $l = 2b = 200$ mm; for manual electrode arc welding, the remelting factor is $\beta = 0.7$.

From the strength condition $\tau = \frac{F}{\beta kl} \leq k_t'$

$$[F] \leq k_t' \beta kl = 0.7 \cdot 96 \cdot 6 \cdot 200 = 80640 \text{ N}$$

Answer: $[F] \leq 80640$ N .

Example 3.5. Calculate the length of the combined weld (Fig. 3.5, a) overlapped by manual arc welding with E42 electrodes, with constant load $F = 78$ kN, plate thickness $s_1 = 5$ mm, $s_2 = 10$ mm, steel strip material S15 with k_r =160 MPa, plate width $b_1 = 100$ mm, $b_2 = 150$ mm.

Data:
Electrode type E42
F=78 kN
$b_1 = 100$ mm
$b_2 = 150$ mm
$s_1 = 5$ mm
$s_2 = 10$ mm
material – steel S215
kr =160 MPa
constant loading
$\beta = 0.7$

Searched for:
l_{gen} - ?

Solution

1. Determine the permissible stresses for the weld.

Taking into account that the load acting in the weld causes shear stresses, from Table D.3 for S215 steel

$$k_t' = 0.6k_r = 0.6 \cdot 160 = 96 \text{ MPa}$$

2. From the strength condition we determine the length of the weld.

We predetermine the angular length of the weld equal to the smaller of the two sheet thicknesses, i.e. $k = s_1 = 5$ and for manual arc welding, we assume a remelting factor of.

From the strength condition $\tau = \frac{F}{\beta k l} \le k_t'$, we have

$$l_{gen} \ge \frac{F}{\beta k k_t^{'}} = \frac{78 \cdot 10^3}{0.7 \cdot 5 \cdot 96} = 232 \text{ mm}$$

Answer: $l_{gen} \ge 232$ mm.

Example 3.6. Determine the length of the weld joint at the angle bar 75 × 75 × 8 (Fig. 3.11). Alternating axial tensile load F = 138 kN, cycle characteristic R = −1. Manual arc welding with E50A. Angle and bevel material S215 steel with k_r = 160 MPa.

Data:
Angle 74 × 75 × 8
Electrode type E50A
F = 65 kN
R = -1
material - S215 steel
k_r = 160 MPa
variable loading

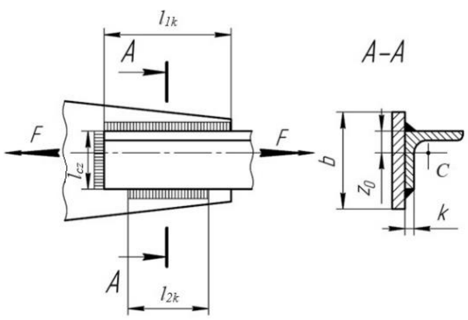


Fig. 3.11. Calculation scheme to Example 3.6

Searched for:
l_k - ? l_{cz} - ?

Solution

1. To reduce the length of the overlap between the angle and the haunch, we use a combined corner weld with a normal section.

From Table D.60, we extract the distance of the centre of gravity to the edge from z_0 = 21.5 mm.

2. Determine the permissible stresses for the weld.

From Table D.3 for angle welds under alternating load (Table D.3)

$$k_t^{'} = \gamma 0.65 k_r$$

The coefficient taking into account the effect of variable load is determined by the formula:

$$\gamma = \frac{1}{(0.6K_{ef} + 0.2) - (0.6K_{ef} - 0.2)R}$$

From table D.4 K_{ef} =3.5 (less favourable option), then

$$\gamma = \frac{1}{(0,6 \cdot 3.5 + 0,2) - (0.6 \cdot 3.5 - 0.2) \cdot (-1)} = 0.23$$

So

$$k_t^{'} = \gamma 0.65 k_r = 0.23 \cdot 0.65 \cdot 160 = 24 \text{ MPa}$$

3. From the strength condition we determine the design length of all welds.

The weld bead thickness is taken to be equal to the thickness of the angle side

$$k = s = 8 \text{ mm}$$

For manual electric arc welding, the remelting coefficient

$$\beta = 0.7$$

From the strength condition

$$\tau = \frac{F}{\beta k l} \leq k_t^{'} \text{ we have } l_{gen} \geq \frac{F}{\beta k k_t^{'}} = \frac{65 \cdot 10^3}{0.7 \cdot 8 \cdot 24} = 484 \text{ mm}$$

4. Determine the dimensions of the welds:

a) assume a butt weld length equal to the width of the angle bracket

$$l_{cz} = b = 75 \text{ mm}$$

b) the length of the side weld (using the lever principle)

$$l_b = l_{gen} - l_{cz} = 484 - 75 = 409 \text{ mm}$$

c) the length of the side weld (using the lever principle)

$$l_{k2} = l_{k1} \frac{z_0}{b} = 409 \cdot \frac{21.6}{75} = 117.25 \text{ mm}$$

then

$$l_{b1} = l_b - l_{b2} = 409 - 117.25 = 291.75 \text{ mm}$$

Given the poor quality of the weld at the end and beginning, we attribute:

$$l_{b2} = 130 \text{ mm}; \; l_{b1} = 310 \text{ mm} < l_{bmax} = 50 \div 60k = 400 \div 480 \text{ mm}$$

The corner profile is often welded completely along the fitting contour.

Answer: l_{cz} = 75 mm; l_{k2} = 130 mm; l_{k1} = 310 mm.

Example 3.7. Calculate bracket and weld (Fig. 3.5, b) F = 10 kN, M = 8 kNm, static load, plate thickness s = 12 mm. Sheet material S215 steel. Welding - manual with electrode E42.

Data:
Electrode load E42
F = 10 kN
M = 8 kNm
material - S215 steel
static load

Searched for:
b - ? l_b - ? l_{cz} - ?

Solution

1. From table D.3 we assume for S215 steel k_r = 160 MPa.

2. Considering only the basic load (bending moment), we determine the width of the cantilever from the strength condition. Convert the moment values Nm into Nmm, hence M = 8 kNm = 8·10⁶ Nmm.

From the strength condition $\sigma = \frac{M}{W_{oc}} = \frac{6M}{sb^2} \le k_r$ we have

$$b \ge \sqrt{\frac{6M}{sk_r}} = \sqrt{\frac{6 \cdot 8 \cdot 10^6}{12 \cdot 160}} = 158 \text{ mm}$$

Taking into account the F load, we assume b =165 mm.

3. Check strength under total load

$$\sigma = \frac{6M}{sb^2} + \frac{F}{sb} = \frac{6 \cdot 8 \cdot 10^6}{12 \cdot 165^2} + \frac{10^4}{12 \cdot 165} \approx 152 \ \frac{\text{N}}{\text{mm}^2} \ = 152 \text{ MPa} < k_r = 160 \text{ MPa}$$

The strength condition is met.

4. Determine the permissible stresses for the weld.

From Table D.3

$$k_t^{'} = 0.6k_r = 0.6 \cdot 160 = 96 \text{ MPa}$$

5. Determine the dimensions of the weld

Accept $l_{cz} = b = 165$ mm, $k = s = 12$ mm.

Based on the strength conditions, we determine the length of the butt weld in advance, only according to the main load.

$$l_{cz} = \frac{6\text{M} - \beta k l_{cz}^2 k_t^{'}}{6\beta k l_{cz} k_t^{'}} = \frac{6 \cdot 8 \cdot 10^6 - 0{,}7 \cdot 12 \cdot 165^2 \cdot 96}{6 \cdot 0.7 \cdot 12 \cdot 165 \cdot 96} = 33 \text{ mm}$$

Given the poor quality of the weld at the end and the beginning, we ultimately assume a side weld length of l_b = 50 mm.

6. Check the strength of welds after total load.

$$\tau_F = \frac{F}{\beta k(2l_k + l_{cz})} = \frac{10^4}{0{,}7 \cdot 12(2 \cdot 50 + 165)} = 4.5 \text{ N/mm}^2 \ = \ 4.5 \text{ MPa}$$

$$\tau_M = \frac{\text{M}}{(\beta k l_b l_{cz} + \beta k l_{cz}^2/6)} = \frac{8 \cdot 10^6}{(0.7 \cdot 12 \cdot 50 \cdot 165 + 0.7 \cdot 12 \cdot 165^2/6)} \approx 75 \text{ N/mm}^2 \ = 75 \text{ MPa}$$

$$\tau = \tau_F + \tau_M = 4.5 + 75 = 80 \text{ MPa} < \ k_t^{'} = 96 \text{ MPa}$$

Strength condition is met.

Answer: b= 165 mm; l_b = 50 mm; l_{cz} = 165 mm.

Example 3.8. Calculate a spot welded joint (Fig. 3.8, *a*). Calculate a spot welded joint (R = -0.5), F = 3 kN, plate thickness s = 3 mm, material – C10 steel (R_{-1} = 160 MPa).

Data:
Resistance spot welding
F = 3 kN
R = -0.5
material – C10 steel
R_{-1} = 160 MPa
variable loading

Searched for:
Welded joint - ?

Solution

1. Determine the permissible stresses for the plate.

Assuming n = 1.5 (Table D.3) we calculate

$$k_r = \frac{R_{-1}}{n} = \frac{160}{1.5} = 107\ \text{MPa}$$

2. Determine the coefficient, taking into account the variable load. From Table D.4 K_{ef} =7.5

$$\gamma = \frac{1}{(0.6K_{ef} + 0.2) - (0.6K_{ef} - 0.2)R} = \frac{1}{(0.6 \cdot 7.5 + 0.2) - (0{,}6 \cdot 7.5 - 0.2)(-0.5)} = 0.146$$

3. Determine the permissible stresses for the plate

$$k = \gamma k_r = 0.146 \cdot 107 = 15.6\ \text{MPa}$$

4. From the tensile strength condition we determine the width of the plate

$$b = \frac{F}{sk} = \frac{3 \cdot 10^3}{3 \cdot 15.6} = 64\ \text{mm}$$

We assume b = 65 mm.

5. Determine the dimensions of the joint:

a) the diameter of a point $d = 1.2s + 4 = 1.2 \cdot 3 + 4 = 7.6\ \text{mm}$.

We assume d = 8 mm;

b) step $t = 3d = 3 \cdot 8$ mm; distance between edges

$$t_1 = 2d = 2 \cdot 8 = 16\ \text{mm};\ t_2 = 1.5d = 1.5 \cdot 8 = 12\ \text{mm};$$

c) the number of points from the strength condition.

Determine in advance the permissible stresses for the welding points taking into account the effect of the alternating load from Table D.3 we have

$$k_t^{'} = \gamma \cdot 0.6k_r = 0.146 \cdot 0.6 \cdot 107 = 9.4\ \text{MPa}$$

Take the number of points in two rows i = 1

$$z = \frac{4F}{\pi d^2 k_t^{'} \cdot i} = \frac{4 \cdot 3 \cdot 10^3}{3.14 \cdot 8^2 9.4 \cdot 1} = 6.35$$

Take the number of points in two rows z = 8.

6. Finally, we determine the width of the plate

$$b = 3t + 2t_1 = 3 \cdot 24 + 2 \cdot 16 = 104\ \text{mm}$$

We assume b = 105 mm.

Answer: b= 65 mm; t = 24 mm; t_1 = 16 mm; t_2 = 12 mm; z = 8.

Individual tasks

(calculation)

Task 3.1. Check the strength of the weld (Fig. 3.5, a), which is subjected to a tensile force, the weld was made on one side. The data to calculations is shown in Table 3.1.

Table 3.1. Initial data for Task 3.1

Var. no	Force, kN	Joint length, mm	Sheet thickness, mm		Top plate width, mm	Electrode	Steel
	F	l	s_1	s_2	b		
1	70	100	5	6	80	E50A	C10
2	60	120	6	8	100	E42	S215
3	50	115	6	6	120	E42A	09G2S
4	80	125	5	6	80	E50A	C10
5	90	135	8	6	85	E42	S215
6	113	155	8	8	125	E42A	09G2S
7	143	145	8	10	142	E50A	C10
8	135	165	6	8	150	E42	S215
9	132	185	8	6	130	E42A	09G2S
10	128	135	8	8	135	E50A	C10
11	151	140	8	10	160	E42	S215
12	154	125	6	8	155	E42A	09G2S
13	130	135	8	6	165	E50A	C10
14	140	100	8	8	180	E42	S215
15	150	120	8	10	170	E42A	09G2S
16	160	115	6	8	200	E50A	C10
17	135	125	8	6	185	E42	S215
18	125	135	8	8	125	E42A	09G2S
19	140	155	8	10	120	E50A	C10
20	165	145	10	8	185	E42	S215
21	155	165	8	8	210	E42A	09G2S
22	156	185	8	6	190	E50A	C10
23	174	135	8	8	145	E42	S215
24	185	140	8	10	135	E42A	09G2S
25	166	125	10	8	165	E50A	C10
26	138	135	8	8	155	E42	S215
27	144	100	8	6	145	E42A	09G2S
28	153	120	6	8	200	E50A	C10
29	164	115	8	6	160	E42	S215
30	136	125	6	8	180	E42A	09G2S

Task 3.2. Determine the length of the welds that connect the T-bar to the base (Fig. 3.11). The data to calculations is shown in Table 3.2.

Table 3.2. Initial data for Task 3.2

Var. no	Load, kN F	T-bar size, mm $b \times b \times s$	Electrode	Material	Load character
1	70	45 × 45 × 3	E50A	C10	Variables (R = -0.5)
2	60	50 × 50 × 4	E42	S215	Constant
3	50	56 × 56 × 5	E42A	09G2S	Variables (R = -0.6)
4	80	63 × 63 × 6	E50A	C10	Constant
5	90	56 × 36 × 3.5	E42	S215	Variables (R = -0.7)
6	113	63 × 63 × 6	E42A	09G2S	Constant
7	143	70 × 70 × 5	E50A	C10	Variables (R = -0.55)
8	135	80 × 80 × 6	E42	S215	Constant
9	132	75 × 75 × 7	E42A	09G2S	Variables (R = -0.58)
10	128	63 × 63 × 4	E50A	C10	Constant
11	151	80 × 80 × 5.5	E42	S215	Variables (R = -0.85)
12	154	90 × 90 × 6	E42A	09G2S	Constant
13	130	63 × 40 × 4	E50A	C10	Variables (R = -0.8)
14	140	70 × 45 × 4.5	E42	S215	Constant
15	150	75 × 50 × 5	E42A	09G2S	Variables (R = -0.9)
16	160	90 × 56 × 6	E50A	C10	Constant
17	135	50 × 50 × 4	E42	S215	Variables (R = -1)
18	125	56 × 56 × 5	E42A	09G2S	Constant
19	140	63 × 63 × 6	E50A	C10	Variables (R = -0.95)
20	165	56 × 36 × 3.5	E42	S215	Constant
21	155	63 × 63 × 6	E42A	09G2S	Variables (R = -0.78)
22	156	70 × 70 × 5	E50A	Stal C10	Constant
23	174	50 × 50 × 4	E42	S215	Variables (R = -0.85)
24	185	56 × 56 × 5	E42A	09G2S	Constant
25	166	63 × 63 × 6	E50A	C10	Variables (R = -0.75)
26	138	56 × 36 × 3.5	E42	S215	Constant
27	144	63 × 63 × 6	E42A	09G2S	Variables (R = -0.6)
28	153	70 × 70 × 5	E50A	C10	Constant
29	164	80 × 80 × 5.5	E42	S215	Variables (R = -0.7)
30	136	90 × 90 × 6	E42A	09G2S	Constant

Task 3.3. Calculate the point welded joint (Fig. 3.8a). The data to the calculations is shown in Table 3.3.

Table 3.3. Initial data for Task 3.3

Var. no	Load, kN	Sheet thickness, mm	Sheet material	Load character
	F	s		
1	7	3	C10	Constant
2	6	4	S215	Variables (R = -0.5)
3	5	5	09G2S	Constant
4	8	6	C10	Variables (R = -0.4)
5	9	3	S215	Constant
6	6.3	4	09G2S	Variables (R = -0.3)
7	4.3	5	C10	Constant
8	3.5	6	S215	Variables (R = -0.2)
9	3.2	3	09G2S	Constant
10	2.8	4	C10	Variables (R = -0.6)
11	5.1	5	S215	Constant
12	5.4	6	09G2S	Variables (R = -0.7)
13	3	3	C10	Constant
14	4	4	S215	Variables (R = -0.8)
15	5	5	09G2S	Constant
16	6	6	C10	Variables (R = -0.9)
17	3.5	3	S215	Constant
18	2.5	4	09G2S	Variables (R = -1)
19	4	5	C10	Constant
20	6	6	S215	Variables (R = -0.75)
21	5	3	09G2S	Constant
22	6	4	C10	Variables (R = -0.85)
23	7	5	S215	Constant
24	5	6	09G2S	Variables (R = -0.65)
25	6	3	C10	Constant
26	3.8	4	S215	Variables (R = -0.55)
27	4	5	09G2S	Constant
28	3	6	C10	Variables (R = -0.5)
29	4	3	S215	Constant
30	6	4	09G2S	Variables (R = -0.95)

3.2. Calculation of threaded connections

General information

Threaded joints are detachable connections made using threads directly applied to the parts to be joined or threaded fasteners such as bolts, screws, pins and nuts.

Basic thread parameters

The basic parameters of the thread (Fig. 3.12) are:

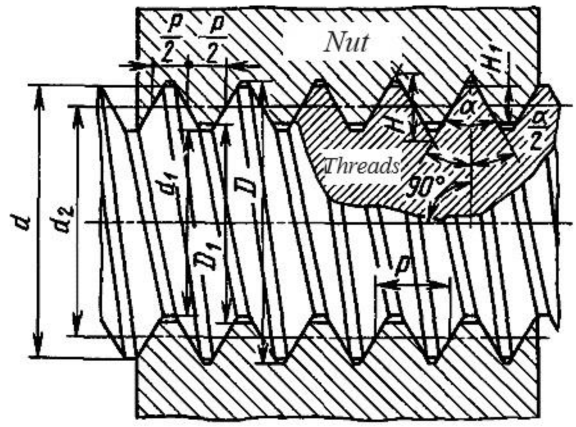


Fig. 3.12. Basic thread parameters

Thread diameter (screw and nut): **external** - nominal thread diameter *d, D;* **central** d_2, D_2, i.e. the diameter of the imaginary cylinder, the base of which intersects the thread at the point where the width of the projection is equal to the only groove (if the value is not given in the table, it can be determined according to the formula: $d_2 = \frac{(d+d_1)}{2}$); **internal** d_1 , D_1 . The diameter of the screw, as the closing part, was indicated by lower case letters, the diameter of the screw, as the closing part, by upper case letters.

The most important feature of a thread is the *thread pitch p* (*t, S*) – the distance between two adjacent thread turns measured parallel to the axis of the screw.

The profile of a thread is the profile of the projection and furrow in the plane of its central section.

Profile angle α - the angle between adjacent sides of a thread in axial section.

The thread profile is also characterised by:

a) *the height of the initial triangle of thread H,* i.e. the triangle whose vertices are formed by the points of intersection of the extended thread profiles;

b) the *working height of* the *thread profile* $H_{1(h)}$ - along which the thread sides of the bolt and nut meet;

Thread pitch $Ph(S_1)$ *- the* distance between two adjacent thread turns measured parallel to the axis of the bolt, or otherwise is the axial displacement after one revolution of the bolt *t* (Fig. 3.12):

for a single thread $S_1 = S$,

for multi-pass $S_1 = zS$, where *z is the* number of thread turns.

The angle of elevation of the thread line φ - the angle of elevation of the thread line after the average diameter.

$$tg\varphi = \frac{p}{\pi d_2} \text{ or } \varphi = arctg\frac{p}{\pi d_2},$$

where φ - is the helix angle in degrees.

These parameters can be considered in a general way, as all profiles have common elements and can be achieved by changing the profile angle, profile height and radius of curvature. For example, by decreasing the profile angle, one can go from a triangular thread to a trapezoidal thread and then to a rectangular thread. Threads, due to having gaps, cannot be used as centring elements.

All geomteric thread parameters and their tolerances are standardised.

Strength classes of threaded fasteners

Steel bolts and screws according to EN ISO 898-2:2023-03 are manufactured in **12** strength **classes** 3.6, 4.6, 4.8, 5.6, 5.8, 6.6, 6.8, 6.9, 8.8, 10.9, 12.9, 14.9 (in order of increasing strength). The strength class is indicated as two numbers separated by a dot. The first number multiplied by 100 indicates the minimum strength limit (MPa) and the first number multiplied by the second and still multiplied by 10 indicates the yield strength limit (MPa). The strength class of bolt 5.6 is read as follows: the bolt material has a strength limit of $5 \cdot 100 = 500$ MPa and a yield strength limit of $5 \cdot 6 \cdot 10 = 300$ MPa. Each strength class corresponds to a specific steel grade, for example, for strength class 3.6 the corresponding steels are S215, C10, etc.

Nuts according to EN ISO 898-2:2023-03 are produced in **7** strength **classes** 4, 5, 6, 8, 10, 12, and 14 (in order of increasing strength). The number multiplied by 100 indicates the maximum load value.

Basic calculation formulae

The value of the peripheral driving force (Fig. 3.13)

$$F_t = Ftg(\varphi + \rho),$$

where F – axial force on the bolt, N;

$\varphi = arctg\frac{p}{\pi d_2}$ – thread angle of elevation, degrees;

$\rho = arctg\frac{f}{\cos\alpha/2}$ – thread friction angle, degrees;

P - thread pitch, mm;

α - thread pitch, mm;

f – friction coefficient;

d_2 – thread centre diameter, mm.

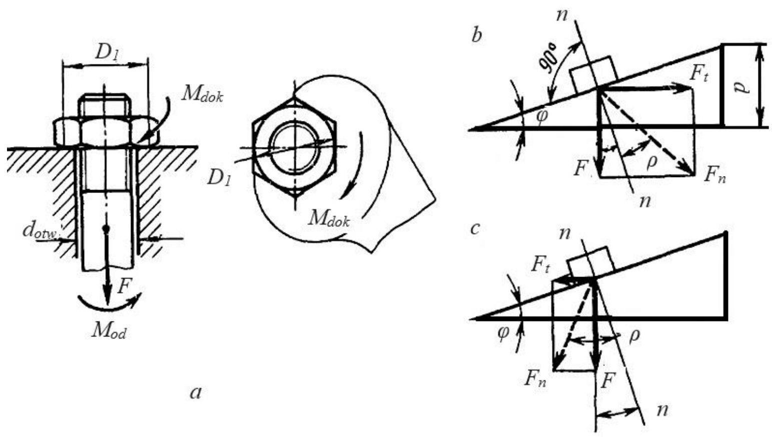


Fig. 3.13. Interaction forces between bolt and nut

Tightening torque for bolt or nut (Fig. 3.13, a, b)

$$M_{dok} = M_{Tg} + M_{Tn},$$

where M_{Tg} – thread friction torque, Nm:

$$M_{Tg} = F_t\frac{d_2}{2} = Ftg(\varphi + \rho)\frac{d_2}{2}$$

M_{Tn} – friction torque at the supporting end of the nut or bolt, Nm:

$$M_T = Ff\frac{D_{av}}{2}$$

D_{cen} = $(D_1+d_h)/2$ or – D_{cen} =1.4d – the central diameter of the screw's retaining surface (thread); d_h – the diameter of the screw hole.

the diameter of the screw hole M_p and M_T we obtain

$$M_{dok} = F\frac{d_2}{2}\left[\frac{D_{cen}}{d_2}f + tg(\varphi + \rho)\right]$$

Torque to loosen bolt or nut (Fig. 3.13, *c*)

$$M_{od} = F\frac{d_2}{2}\left[\frac{D_{cen}}{d_2}f + tg(\varphi - \rho)\right]$$

Thread calculations

The main types of thread failure: are fastening *threads* - **thread shear**, and movable **threads - thread wear**. Because of this, the main performance and calculation criteria for fastening threads are the strength associated with shear stresses, and for movable threads, the wear resistance associated with compressive stresses (Fig. 3.14).

Strength conditions for threads with shear stress

for screws $\tau_c = \frac{F}{(\pi d_1 HKK_m)} \le k_t$,

for nuts $\tau_c = \frac{F}{(\pi dHKK_m)} \le k_t$,

where F – force;

H – the height of the screw or the depth of the screw into the component;

$K = ab/p$ or $K = ce/p$ – coefficient of thread completeness; for a triangular thread K=0.87, for a rectangular thread K = 0.5, for a rectangular thread $K \approx 0.65$; K_m =0.55 ÷ 0.75 – coefficient of non-uniformity of load along the thread turns (higher value for large metric threads and provided the bolt material is stronger than the nut material); *a*, *b*, c, *e*, *p* - correction factors;

k_t – allowable shear stresses $k_t = 0.4R_e$ – constant load;

k_t = (0.2 ÷ 0.3)R_m – variable load.

If the materials of the bolt and the nut are the same then the shear stresses are calculated for the bolt thread only.

Condition for wear resistance of the running thread under compressive stresses:

$$\sigma_c = \frac{F}{(\pi d_2 hz)} \le k_c,$$

where d_2 – centre diameter of the thread, mm;

h – centre diameter of thread, mm;

$z = \frac{H}{p}$ – number of working turns of the screw or nut;

k_c – permissible compressive stress for the lower strength part of the threaded pair. Assumes $k_c = (0.3 \div 0.4)R_m$.

The formula is the same for a bolt and the nut. The coefficient K_T is assumed equal to unity, taking into account thread lapping.

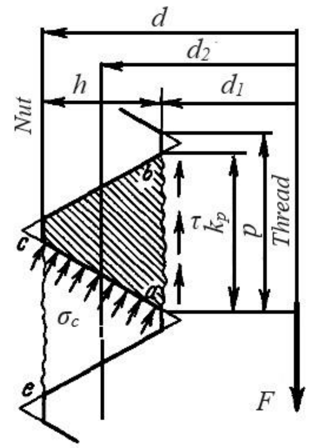


Fig. 3.14. Force diagram for calculating thread strength and wear resistance

Performance of a threaded connection without taking into account frictional forces at the end of the nut or bolt

$$\eta_{p.g} = \frac{A_p}{A_3} = \frac{tg\varphi}{tg(\varphi + \rho)}$$

Bolt performance including friction at the nut end or thread end

$$\eta_{p.g} = \frac{A_p}{A_3} = \frac{tg\varphi}{tg(\varphi + \rho) + \frac{f_{av}}{d_2}}$$

Strength calculation of threaded connections under different types of loading

The main performance criterion for threaded connections is strength. All standard bolts, screws and studs are made *to have equal tensile strength of the bar after threading, thread shear and head detachment* (Fig. 3.15), so calculations of the strength of a threaded connection are usually carried out against only one performance criterion - the *strength of the threaded part of the bar, taking into account the internal thread diameter d_1.*

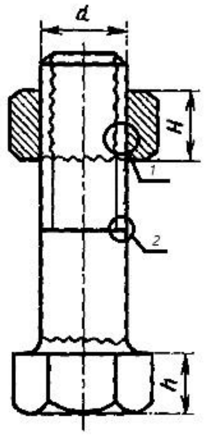


Fig. 3.15. Areas of possible damage to fixing connections

The length of the bolt, dowel or the height of the nut is taken according to the thickness of the parts to be joined. The other dimensions of the components of the threaded connection (nut, washer, etc.) are assumed to depend on the thread diameter according to the standard.

Head shear strength (Fig. 3.15)

$$\tau_c = \frac{F}{\pi d h} \leq k_t,$$

where h – bolt head height, mm.

Calculation of a bolt loaded by an axial tensile force *F*. The nut is screwed but not tightened. The bolt is not tightened.

This case is rare. An example is the bolted connection of the bracket, block and hook end section of crane mechanisms (Fig. 3.16). The calculation boils down to determining the internal thread diameter d_1 from the tensile strength condition

$$\sigma_r = \frac{4F}{\pi d_1^2} \leq k_r$$

where

$$d_1 = \sqrt{\frac{4F}{\pi k_r}}$$

where F – current force, N;

d_1 – internal thread diameter, mm;

k_r – allowable tensile stresses, MPa; $k_r = 0.6R_m$ without tightening screws.

The resulting value of the inner diameter d_1 o is rounded up to the largest standardised value, to which the value of the outer diameter is matched.

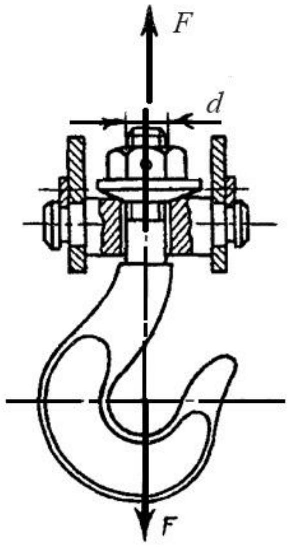


Fig. 3.16. Diagram of forces in a thread loaded with an axial force

Calculation of a thread loaded axially and by torque. An example is a threaded band during tightening (Fig. 3.17)

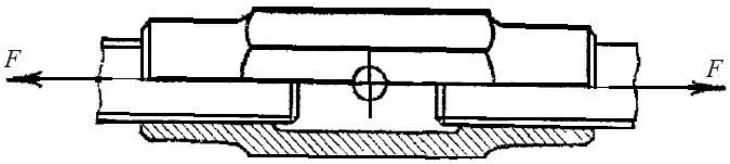


Fig. 3.17. Forces in a threaded connection

in this case, the strength condition becomes

$$\sigma_{red} = \frac{1{,}3 \cdot 4F}{\pi d_1^2} \leq k_r$$

then

$$d_1 = \sqrt{\frac{5.2F}{\pi k_r}}$$

where 1.3 – is a factor that takes into account the torsional stresses in the thread due to friction in the thread;

$k_r = \frac{R_m}{[n]}$– allowable tensile stresses, MPa;

$[n]$ – safety factor.

Calculation of a welded joint loaded with critical shear stresses

A prerequisite for the reliability of the connection is that there is no displacement of the components at the connection point. Two cases can be considered:

The bolt is tightened without a gap (Fig. 3.18). In this case, the bolt is driven into a calibrated hole with a reamer, and the bolt core is made to a tolerance that allows a gap-free fit.

The reliability (immobilisation of the parts to be joined) of the joint is ensured by the bolt core. The calculations refer to the shear and crushing of the core. Frictional forces are not included in the calculation, as tightening is not mandatory. In the general case, the bolt can be replaced by a pin of the two types of stresses, shear stresses are the most dangerous, so only shear stress calculations are usually carried out.

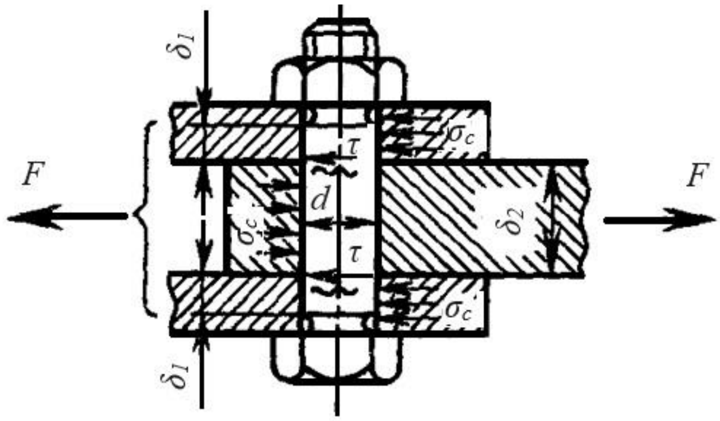


Fig. 3.18. Diagram of the calculation of bolts placed in a hole without a slot

Shear strength condition

$$\tau_c = \frac{4F}{\pi d^2 zi} \leq z$$

then

$$d = \sqrt{\frac{4F}{\pi k_t zi}},$$

where i – number of shear planes; i = n - 1, where n – number of joined elements;

z – number of connected bolts;

k_t – allowable shear stresses, MPa.

The resulting value is rounded up to the larger normalized value.

Compressive strength condition

general formula

$$\sigma_c = \frac{F}{d \cdot z \cdot \delta_{min} k_c}$$

then $d = \frac{F}{k_g \delta_{min}}$

for the central element (Fig. 3.18)

$$\tau_s = \frac{F}{(d \cdot \delta_2)z} \leq k_c$$

$$\text{then } d = \frac{F}{k_c \delta_2 z}$$

for the outermost element

$$\sigma_c = \frac{F}{(2d\delta_1 z)} \leq k_c$$

then

$$d = \frac{F}{2 \cdot k_c \delta_1 z}$$

where k_c – permissible compressive stress of lower strength material, MPa;

δ_{min} – minimum thickness of joined parts, mm;

δ_1, δ_2 – thickness of joined parts, mm.

The disadvantage of such connections is their high cost due to the complexity of the production technology (precise marking, positioning and accuracy of bolt production).

The bolt is tightened with a gap (Fig. 3.19). In this assembly, reliability is provided by the frictional forces resulting from the tightening of the bolt, but it should not be subjected to external load. If the bolt is subjected to an external load in this assembly, reliability is compromised and such an assembly is not valid.

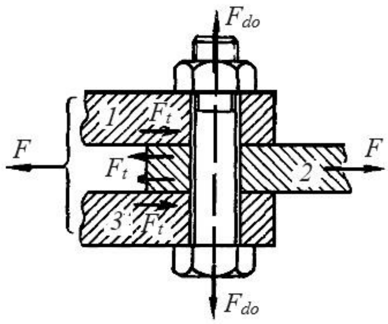


Fig. 3.19. Diagram for calculating bolts in clearance holes

In joints with a gap external loads do not act on the bolt. Therefore, the bolt is only calculated for static strength against a tightening force, even if the external load is variable. The effect of a variable load is calculated by selecting increased values for the safety factor.

The no displacement condition can be written as:

$$F \leq i \cdot F_t = i \cdot F_{do} \cdot f$$

where i – number of shear planes (at Fig. 3.19 i = 2, when two elements are joined i=1);

F_t – frictional force between connected parts due to tightening, N;

f – coefficient of friction at the joint (f ≈ 0.15 ÷ 0.20 for steel and cast iron dry surfaces);

F_{do} – screw tightening force, N;

$$F_{do} = \frac{KF}{zif},$$

where K – coefficient of adhesion reserve (K = 1.3 ÷ 1.5 under static load, K = 1.8 ÷ 2 under variable load);

z – number of bolts in the assembly.

The strength condition will take the form

$$\sigma_{red} = \frac{1{,}3 \cdot 4F_{do}}{\pi d_1^2} \leq k_r$$

then

$$d_1 = \sqrt{\frac{5{,}2F_{do}}{\pi k_r}}$$

When comparing the cases of placing bolts with and without a gap, it is worth noting that the first case is cheaper, as it does not require the accuracy of the bolt and hole dimensions. However, the working conditions of the bolt, placed with a gap, are worse than without. The design load on the bolt with a gap is 5 ÷ 7.5 higher than the external stress. In addition, as a result of the instability of the friction coefficient and the complex control of tightening, the operation of such connections at offsets is insufficiently stable.

Calculation of pre-tightened joints when assembled and loaded with external tensile force

This case is often found in mechanical engineering for the attachment of gearbox covers, tanks, cylinders, bearings (Fig. 3.20) etc. Here, two cases are also considered.

There is no additional tightening of the bolt, so the design load is

$$F_{calc} = [1{,}3K(1-\chi) + \chi]F,$$

where χ – external load factor characterising the susceptibility of the joint components (χ = 0.2 ÷ 0.3 without seals; χ = 0.4 ÷ 0.5 with seals).

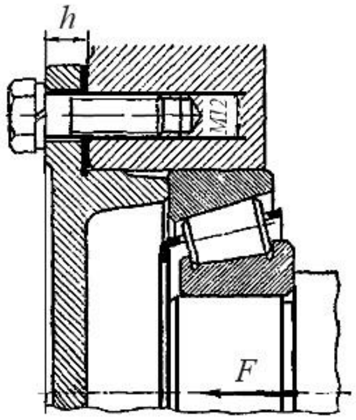


Fig. 3.20. Strength distribution of the tightened connection

It is possible to tighten the bolt additionally under full external load, in which case the design load is

$$F_{calc} = 1{,}3F[K(1-\chi)+\chi],$$

Strength condition

$$\sigma_{red} = \frac{4F_{ob}}{\pi d_1^2 z} \le k_r,$$

then

$$d_1 = \sqrt{\frac{4F_{ob}}{\pi k_r z}},$$

where i is the number of bolts in the assembly.

Calculation of a torque-loaded joint (couplings, complex gears, etc.). This case is similar to the transverse force loading case, the connected components are displaced by a circumferential force (Fig. 3.21). Here, two cases are also considered (bolts placed without a gap and with a gap).

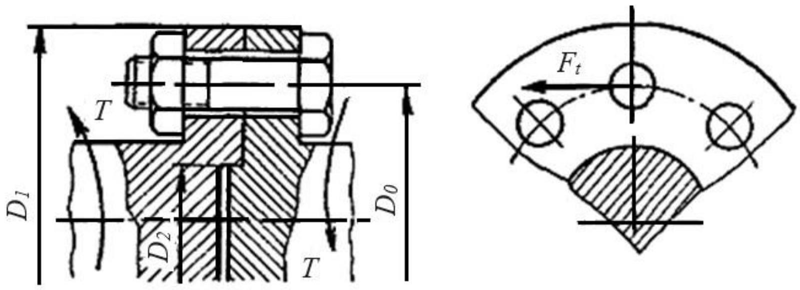


Fig. 3.21. Diagram of a torque loaded threaded connection

The peripheral force will be

$$F_t = \frac{2T}{D_0},$$

where T – torque, Nm;

D_0 – diameter of bolt axis, m.

The peripheral force F_t is replaced by F in the formulas for connections loaded transverse force.

Calculation of assemblies loaded by centrifugal force

Eccentric loading on the bolt occurs due to the non-parallelism of the bearing surfaces of the parts to be joined and the nut or the head of the screw, for example, due to the inclination of the channel flange (Fig. 3.22, a), errors in the manufacture of bolts, nuts, the use of slotted-head screws (Fig. 3.22, b) etc. In all cases, there are bending stresses in the bolt core in addition to tensile stresses.

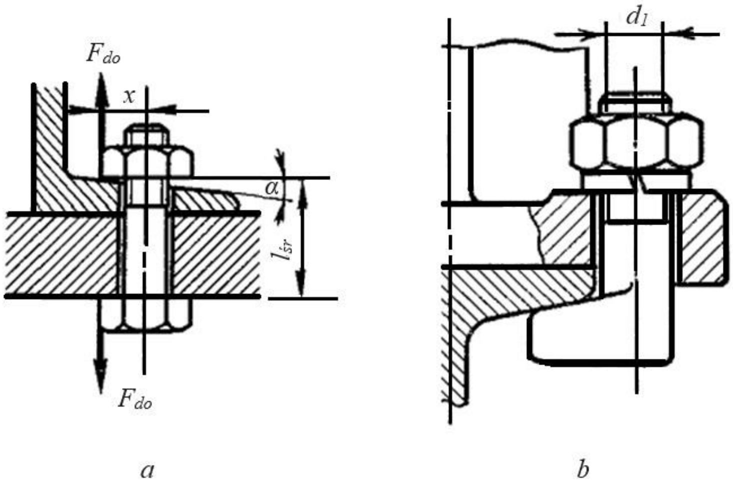


Fig. 3.22. Loading of the joint with centrifugal force

The strength condition will take the form

$$\sigma_{red} = \sigma_r + \tau_s = \frac{4F_{ob}}{\pi d_1} + \frac{32F_{calc}\mathrm{x}}{\pi d_1^3} = \frac{4F}{\pi d_1^2}\left(1 + \frac{8x}{d_1}\right) \le k_r$$

where x – eccentricity value, mm.

The value of the calculated load F_{calc} is determined according to the formulae for pre-tightened connections when assembled before the external load is applied.

If x = 0.5 d, the thread diameter can be determined

$$d_1 = 2{,}24\sqrt{\frac{4F_{calc}}{\pi k_r z}},$$

where z – is the number of bolts in connection.

Eccentric loading requires an increase in bolt diameter and a reduction in connection strength. In the design and manufacture of the connection construction, it is necessary to avoid eccentric loading or to take measures to reduce eccentric loading (planning the bearing surfaces of nuts and screw heads, bolts and the use of standard bevel washers).

Recommended values for permissible stresses, safety factors and dimensions for metric threads are given in D.7 ÷ D.9 in the appendix.

Examples of calculations

Example 3.9. Determine the diameter of the cut end of the hook (Fig. 2.5) if: acting pulsating alternating load F = 10 T; hook material - C35 steel; nut is screwed on but not tightened.

Data:	*Searched for:*
F= 10 T	d - ?
material – C35 steel	
pulsating variable load	

Solution

1. Determine the permissible stresses.

Tables D.2 and D.7 for C35 steel taking into account the effect of a pulsating load k_r = 125 MPa.

2. From the strength condition we calculate the internal diameter of the thread:

$$d_1 = \sqrt{\frac{4F}{\pi k_r}} = \sqrt{\frac{4 \cdot 100 \cdot 10^3}{3.14 \cdot 125}} = 31.9 \text{ mm}$$

Choose a metric thread that can withstand high loads and has high friction. According to Table D.8, we take the nearest larger value of the internal diameter d = 37.129 mm with a step of p = 4.5 mm, the external diameter of the thread d = M42.

Answer: d= M42.

Example 3.10. From the strength condition, determine the diameter of bolts in a threaded connection loaded with a variable transverse force F = 20 kN. Number of bolts $z = 2$, number of elements in assembly $n = 3$, bolt strength class 4.8, bolt tightening is uncontrolled. In the first case, the bolts are placed without a gap (Fig. 3.18), and in the second case with a gap (Fig. 3.19).

Data:	*Searched for*:
F = 20 kN	d - ?
strength class 4.8	
z = 2	

n = 3
uncontrolled tightening
case 1 – without gap
case 2 – with gap

Solution

Cas 1 – bolts without gap (Fig. 3.18).

In a shear-loaded assembly, strength is provided by the bolt core.

1. Determine the allowable stresses.

For bolts of strength class 4.8, the yield strength is $R_e = 4 \cdot 8 \cdot 10 = 320$ MPa, so from Table D.7 we will determine the allowable shear stress from the formula:

$$k_c = (0.2 \div 0.3)R_e = (0.2 \div 0.3) \cdot 320 = 64 \div 96 \text{ MPa}$$

Accept $k_c = 64$ MPa.

2. From the strength condition we determine the diameter of the bolt.

Several shear planes $i = n - 1 = 3 - 1 = 2$.

$$d = \sqrt{\frac{4F}{\pi k_c z i}} = \sqrt{\frac{4 \cdot 2 \cdot 10^3}{3.14 \cdot 64 \cdot 2 \cdot 2}} \approx 10 \text{ mm}$$

From Table D.53, we adopt a bolt with increased accuracy for mounting from under the reamer d_1 = 11 mm, at the end of which the thread d = M10 is arranged.

Case 2 – bolts with gap (Fig. 3.19).

When a bolt is positioned with a slot, the immobility of the assembly is determined by the frictional forces generated when the bolts are tightened. The bolts are subjected to combined loads (tension and torsion), so calculations are based on the determination of equivalent stresses.

3. Determine the allowable stresses.

Taking into account alternating stress and uncontrolled tightening, and assuming that the bolt diameter will be in the range M16 ÷ M30 we assume [n] = 6.5 (Table D.58).

$$k_r = \frac{R_e}{[n]} = \frac{320}{6.5} = 49 \text{ MPa}$$

4. Determine the tightening force of the screw.

Steel-steel friction coefficient $f \approx 0.17$; adhesion reserve coefficient under alternating load K = 1.8.

$$F_{do} = \frac{KF}{zif} = \frac{1.8 \cdot 20 \cdot 10^3}{2 \cdot 2 \cdot 0.17} = 52941 \text{ N}$$

5. From the strength condition we determine the internal diameter of the thread

$$d_1 = \sqrt{\frac{5{,}2F_{do}}{\pi k_r}} = \sqrt{\frac{5.2 \cdot 52941}{3.14 \cdot 49}} = 42.29 \text{ mm}$$

From Table D.8, we take the nearest larger value $d_1 = 42.587$ mm, for which d = M48.

Answer: case 1: d_1 = 11mm; case 2: d = M48.

Example 3.11. Determine the diameter of the bolts of the flange coupling (Fig. 3.21). Variable load, torque moment T = 1 kNm, bolts of strength class 5.6, uncontrolled tightening, number of bolts z = 4, bolt axis core diameter D_0 =200 mm. In the first case, the bolts are set without a gap and in the second case with a gap.

Data:	*Searched for*:
T = 1 kNm	d – ?
strength class 5.6	
z = 4	
D_0 = 200 mm	
uncontrolled tightening	
case 1 – without gap	
case 2 – with gap	

Solution

Case1 – bolts without gap

1. Determine the allowable stresses

For bolts of strength class 5.6, the yield strength is

$$R_e = 5 \cdot 6 \cdot 10 = 3000 \text{ MPa},$$

so from Table D.7 we determine the allowable shear stress from the ratio

$$k_t = (0.2 \div 0.3)R_e = (0.2 \div 0.3) \cdot 300 = 60 \div 90 \text{ MPa}$$

We assume k_t =60 MPa.

2. Determine the peripherical force acting on the assembly:

1000 Nm = 10^6 Nmm,

$$F_t = \frac{2\text{T}}{D_0} = \frac{2 \cdot 10^6}{200} = 10000 \text{ N}$$

3. From the strength condition we determine the diameter of the bolt

Bumble of shear planes $i = n - 1 = 2 - 1 = 1.$

$$d = \sqrt{\frac{4F_t}{\pi \tau_{av} z i}} = \sqrt{\frac{4 \cdot 10^4}{3.14 \cdot 60 \cdot 4 \cdot 1}} \approx 7.3 \text{ mm}$$

From Table D.53, we adopt a bolt with increased accuracy for mounting from under the reamer d_1 = 9 mm at the end of which the thread d = M8 is arranged.

Case 2 – bolts with a gap (Fig. 3.21)

1. Determine the allowable stresses

Taking into account variable stress and uncontrolled tightening, and assuming that the bolt diameter will be in the range M16 ÷ M30, we assume [n] = 6.5 (Table D.58).

$$k_r = \frac{R_e}{[n]} = \frac{300}{6.5} = 46 \text{ MPa}$$

2. Determine the bolt tightening force.

Steel-steel friction coefficient $f \approx 0.17$; adhesion reserve coefficient under alternating load K = 1.8.

$$F_{do} = \frac{KF_t}{zif} = \frac{1.8 \cdot 10^4}{4 \cdot 1 \cdot 0.17} = 26471 \text{ N}$$

3. From the strength condition we determine the internal diameter of the thread

$$d_1 = \sqrt{\frac{5{,}2F_{do}}{\pi k_r}} = \sqrt{\frac{5{,}2 \cdot 26471}{3{,}14 \cdot 46}} = 30{,}87 \text{ mm}$$

From Table D.8, we take the nearest higher value of d_1 = 31.670 mm, for which d = M36.

This case demonstrates the desirability of installing bolts in target couplings without a gap.

Answer: case 1: d_1 = 9 mm; case 2: d_1 = M36.

Example 3.12. Determine the number of bolts in an assembly, loaded with a constant transverse tension F = 50 kN. The bolts are assembled with a gap (Fig. 3.19), number of fasteners n = 3, bolt diameter d = M24, uncontrolled tightening, bolt material C10 steel.

Data:	*Searched for:*
F = 50 kN	z - ?
d = M24	
n = 3	

constant load
uncontrolled tightening

Solution

1. Determine the allowable stresses.

From Table D.1 for C10 steel R_e = 210 MPa.

From Table D.6 considering constant load and uncontrolled tightening for carbon steels in the size range M16 ÷ M30 we assume $[n]$ = 3, then the allowable stresses

$$k_r = \frac{R_e}{[n]} = \frac{210}{3} = 70 \text{ MPa}$$

2. From the strength condition we specify the number of bolts

Steel-steel friction coefficient $f \approx 0.17$; coefficient of adhesion under constant load K = 1.3; number of shear planes $i = n - 1 = 3 - 1 = 2$. From the Table D.9 for bolt M24 d_1 = 20.752 mm.

$$z = \frac{5{,}2KF}{if\pi d_1^2 k_r} = \frac{5.2 \cdot 1.3 \cdot 50 \cdot 10^3}{2 \cdot 0.17 \cdot 3.14 \cdot 20.752^2 \cdot 70} = 10.5$$

We assume the number of bolts at the assembly z = 12.

Answer: z = 12.

Example 3.13. What is the maximum load that a screw connection loaded with a constant transverse force can withstand, where bolts are installed without gap (Fig. 3.18). Number of bolts z = 4, bolt diameter d = 17 mm, material of bolt – C35 steel (R_e = 320 MPa), number of elements in connection n = 2.

Data:
d = 17 mm
z = 4
material – C35 steel
R_e = 320 MPa
n = 2
constant load

Searched for:
F - ?

Solution

1. Calculate the permissible stresses.

With static load from the Table D.7

$$k_t = 0.4R_e = 0.4 \cdot 320 = 128 \text{ MPa}$$

2. From the strength condition we determine the permissible load

$$F \le \frac{k_t \pi d_1^2 z i}{4} = \frac{128 \cdot 3.14 \cdot 17^2 \cdot 4 \cdot 1}{4} = 116155 \text{ N}$$

Answer: F = 116155 N.

Example 3.14. Determine the diameter of the bearing connection shield bolts (Fig. 3.20), subjected to a constant axial load of F = 12.5 kN. A number of bolts z = 6, material – C35 steel (R_e = 320 MPa), bolts were installed with a gap and tightened before the load was applied. Consider two cases: case 1 – without tightening of bolts under load; case 2 – with tightening of bolts under load.

Data:
F = 12.5 kN
z = 6
material – C35 steel
R_e = 320 MPa
constant load

Searched for:
d -?

Solution

1. Determine the allowable stresses

From Table D.6 with consideration of constant load and uncontrolled tightening for carbon steels in the size range M6 ÷ M16 we assume [n] = 4, then the allowable stresses

$$k_r = \frac{R_e}{[n]} = \frac{320}{4} = 80 \text{ MPa}$$

2. Determine the design force for case 1 – without tightening the bolts under load.

We take into account the soft gasket and assume χ = 0.4, the adhesion reserve factor under constant load is assumed to be K = 1.3.

$$F_{calc} = [1.3K(1-\chi)+\chi]F = [1.3 \cdot 1.3(1-0.4)+0.4] \cdot 12.5 \cdot 10^3 = 17675 \text{ N}.$$

3. From the strength condition determine the inner diameter of the screw thread

$$d_1 = \sqrt{\frac{4F_{calc}}{\pi k_r z}} = \sqrt{\frac{4 \cdot 17675}{3.14 \cdot 80 \cdot 6}} = 6.85 \text{ mm}$$

From Table D.8, we take the nearest larger value of d_1 = 8.376 mm, which corresponds to the outside diameter of the M10 thread.

4. Determine the design force for case 2 – with bolts tightened under load

$$F_{calc} = 1.3F[K(1-\chi)+\chi] = 1.3 \cdot 12.5 \cdot 10^3[1.3(1-0.4)+0.4] = 19175 \text{ N}$$

5. From the strength condition we determine the internal diameter of the screw thread

$$d_1 = \sqrt{\frac{4F_{calc}}{\pi k_r z}} = \sqrt{\frac{4 \cdot 19175}{3.14 \cdot 80 \cdot 6}} = 7.13 \text{ mm}$$

From Table D.8, we take the nearest larger value d_1 = 8.376 mm, which corresponds to the outside diameter of the M10 thread. The bolt diameter is within the range M6 ÷ M16 for which a safety factor has been determined.

Answer: for cases 1 and 2 d = M10.

Example 3.15. Determine the force that must be applied to the spanner when turning the nut (Fig. 3.13) for the bolt to reach its yield point R_e = 210 MPa (C10 steel). Perform the calculation for an M24 bolt. Assume l = 15d, for the handle length of the spanner, friction coefficient in the thread at the end of the nut f = 0.15.

Data:	*Searched for:*
d = M24	F_k - ?
material - C10 steel	
R_e = 210 MPa	
f = 0.15	
l = 15d	

Solution

1. From Table D.8 take the necessary dimensions for the calculation: d = 24 mm; d_1 = 20.752 mm; d_2 = 22.051 mm; p = 3 mm, we determine the angle of elevation of the thread according to the formula

$$\varphi = arctg\frac{p}{\pi d_2} = arctg\frac{3}{3.14 \cdot 22.051} = 2°30'$$

2. From the strength condition, determine the tightening force for the bolt at which there is a stress in the core equal to the yield strength

$$F_{do} = \frac{\pi d_1^2 R_e}{5.2} = \frac{3.14 \cdot 20.752^2 \cdot 210}{5.2} = 54625 \text{ N}$$

3. Determine the tightening torque applied to the nut

Before that, we determine the friction angle

$$\rho = arctg\frac{f}{cos\,\alpha} = arctg\frac{0{,}15}{cos\,60°/2} = 9°50'$$

$$M_{zak} = F_{do}\frac{d_2}{2}\left[\frac{D_{av}}{d_2}f + tg(\varphi + \rho)\right] =$$

$$= 54625\frac{22.051}{2}\left[\frac{33.6}{22.051}\cdot 0.15 + tg(2°30' + 9°50')\right] = 258975\ \text{Nmm} \approx 259\ \text{Nm}$$

4. Determine the force to be applied

$$F_k = \frac{\mathrm{M}_{zak}}{l} = 258975/15{\cdot}24 = 719.4\ \text{N}$$

Yield in strength $\frac{F_{do}}{F_k} = \frac{54625}{719.4} \approx 76$ times

Answer: F_k = 719.4 N.

Individual task
(calculation)

Task 3.4. Determine the force to be applied to the spanner when turning the nut (Fig. 3.13) so that the stress in the bolt rod reaches its yield point. Take the length of the handle of the spanner to be l = 15d, coefficient of friction at the end in the thread of the nut f = 0,15. The data to calculation is shown in Table 3.4.

Table 3.4. Initial data for Task 3.4

Var. no	Diameter mm	Thread type	R_e , MPa
1	M8	metric	210
2	M10	metric	230
3	M12	metric	240
4	M16	metric	260
5	M20	metric	280
6	M22	metric	320
7	M30	metric	340
8	M32	metric	360
9	M36	metric	380
10	M42	metric	220
11	M16	metric	180
12	M20	metric	210
13	M22	metric	360
14	M30	metric	400
15	M10	metric	420
16	M10	metric	460
17	M12	metric	480
18	M16	metric	500
19	M20	metric	315
20	M22	metric	215
21	M30	metric	415
22	M32	metric	435
23	M36	metric	265
24	M42	metric	245
25	M16	metric	325
26	M8	metric	235
27	M10	metric	225
28	M12	metric	185
29	M16	metric	210
30	M20	metric	200

Task 3.5. Determine the diameter of the flange coupling bolts (Fig. 3.21). Uncontrolled tightening. In the first case, the bolts are set without a gap, and in the second case with a gap. The initial data is shown in Table 3.5.

Table 3.5. Initial data for Task 3.5

Var. no	Load kNm	Number of screws	D_0, mm	Bolt strength class	The character of the load
	T	z			
1	0.5	4	220	3.6	Constant
2	0.6	6	230	4.6	Variable
3	0.7	8	240	4.8	Constant
4	0.8	4	250	5.6	Variable
5	0.9	6	260	5.8	Constant
6	1	8	280	6.6	Variable
7	1.2	4	290	6.8	Constant
8	1.3	6	300	6.9	Variable
9	1.4	8	310	8.8	Constant
10	1.5	4	315	10.9	Variable
11	1.6	6	320	3.6	Constant
12	0.3	8	325	4.6	Variable
13	0.6	4	330	4.8	Constant
14	0.8	6	340	5.6	Variable
15	0.5	8	350	5.8	Constant
16	0.6	4	345	6.6	Variable
17	0.7	6	360	6.8	Constant
18	0.8	8	200	6.9	Variable
19	0.9	4	210	8.8	Constant
20	1	6	220	10.9	Variable
21	1.2	8	230	3.6	Constant
22	1.3	4	240	4.6	Variable
23	1.4	6	250	4.8	Constant
24	1.5	8	260	5.6	Variable
25	1.6	4	280	5.8	Constant
26	0.3	6	290	6.6	Variable
27	0.6	8	300	6.8	Constant
28	0.8	4	310	6.9	Variable
29	0.5	6	315	8.8	Constant
30	0.6	8	320	10.9	Variable

Task 3.6. Determine the diameter of the bearing unit cover screws (Fig. 3.20) and check the strength of the thread and screw head. The screws were set with a gap and tightened before the load was applied. The depth of the screw was taken to be H = 1.$2d$. Consider two cases: case 1 – no tightening under load; case 2 - no tightening of screws under load. The data to calculations is shown in Table 3.6.

Table 3.6. Initial data for Task 3.6

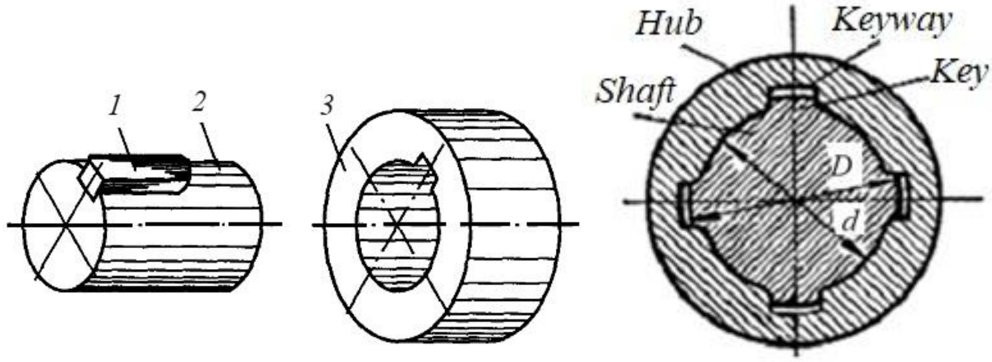
Var. no	Load kN	Number of screws	Bolt strength class	The character of the load
	F_a	z		
1	10	4	5.6	Constant
2	11	6	5.8	Variable
3	12	8	6.6	Constant
4	12.5	10	6.8	Variable
5	14	4	6.9	Constant
6	14.5	6	8.8	Variable
7	15	8	10.9	Constant
8	16	10	3.6	Variable
9	10.5	4	4.6	Constant
10	11	6	4.8	Variable
11	8	8	5.6	Constant
12	10	10	5.8	Variable
13	9	4	6.6	Constant
14	10	6	6.8	Variable
15	11	8	3.6	Constant
16	12	10	4.6	Variable
17	12.5	4	4.8	Constant
18	14	6	5.6	Variable
19	14.5	8	5.8	Constant
20	15	10	6.6	Variable
21	16	4	6.8	Constant
22	10.5	6	6.9	Variable
23	11	8	8.8	Constant
24	8	10	10.9	Variable
25	10	4	3.6	Constant
26	9	6	4.6	Variable
27	10	8	4.8	Constant
28	11	10	5.6	Variable
29	12	4	5.8	Constant
30	12.5	6	6.6	Variable

3.3. Calculation of keyed and splined connections

General information

Keyed and splined connections are used to connect shafts and rotating axles (gears, pulleys, sprockets and other components), to transmit torque from the shaft to the hub of the mounted component and vice versa, and to move workpieces along shafts along an axis.

Keyed connections (Fig. 3.23, a) comprise a shaft (2), a key (1) and a hub (3) (wheel, pulley or other component). The key is a steel wedge that is inserted into the grooves of the shaft and hub.



a b

Fig. 3.23. Keyed (*a*) and splined (*b*) connections

Spline connections (fig. 3.23, b) are formed by the specific shape of the projections (keys) on the shaft and the corresponding pits (splines) in the hub. The working surfaces are the sides of the keys. These connections can be considered splines if the grooves are made as a whole with the shaft.

Basic calculation formulae

Keyway connections

The primary performance criterion for keyed connections is **strength**. From the strength condition, verification calculations can be carried out by determining the design stresses and comparing them with the allowable or determining allowable moment, and design calculations by determining the geometrical dimensions of the connections (usually the length of the keyway is determined).

The prismatic keyway connection (Fig. 3.24) is calculated from the wedge compressive strength condition.

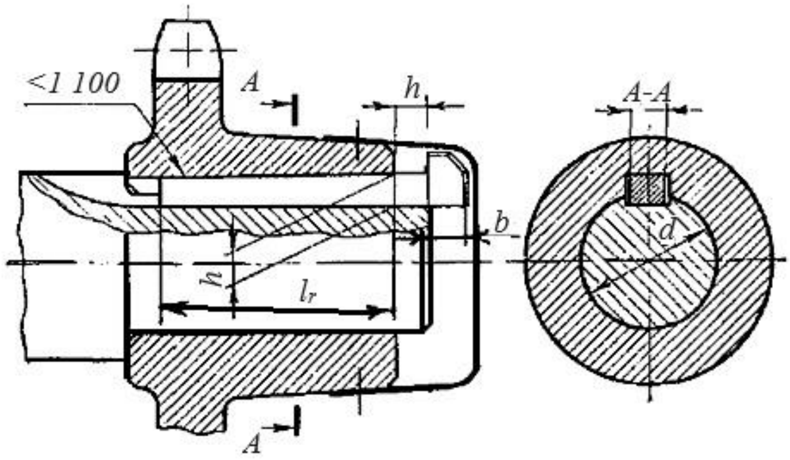


Fig. 3.24. Prismatic key connection

Strength condition

$$\sigma_c = \frac{12T \cdot 10^3}{(b + 6fd)b \cdot l_r} \leq k_c,$$

where T – torque, Nm;

d – shaft diameter, mm;

b – wedge width, mm;

l_r – length of the working part of the wedge, mm;

f – coefficient of friction; for steel and cast iron 0.15 ÷ 0.2;

k_c – allowable compressive stresses, MPa (Table D.9).

The complexity of making wedges and grooves, the occurrence of assembly stresses, radial displacement and skewing of products limits their use.

The tenon groove connection (Fig. 3.25) is calculated from the wedge compressive strength condition.

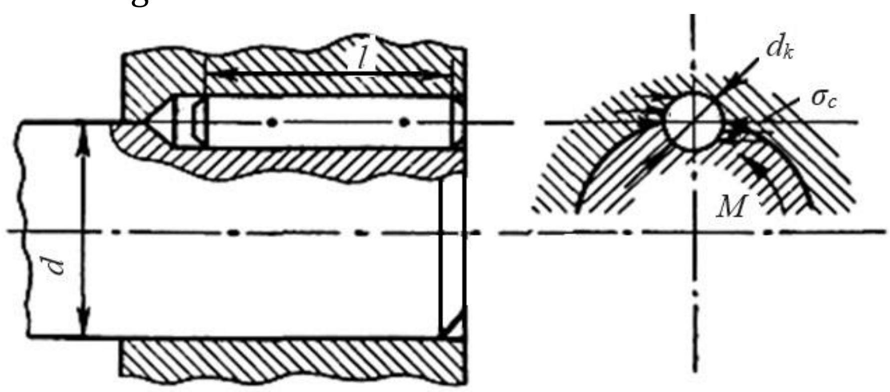


Fig. 3.25. Tenon groove connection

Strength condition:

$$\sigma_c = \frac{4T \cdot 10^3}{d d_k l_k} \leq k_c,$$

where d – shaft diameter, mm;

d_k – wedge diameter, mm;

l_k – key length, mm.

The geometric dimensions of the key are either determined from the strength condition or taken from the relationship:

Key diameter

$$d_k = (0{,}13 \div 0{,}16) d_w,$$

where d_w - shaft diameter, mm.

The length of the wedge is taken as l = (3 ÷ 4) d_k or determined by the length of the hub.

Tenon wedges are manufactured by PN-EN ISO 2338:2003 and PN-EN ISO 8735:2003. For heavy loads, two 180 ° or three 120 ° keys are used. It is worth bearing in mind that this considerably weakens the cross-section of the shaft, especially under impact and fluctuating loads.

The tangential keyway connection (Fig. 3.26) is also calculated from the compressive strength condition.

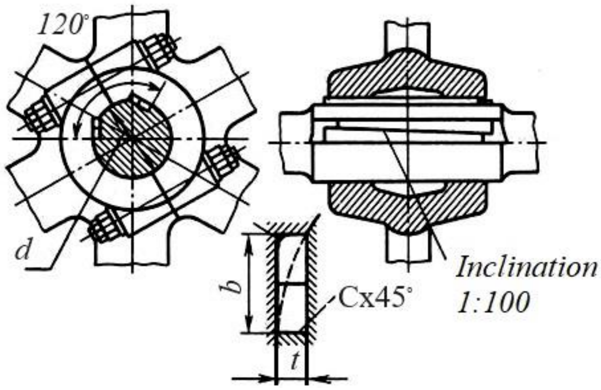


Fig. 3.26. Tangential groove connection

Strength condition for tangential keyway connection:

$$\sigma_c = \frac{T \cdot 10^3}{\left(0.45 + \frac{2}{\pi} f\right) d l_r (t - c)} \leq k_c$$

where t – is the width of the working edge of the wedge, it is equal to the depth of the keyway on the shaft, mm;

c – chamfer of wedge, mm.

In such connections, the keyway is subjected to compressive stresses, i.e. it operates under more favourable conditions than other wedges. The dimensions of the wedges and keys are selected according to ISO 3117:1977. The stress is applied by the relative axial displacement of the wedges. Ordinary wedges are positioned at an angle of 120 ÷ 135 °.

Prismatic keyway connections (Fig. 3.27)

For the transmission of torque, there are compressive stresses on the lateral surfaces of the wedges and keyways σ_c and shear stresses in the cross-section of the wedge τ_c. Since the dimensions of the keyways and keys in the standard are selected according to the compressive strength condition, the primary calculation is a compression calculation. Shear calculations are in most cases not carried out.

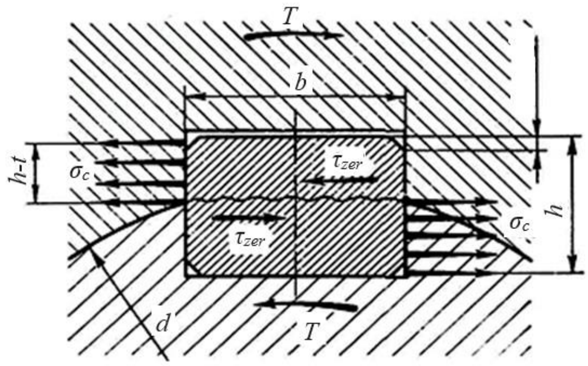


Fig. 3.27. Calculation diagram for a prismatic keyway connection

With precise calculations, the strength condition is as follows:

$$\sigma_c = \frac{2T \cdot 10^3}{d(0{,}95h - t)l_r} \leq k_c$$

where T – torque, Nm;

d – shaft diameter, mm;

h – key height, mm;

t – shaft keyawy depth, mm (h and t from the Table D.11);

l_r – working length of wedge, mm:

- for wedges with rounded edges $l_r = l - b$ (Fig. 3.28, c);
- for wedges with rounded edges $l_r = l$ (Fig. 3.28, a);
- for wedges with one flat end and one rounded end $l_r = l - \frac{b}{2}$ (Fig. 3.28, b),

where l – total length of the wedge (Table D.12), mm;

b – ker width, mm;

0.95 –reduction factor for the chamfer height of the working wedge, $f \approx 0{,}05h$.

With an average precision of calculation, the strength condition is as follows:

$$\sigma_c = \frac{2T \cdot 10^3}{d(h-t)l_r} \le k_c$$

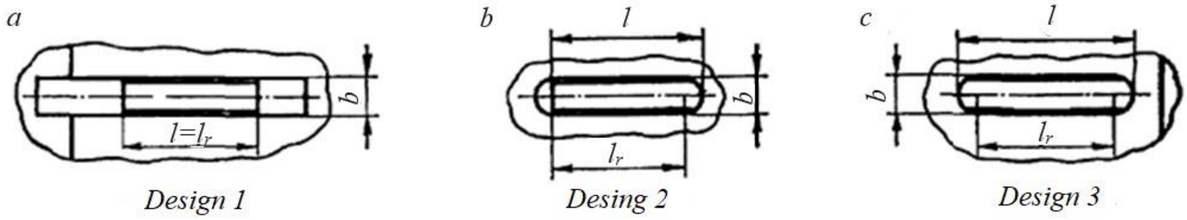


Fig. 3.28. Basic types of prismatic inlets

Shuttle keyway connection (Fig. 3.29)

Such a joint is verified for compressive and shear strength because such a wedge is narrow (the height is significantly greater than the width of the wedge) and there is a danger of shearing.

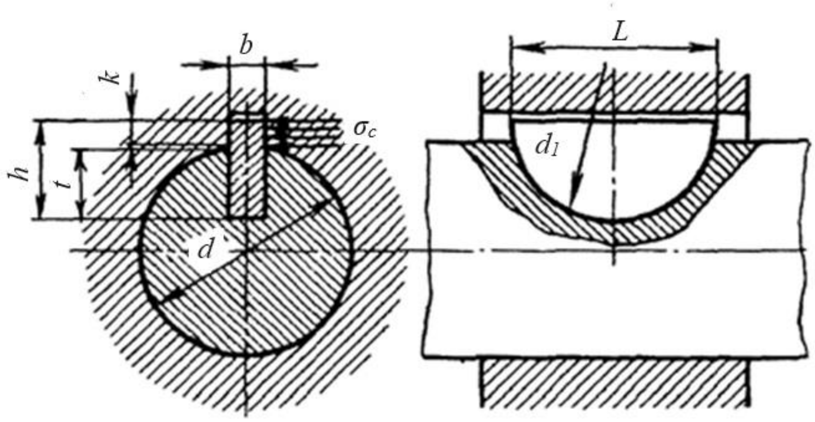


Fig. 3.29. Calculation diagram for shuttle keyway connection

Compressive strength condition:

$$\sigma_c = \frac{2T10^3}{d(h-t)l} \le k_c,$$

where h – key high, mm; t – shaft keyway depth, mm; l – wedge lenght, mm.

Shear strength condition (can be also applied to prismatic wedges):

$$\tau_c = \frac{2T10^3}{dbl} \le k_t,$$

where b – wedge width, mm;

k_t – allowable shear stresses, MPa (Table D.9);

for prismatic wedges, *l* is used instead of l_r – wedge working length, mm.

The key sizes (including length) and grooves are selected according to shaft diameter PN 85008 (Table D.13). The working edges are the edges of the wedge.

Spline connections (Fig. 3.30)

The primary performance criterion for spline connections is **strength**. Spline connections fail due to damage to the working surfaces of the teeth: wear, crushing, galling and fracture of the spline shafts and teeth. The basic dimensions of the connection are selected from standardised tables depending on the shaft diameter and then checked by calculation.

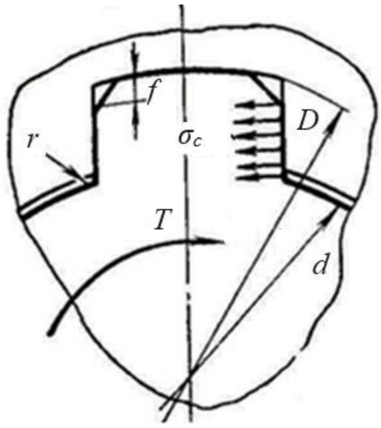


Fig. 3.30. Calculation scheme for spline connections

The tooth dimensions in the standards are taken from the compressive strength condition, so the primary calculation to check spline joints is in compression. Spline joints are not checked in shear. When calculating for strength, it is assumed that the loads are distributed uniformly in the lateral surfaces of the splines, but due to manufacturing inaccuracies, 0.75 of the total number of splines are involved in the work.

Condition for the compressive strength of a splined connection:

$$\sigma_c = \frac{2T \cdot 10^3}{0{,}75 z d_{av} h l} \leq k_c,$$

where T – torque, Nm;

0.75 – uneven load factor between splines;

z – number of inlets;

d_{av} – average diameter of the connection, mm:

- for rectangular profile $d_{av} = \frac{D+d}{2}$;

- for an involute profile $d_{av} = m \cdot z$;

D – external diameter of wedges, mm;

h – splines contact surface height, mm:

- for rectangular profile $h = \frac{D-d}{2} - 2 \cdot f$;

- for an involute profile $f = m$;

f – chamfering the wedge;

l – length of the tooth contact surface, which is equal to the length of the hub, mm;

k_c – allowable compressive stress of the wedge material, MPa (Table D.10).

The dimensions *D, d, z, m, and f* are selected from Tables D.14 and D.15.

Examples of calculations

Example 3.16. Check the strength condition in formed joints:

1. tenon wedge (Fig. 3.25);
2. prismatic wedge, with rounded edges (Fig. 3.27 and Fig. 3.28, c);
3. shuttle wedge (Fig. 3.29);
4. splines with rectilinear profile (Table D.14);
5. splines with a revolute profile (Table D.15).

If: torque T = 1.2 kNm; shaft diameter d = 40 mm; hub width B = 60 mm; hub material – steel. Stationary connections, variable load; transition keyways; surfaces without heat treatment.

Data:	*Searched for:*
T = 1.2 kNm	σ_c - ?
d = 40 mm	τ_c - ?
B = 60 mm	
material – steel	
transition keyways	
variable load	
stationary connection	

Solution

1. Write down the strength conditions depending on the type of connection:

(a) pivot key

$$\sigma_c = \frac{4T \cdot 10^3}{d d_k l_k} \leq k_c$$

(b) prismatic key

$$\sigma_c = \frac{2T \cdot 10^3}{d(h-t)l_r} \leq k_c$$

(c) shuttle key

$$\sigma_c = \frac{2T \cdot 10^3}{d(h-t)l} \leq k_c$$

$$\tau_c = \frac{2T \cdot 10^3}{dbl} \leq k_t$$

(d) splined connection

$$\sigma_c = \frac{2\mathrm{T} \cdot 10^3}{0{,}75 z d_{av} h l} \leq k_c$$

2. Determine the unknown values necessary to calculate the strength condition.

2.1. Determine the allowable stresses for the wedges:

Taking into account the nature of the connections, the load and the hub material from Table D.10: k_c = 100 MPa; k_t = 70 MPa.

2.2. Determine the allowable stresses for spline connections:

- taking into account the nature of the connection, the load, the hub material and the surface condition from Table D.10: k_c = 60 MPa;

2.3. Define the geometrical parameters of the keys:

(a) pivot

- the diameter of the gully is determined by the relationship:

$$D_k = (0.13 \div 0.16)d_w = (0.13 \div 0.16) \cdot 40 = 5.0 \div 6.4 \text{ mm}$$

Taking into account the high load for such a shaft diameter, we adopt a keyway diameter to reduce the weakening of the shaft cross-section:

$$d_k = 10 \text{ mm}$$

- the keyway length is assumed to be the width of the component hub:

$$l_k = B = 60 \text{ mm}$$

(b) prismatic with rounded edges

- for shaft diameter d = 40 mm from the Table D.11 we take b = 12 mm; h = 8 mm; t = 5 mm.

- the length of the keyway is taken into account about the hub width of the component for the standard length series in Table D.12: l = 56 mm.

Given that the wedge has rounded edges, the working length will be $l_r = l - b = 56 - 12 = 44$ mm.

(c) shuttle

- for shaft diameter d = 40 mm from the Table D.13 we take: b = 12 mm; h = 19 mm, l = 59,1 mm; t = 16 mm.

2.4. Determine the geometric parameters of the spline connection:

(a) with a rectangular profile

Given the heavy load for such a shaft diameter, we adopt a heavy series. For a shaft diameter of D = 40 mm (the external diameter of the spline connection is denoted by D), we take

$$z \cdot d \cdot D = 10 \cdot 32 \cdot 42 \text{ mm}; \; f \; = \; 0.4 \text{ mm}.$$

- the length of the keyways is assumed to be equal to the hub width l = B = 60 mm.

- average diameter of connection:

$$d_{av} = \frac{D + d}{2} = \frac{40 + 32}{2} = 36 \text{ mm}$$

- height of wedge contact surface:

$$h = \frac{D-d}{2} - 2 \cdot f = \frac{40-32}{2} - 2 \cdot 0.4 = 3.2 \text{ mm}$$

(b) with an evolvente profile

For a shaft diameter of d = 40 mm from Table D.15 we take z = 18; m = 2 mm (taking into account the heavy load). We take the length of the wedges equal to the hub width $l = B$ = 60 mm.

- average diameter of connection:

$$d_{av} = m \cdot z = 2 \cdot 18 = 36 \text{ mm}$$

- contact key surface height:

$$h \approx m = 2 \text{ mm}$$

3. We determine the design stresses and compare them with the allowable stresses:

(a) pivot key

$$\sigma_c = \frac{4T \cdot 10^3}{d d_k l_k} = \frac{4 \cdot 1.2 \cdot 10^6}{40 \cdot 10 \cdot 60} = 200 \frac{\text{N}}{\text{mm}^2} = 200 \text{ MPa} > k_c = 100 \text{ MPa}$$

condition is not met

(b) prismatic key

$$\sigma_c = \frac{2T \cdot 10^3}{d(h-t)l_r} = \frac{2 \cdot 1.2 \cdot 10^6}{40 \cdot (8-5) \cdot 44} = 455 \frac{\text{N}}{\text{mm}^2} = 455 \text{ MPa} > k_c = 100 \text{ MPa}$$

condition is not met

(c) shuttle key

$$\sigma_c = \frac{2T \cdot 10^3}{d(h-t)l} = \frac{2 \cdot 1.2 \cdot 10^6}{40 \cdot (19-16) \cdot 59.1} = 339 \frac{\text{N}}{\text{mm}^2} = 455 \text{ MPa} > k_c = 100 \text{ MPa}$$

condition is not met

$$\tau_c = \frac{2\text{T} \cdot 10^3}{dbl} = \frac{2 \cdot 1{,}2 \cdot 10^6}{40 \cdot 12 \cdot 59.1} = 85 \frac{\text{N}}{\text{mm}^2} = 85 \text{ MPa} > k_t = 70 \text{ MPa}$$

condition is not met

(d) spline connection

- with a rectangular profile

$$\sigma_c = \frac{2 \cdot 1.2 \cdot 10^6}{0.75 \cdot 10 \cdot 36 \cdot 3.2 \cdot 60} = 46 \frac{\text{N}}{\text{mm}^2} = 46 \text{ MPa} < k_c = 60 \text{ MPa}$$

condition is met

- with an involute profile

$$\sigma_c = \frac{2\text{T} \cdot 10^3}{0{,}75 z d_{av} h l} = \frac{2 \cdot 1.2 \cdot 10^6}{0.75 \cdot 18 \cdot 36 \cdot 2 \cdot 60} = 41 \frac{\text{N}}{\text{mm}^2} = 41 \text{ MPa} < k_c = 60 \text{ MPa}$$

condition is met

Conclusion: Only spline connections can be used for the assumed load and operating mode.

Example 3.17. Determine the torque which can transmit the shuttle keyway connection (Fig. 3.29) with shaft diameter d = 25 mm. Hub material - steel, constant load.

Data:	*Searched for:*
d = 25 mm	$[T]$ - ?
material – steel	
constant load	

Solution

1. Define the geometrical parameters of the connection.

From Table D.13 for a shaft diameter of d = 25 mm we takey b = 8; h = 11 mm; l = 27.3 mm; t = 8 mm.

2. Determine the permissible torque:

(a) from the compressive strength condition:

From Table D.9 we take k_c = 150 MPa; i= 100 MPa;

$$[T] \leq \frac{d(h-t)lk_c}{2} = \frac{25 \cdot (11-8) \cdot 27.3 \cdot 150}{2} = 153.5 \cdot 10^3 \ = 153 \text{ Nm}$$

(b) from the shear strength condition:

$$[T] \leq \frac{dblk_t}{2} = \frac{25 \cdot 8 \cdot 27.3 \cdot 100}{2} = 273 \cdot 10^3 \text{ Nmm} = 273 \text{ Nm}$$

Answer: Largest torque that can be carried by a shuttle key connection $[T]$ ≤ 153.5 Nm (we assume lower).

Example 3.18. From the strength condition, determine the length of a prismatic key with rounded edges (Fig. 3.28, a). Torque T = 290 Nm, diameter d = 40 mm, hub material cast iron, variable load.

Data:	*Searched for:*
T = 290 Nm	l - ?
d = 40 mm	
material – cast iron	
variable load	

Solution

1. Define the geometrical parameters of the connection.

From Table D.12 for a shaft diameter d = 40 mm, we assume

b = 12; h = 11 mm; t = 7 mm.

2. Determine the allowable stresses.

From Table D.9 we take k_c = 60 MPa.

3. From the compressive strength condition we determine the width of the keyway

$$l_r \geq \frac{2\mathrm{T}\cdot 10^3}{d(h-t)k_c} = \frac{2\cdot 290\cdot 10^3}{40\cdot(11-7)\cdot 60} = 60.4\ \mathrm{mm}$$

The final value is taken with a standard length series, according to the form of the wedge edge, then

$$l = l_k + b = 60.4 + 12 = 72.4\ \mathrm{mm}$$

Taking into account the standard values for wedge lengths, we assume

$$l = 80\ \mathrm{m}$$

Answer: l = 80 mm.

Example 3.19. Check the strength of a keyed joint with a flat-edged wedge (Fig. 3.24). Shaft diameter d = 80 mm, torque T = 2000 Nm, hub length l = 50 mm, hub material cast iron, constant load.

Data:
T = 2000 Nm
d = 80 mm
l = 50 mm
material – cast iron
constant load

Searched for:
σ_c - ?

Solution

1. We define the geometrical parameters of the connection.

According to ISO/R 774:1996–80 for shaft diameter d = 80 mm, we assume b = 22 mm; h = 14 mm; the working length is assumed to be 5 mm less than the hub length l_r = 45 mm.

2. Determine the allowable stresses.

From Table D.9 we assume k_c = 90 MPa.

3. We determine the design stresses and compare them with the permissible ones (friction coefficient of steel against cast iron f = 0,18):

$$\sigma_c = \frac{12\mathrm{T}\cdot 10^3}{(b+6fd)b\cdot l_r} = \frac{12\cdot 2\cdot 10^6}{(22+6\cdot 0.18\cdot 80)22\cdot 45} = 224\frac{\mathrm{N}}{\mathrm{mm}^2} = 224\ \mathrm{MPa} > k_c = 90\ \mathrm{MPa}$$

The strength condition is not met.

Conclusion: the considered connection will not work under these conditions.

Individual tasks
(calculation)

Task 3.7. Check the strength of the keyway connection. The data for the calculations is shown in Table 3.7.

Table 3.7. Initial data for Task 3.7

Var. no	Shaft diameter *d*, mm	Torque *T*, Nm	Key type	Hub material	Hub length *l*, mm	The character of the load
1	45	100	Prismatic with rounded edges	Steel	60	Constant
2	55	128			70	
3	75	205			70	
4	60	145			50	
5	85	230			75	
6	30	95	Round	Cast iron	-	Variable
7	50	125			-	
8	70	260			-	
9	60	300			-	
10	80	450			-	
11	15	75	Dugout	Steel	40	Constant
12	20	80			50	
13	25	60			60	
14	32	110			40	
15	42	220			60	
16	115	400	Prismatic with flat edges	Cast iron	80	Variable
17	52	163			60	
18	62	95			75	
19	20	80			40	
20	28	90			100	
21	18	65	Dugout	Steel	40	Constant
22	44	85			50	
23	30	50			42	
24	35	145			38	
25	12	65			60	
26	165	620	Prismatic with flat edges on one side and a rounded end on the other	Cast iron	75	Variable
27	100	530			110	
28	125	480			115	
29	90	280			130	
30	145	800			95	

Taske 3.8. From the strength condition, determine the greatest moment that the given connection can transmit. The data for calculations is shown in Table 3.8.

Table 3.8. Initial data for Task 3.8

Var. no	Shaft diameter *d*, mm	Key type	Hub material	Hub length *l*, mm	Load character	Type of connection
1	30	Prismatic with flat edges on one side and rounded end on the other	Cart iron	80	Variable	Stationary
2	50			60		
3	70			75		
4	60			40		
5	80			100		
6	18	Dugout	Steel	40	Constant	Stationary
7	35			50		
8	44			60		
9	30			40		
10	12			60		
11	75	Round	Cart iron	75	Constant	Stationary
12	60			110		
13	85			115		
14	30			130		
15	50			95		
16	65	Prismatic with flat edges	Cart iron	75	Constant	Stationary
17	70			40		
18	85			100		
19	125			40		
20	115			50		
21	22	Dugout	Steel	-	Constant	Stationary
22	32			-		
23	42			-		
24	16			-		
25	20			-		
26	45	Prismatic with rounded edges	Steel	40	Variable	Movable
27	35			55		
28	95			65		
29	100			90		
30	125			85		

Task 3.9. Check the strength of the splined connection. Operating conditions are good. The data for calculationa is shown in Table 3.9.

Table 3.9. Initial data for Task 3.9

Var. no	Shaft diameter *d*, mm	Torque *T*, Nm	Key profile	Tooth surface	Hub length *l*, mm	Load character	Type of connection
1	11	70	Straight line	Without heat treatment	60	Constant	Stationary
2	82	110			70		
3	62	120			70		
4	32	130			50		
5	112	180			75		
6	120	190	Evolvent	With heat treatment	75	Variable	Movable under load
7	200	800			110		
8	15	65			115		
9	70	230			130		
10	30	195			95		
11	21	300	Straight line	Without heat treatment	40	High-frequency vibration	Stationary
12	16	135			50		
13	56	420			60		
14	92	220			40		
15	46	175			60		
16	170	330	Evolvent	With heat treatment	40	Constant	Stationary
17	22	115			50		
18	65	135			60		
19	12	85			40		
20	140	210			60		
21	13	100	Straight line	Without heat treatment	60	Constant	Stationary
22	16	110			70		
23	23	120			70		
24	72	145			50		
25	102	185			75		
26	45	165	Evolvent	With heat treatment	40	Variable	Movable under load
27	50	140			50		
28	95	400			42		
29	130	620			60		
30	13	160			38		

3.4. Calculation of kinematic and force parameters of gearboxes

General information

Gears are mechanisms that are used to transfer energy over a distance, usually with a transformation of the parameters and type of movement.

Depending on the method of power transmission, a distinction is made between *mechanical, electric, pneumatic, hydraulic and combination transmissions*.

Only mechanical transmissions are discussed in this script.

A mechanical transmission is a mechanism that transforms the motion parameters of the motor and transfers the movement to the working parts of the machine.

In simple terms, the mechanical transmission is the intermediate link between the motor and the machine's execution part (Fig. 3.31).

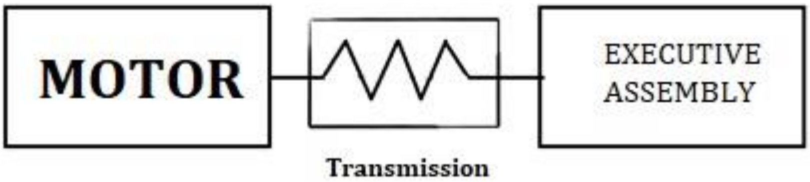


Fig. 3.31. Gearbox location in the machine

Basic calculation formulae

Each mechanical transmission is characterised by *geometric, force and kinematic parameters.*

Geometric parameters of a gearbox include the dimensions of its components (m, mm): diameters (d); lengths (l); widths – (b); inter-axial distances (a) and others.

Gear **force parameters** include forces (F, N); moments(T(M), Nm); powers (N(P), W).

Gearbox **kinematic parameters** include linear velocity [m/s], circumferential velocity [m/s]; angular velocity (ω, rad/s or s^{-1}); rotational speed (n, rpm or min^{-1}).

The derivatives of the basic parameters are:

Conversion efficiency- η:

Conversion efficiency shows the amount of loss in the gearbox and characterises its performance.

$$\eta = \frac{A_u}{A_z} = \frac{N_2}{N_1} < 1$$

where A_u – useful work - work, transferred from the machine to the environment;

A_z – work consumed - work used to do a specific job (including useful work and work to overcome resistance e.g. friction, air resistance etc.);

N_1 – power at the input to the gearbox, W;

N_2 – gearbox output power, W.

The efficiency of multi-stage gearboxes or drives consisting of several widely connected gearbox elements or transmissions is determined by the formula:

$$\eta_{gen} = \eta_1 \cdot \eta_2 \dots \eta_n,$$

where η_1, η_2, η_n – efficiency of a separate kinematic pair (pair of gears, sprockets, pulleys, etc.) or transmission (belt, gears, etc.) and other kinematic elements (bearings, couplings).

The transmission ratio (i) is the ratio of the angular velocity of the driving element to the angular velocity of the driven element. The ratio can be greater than, less than or equal to unity.

The ratio (u) of a gearbox is the ratio of the higher angular velocity to the lower angular velocity. The gear ratio must not be less than one.

Gearboxes with i >1 and $n_1 > n_2$ are called **reduction gearboxes.**

Gearboxes with i <1 and $n_1 < n_2$ are called **multipliers (accelerators).**

Reduction gears are the most common, as the speed of the moving parts of the machines is in most cases lower than the speed of the motor shaft. In this script, reduction gears are discussed.

In reduction gears, *the speed and power in the transfer of motion from the motor to the machine's execution part decreases and the torque increases. Power is reduced by the amount of loss, characterising efficiency. Speed decreases and torque increases by the value of the gear ratio.*

In reduction gears, *the dimensions of the driving elements are smaller than the driven elements.*

The ratio and ratio for different reduction gears can be calculated individually or according to the relationship:

$$i(u) = \frac{d_2}{d_1}; \frac{n_1}{n_2}; \frac{\omega_1}{\omega_2}; \frac{z_2{}^*}{z_1}; \frac{\mathrm{T}_2}{\eta \mathrm{T}_1},$$

where d_1, d_2 – diameters of the driving and driven elements of the transmission, mm (shafts, pulleys, gears, etc.);

* - the ratio of the number of teeth of the driving and driven chain of a sprocket transmission. For pinion gears for a pair of wheels, this number is called the ratio and is denoted by the symbol u;

T_1, T_2 – torque of the driving and driven gear element respectively, Nm;

η - kinematic pair efficiency.

The transmission ratio of gearboxes or drives consisting of several series-connected gear elements is determined by the formula:

$$i_{gen} = i_1 \cdot i_2 \cdot \ldots \cdot i_n \text{ or } u_{gen} = u_1 \cdot u_2 \cdot \ldots \cdot u_n$$

Basic calculation formulas and relationships for mechanical transmissions

Dependence of angular velocity on rotational speed

$$\omega = \frac{\pi n}{30}, \text{ then } n = \frac{30\omega}{\pi} \approx 9{,}55\omega,$$

where 9.55 is the approximate value when dividing 30 by π.

Dependence of rotational speed on angular velocity

$$v = \omega \frac{d}{2 \cdot 1000}\text{, then } \omega = \frac{2 \cdot 1000 \cdot n}{d},$$

where d - the diameter of the gear element, mm (shaft, pulley, gear wheel, etc.); 1000 - is the millimetre-to-metre conversion factor.

Dependence of peripheral speed on rotation frequency

$$v = \frac{\pi n d}{60 \cdot 1000}, \text{m/s then } n = \frac{60 \cdot 1000 \cdot v}{\pi d}, \text{min}^{-1}$$

Expression of power by rotational or linear force and rotational and linear speed

$$N = Fv, \text{then } F = \frac{N}{v},$$

where F – force, N;

v - rotational or linear speed, m/s.

Expression of power by torque and angular velocity

$$N = \mathrm{T}\omega, \text{W then } T = \frac{N}{\omega}, \text{Nm},$$

where T – torque, Nm.

Expression of power by torque and speed

$$N = \frac{Tn}{9.55}, \text{then } T = \frac{9.55N}{n}$$

Engine power in forward and rotary motion

$$N_m = \frac{Fv}{\eta_{gen}} = \frac{\mathrm{T}\omega}{\eta_{gen}},$$

where η_{gen} – the overall efficiency of the gearbox.

Relationship of the power on the driving element to the power on the driven element when transferring motion from the motor to the machine actuator:

$$N_2 = N_1 \cdot \eta,$$

where η - the kinematic efficiency of the gear pair.

Relationship of the torque on the driving element to the torque on the driven element in the direction of power flow from the motor to the machine actuator:

$$T_2 = T_1 \cdot u \cdot \eta, \text{then } T_1 = \frac{T_2}{u \cdot \eta}$$

Relationship between peripheral force and torque

$$F_t = \frac{2T}{d} \text{ then } T = \frac{F_t d}{2}$$

here d in [m], T – [Nm].

Examples of calculations

Example 3.20. Determine the angular and rotational velocity of a shaft with a diameter of d = 80 mm which rotates at a speed of n = 600 mm^{-1}.

Data:
d = 80 mm
n = 600 min^{-1}

Searched for:
υ - ?, ω - ?

Solution

Plot a calculation diagram (Fig. 3.32)

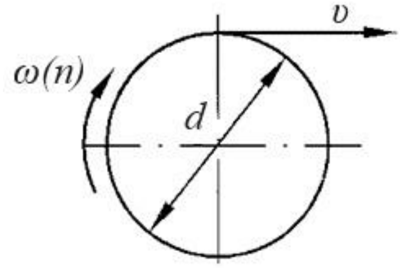


Fig. 3.32. Calculation scheme for the shaft of example 3.20

Determine the rotational and angular speed:

$$\upsilon = \frac{\pi n d}{60 \cdot 1000} = \frac{3.14 \cdot 600 \cdot 80}{60 \cdot 1000} = 2.5, \mathrm{m/s}$$

$$\omega = \frac{\pi n}{30} = \frac{3.14 \cdot 600}{30} = 63, s^{-1}$$

Answer: υ = 2.5 m/s; ω =63 s^{-1}.

Example 3.21. Calculate the angular and rotational speed of the transmission pulleys if: pulley diameters D_1 = 100 mm and D_2 = 400 mm, drive pulley speed n_1 = 100 min^{-1}.

Data:
D_1 = 100 mm
D_2 = 400 mm
n_1 = 100 min^{-1}

Searched for:
υ_1 - ?, υ_2 - ?
ω_1 - ?, ω_2 - ?

Solution

Plot a calculation diagram (Fig. 3.33)

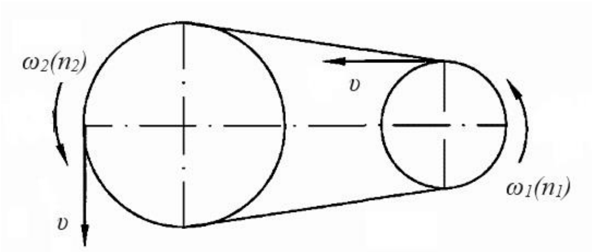


Fig. 3.33. Calculation scheme for the belt transmission to Example 3.21

1. Determine the rotational and angular velocity on the drive wheel:

$$\upsilon_1 = \frac{\pi n_1 D_1}{60 \cdot 1000} = \frac{3.14 \cdot 100 \cdot 100}{60 \cdot 1000} = 0.52 \frac{\mathrm{m}}{\mathrm{s}},$$

$$\omega_1 = \frac{\pi n_1}{30} = \frac{3.14 \cdot 100}{30} = 10.5\ s^{-1}$$

2. Determine the gear ratio:

$$u = \frac{D_2}{D_1} = \frac{400}{100} = 4$$

3. Determine the rotational and angular velocity on the driven wheel:

$$\upsilon_2 = \upsilon_1 = 0.52 \frac{\mathrm{m}}{\mathrm{s}},$$

$$\omega_2 = \frac{\omega_1}{u} = \frac{10.5}{4} = 2.63\ s^{-1}$$

Answer*:* $\upsilon_1 = \upsilon_2 = 0.52$ m/s; $\omega_1 = 10.5$ s^{-1}; $\omega_2 = 2.63$ s^{-1}.

Example 3.22. Determine the torque and power on the working shaft of the machine if: motor power N_{m} = 7.5 kW; torque T_m = 200 Nm; transmission ratio: belt transmission $u_{p.p}$ = 2; pinion transmission $u_{p.z}$ = 15; effiency: belt transmition $\eta_{p.p}$ = 0.96; pinion transmition - $\eta_{p.z}$ = 0.95; coupling η_s = 0.98.

Data:	*Searched for:*
N_m = 7.5 kN	N_p – ? T_p - ?
T_m = 200 Nm	
$u_{p.p}$ = 2	
$u_{p.z}$ = 15	
$\eta_{p.p}$ = 0.96	
$\eta_{p.z}$ = 0.95	
η_s = 0.98	

Solution

1. Determine the power at the output shaft:

- determine the power at the output shaft

- determine overall efficiency:

$$\eta_{gen} = \eta_{p.p} \cdot \eta_{p.z} \cdot \eta_s = 0.96 \cdot 0.95 \cdot 0.98 = 0.89,$$

- power at the machine's working shaft:

$$N_p = N_m \cdot \eta_{gen} = 7.5 \cdot 0.89 = 6.7 \text{ kW},$$

which indicates that the power from the motor to the working shaft is reduced by the amount of loss that characterises efficiency.

2. Determine the torque on the working shaft of the machine:

- we determine the overall values:

$$u_{gen} = u_{p.p} \cdot u_{p.z} = 2 \cdot 15 = 30,$$

- power at the machine's working shaft

$$T_p = T_m.u_{gen} \cdot \eta_{gen} = 200 \cdot 30 \cdot 0.89 = 5340 \text{ Nm},$$

which indicates that the power from the motor to the working shaft is reduced by the amount of loss that characterises efficiency.

Answer: N_p = 6.7 kW; T_p = 5340 Nm.

Example 3.23. Determine the motor power if: rotational force F_t = 10 kN; d = 300 mm; rotation speed n = 750 min^{-1}; overall efficiency η_{gen} = 0.9.

Data:	*Searched for:*
F_t = 10 kN	N_m - ?
d = 300 mm	
n = 750 min^{-1}	
η_{gen} = 0.9	

Solution

1. Formula for determining the power of an engine in rotary motion:

$$N_m = \frac{T\omega}{\eta_{gen}}$$

2. Determine the unknowns in the formula:

(a) we determine the torque:

$$T = 0.5F_t \cdot d = 0.5 \cdot 10000 \cdot 300 = 15 \cdot 10^5 \text{ Nmm} = 1.5 \text{ kNm},$$

(b) we determine the angular velocity:

$$\omega = \frac{\pi n}{30} = \frac{3.14 \cdot 750}{30} = 78{,}5 \text{ min}^{-1}$$

3. We determine the computing power in the engine:

$$N_m = \frac{1.5 \cdot 78.5}{0.9} = 131 \text{kW}$$

Round the specified values to the nearest largest normalized value.

We assume N_m = 150 kW.

Answer: N_m = 150 kW.

Example 3.24. Determine the rotational force and revolutions per minute if power N = 15 kW; diameter d = 80 mm; rotational speed υ = 8 m/s.

Data:
N = 15 kW
d = 80 mm
υ = 8 m/s

Searched for:
F_t - ? n - ?

Solution

1. Determine the angular velocity:

$$\omega = \frac{2 \cdot 1000\upsilon}{d} = \frac{2 \cdot 1000 \cdot 8}{80} = 200 \text{ min}^{-1}$$

2. Determine the rotation per minute:

$$n = \frac{30\omega}{\pi} \approx 9.55\omega = 9.55 \cdot 200 = 1910 \text{ min}^{-1}$$

3. From the power formula we determine the torque:

$$T = \frac{N}{\omega} = \frac{15000}{200} = 75 \text{ Nm}$$

4. Determine the circular force:

$$F_t = \frac{2T}{d} = \frac{2 \cdot 75}{0{,}08} = 1875 \text{ N} \text{ or } F_t = \frac{N}{\upsilon} = \frac{15000}{8} = 1875 \text{ N}$$

Answer: F_t =1875 N; n = 1910 min^{-1}.

Example 3.25. Select an electric motor. Carry out kinematic and force calculations for the transmission drive of a chain conveyor (Fig. 3.34), which consists of an electric motor; a belt transmission; a cylindrical single-stage gear; a clutch; a drive sprocket whose shaft is supported by plain bearings. Thepulling force of the chain F_t = 20 kN, the linear speed of the chain υ = 1.2 m/s, the diameter of the drive sprocket D_3 = 500 mm, and the diameters of the pulleys D_1 = 100 mm and D_2 = 400 mm respectively.

Data:
F_t = 20 kN
D_1 = 100 mm
D_2 = 400 mm

Searched for:
N_m - ? u - ? T_i - ? N_i - ?

$D_3 = 500$ mm

$\upsilon = 1.2$ m/s

Solution

Plot a kinematic diagram of a chain conveyor drive.

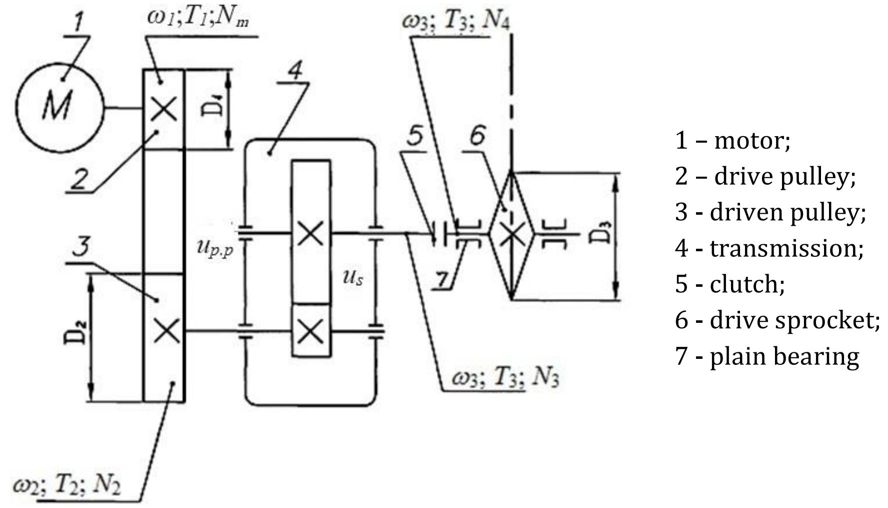


Fig. 3.34. Kinematic diagram of a chain conveyor drive

1. Determine the power of the chain conveyor drive motor

- based on the drive diagram we determine the overall efficiency (Fig. 3.34)

$$\eta_{gen} = \eta_{p.p} \cdot \eta_p \cdot \eta_s \cdot \eta_{s.p}^2$$

From the Table D.17 we assume $\eta_{p.p} = 0.96$; $\eta_p = 0.97$; $\eta_s = 0.98$; $\eta_{s.p} = 0.98$, then

$$\eta_{gen} = \eta_{p.p} \cdot \eta_p \cdot \eta_s \cdot \eta_{s.p}^2 = 0.96 \cdot 0.97 \cdot 0.98 \cdot 0.98^2 = 0.87$$

$$N_m = \frac{F_t \upsilon}{\eta_{gen}} = \frac{20 \cdot 10^3 \cdot 1.2}{0.87} = 27586 \text{ W } \approx 28 \text{ kW}$$

2. Selecting the motor.

When selecting an electric motor, it is important to remember that the lower the speed of the motor shaft, the greater the size, weight and cost. High-speed motors, on the other hand, have smaller dimensions, weight and cost compared to low-speed motors of the same power. However, as the engine speed increases, the overall gear ratio and therefore the cost increases. Therefore, it is usually recommended to use motors with $n_s = 1500$ rpm for drives without shaft reversible rotation and $n_s = 1000$ rpm with reversible rotation, where n_s – synchronous motor speed, rpm. When selecting a low-speed electric motor, its power rating may differ from the required one. In such a case, two considerations must be taken into account: a large motor power reserve leads to reduced power losses (recommended underloading of no more than 10 %) and overloading leads to motor overheating (permissible overloading of no more than 5 %).

Motor selection condition

$$N_{ob.m} \leq N_{nom},$$

where $N_{\text{ob.m.}}$ – calculated motor power, kW;

N_{nom} – Normalized power of the selected motor, kW.

In order to reduce the dimensions of the drive gears from the Table D.18, we adopt a motor of type 4A225M8U3 for wich N_{m} = 30 kW, synchronous speed n_{s} = 750 min^{-1}, slip s = 1.8 %. Permissible overload $N_{p.}$ = 31.5 kW, permissible underload N_n = 27 kW. In further calculations we assume the calculated power.

The nominal speed is determined from the formula:

$$n = n_c\left(1 - \frac{s}{100}\right) = 750\left(1 - \frac{1.8}{100}\right) = 736\ \text{min}^{-1}$$

3. Define the kinematic parameters:

(a) the angular velocity at the shaft of the electric motor (drive wheel):

$$\omega_1 = \frac{\pi n_m}{30} = \frac{3{,}14 \cdot 736}{30} = 77\text{s}^{-1}$$

(b) the angular velocity on the high-speed gear shaft (driven pulley):

- determine the belt transmission ratio

$$u_{p.p} = \frac{D_2}{D_1} = \frac{400}{100} = 4$$

$$\omega_2 = \frac{\omega_2}{u_{p.p}} = \frac{77}{4} = 19.3\ s^{-1}$$

(c) angular velocity of the drive wheel shaft (gearbox output shaft)

$$\omega_3 = \frac{2 \cdot 1000 \cdot v}{D_3} = \frac{2 \cdot 1000 \cdot 1.2}{500} = 4.8\ s^{-1}$$

(d) transmission ratio

$$u_p = \frac{\omega_2}{\omega_1} = \frac{19.3}{4.8} = 4$$

(e) overall gear ratio

$$u_{gen} = u_{p.p} \cdot u_p = 4 \cdot 4 = 16$$

4. Determine the force parameters

(a) motor shaft torque

$$T_1 = \frac{N_m \eta_{gen}}{\omega_1} = \frac{28000 \cdot 0.87}{77} = 316\ \text{Nm}$$

(b) the torque on the high-speed shaft of the transmission

$$T_2 = T_1 \cdot u_{p.p} = 316 \cdot 4 = 1264\ \text{Nm}$$

(c) drive wheel shaft torque

$$T_3 = T_2 \cdot u_p = 1264 \cdot 4 = 5056\ \text{Nm}$$

$$or\ T_3 = T_1 \cdot u_{gen} = 316 \cdot 16 = 5056\ \text{Nm}$$

Answer: electric motor 4A225M8U3: N_m = 30 kW; n_s = 750 min^{-1}; $n = 736$ min^{-1}; $N_m = 28$ kW; ω_1 = 77min^{-1}; ω_2 = 19,3 min^{-1}; ω_3 = 4,8 c^{-1}; $u_{p.p}$ = 4; u_p = 4; u_{gen} = 16; $T_1 = 316$ Nm; T_2 = 1264 Nm; T_3 = 5056 Nm.

Individual tasks

(calculation)

Task 3.10. Determine the motor power. The data for the calculations are shown in Table 3.10.

Table 3.10. Initial data for Task 3.10

Var. no	Shaft working load, kN	Working shaft diameter, mm	Working shaft speed, min^{-1}	Overal efficiency
	F_t	d	n	η_{gen}
1	7	40	300	0.9
2	6	45	315	0.93
3	5	50	425	0.88
4	8	55	520	0.95
5	9	60	635	0.96
6	10	65	552	0.97
7	11	70	722	0.91
8	12	75	433	0.75
9	14	80	638	0.78
10	16	85	551	0.82
11	17	90	665	0.93
12	13	95	530	0.94
13	3	35	815	0.76
14	4	55	918	0.80
15	5	60	1116	0.81
16	6	65	1114	0.85
17	3.5	50	1213	0.92
18	2.5	70	1344	0.93
19	14	75	1432	0.96
20	13	80	744	0.76
21	15	85	548	0.75
22	16	40	354	0.77
23	17	45	462	0.88
24	17.5	50	270	0.98
25	10.5	55	335	0.76
26	11.5	60	338	0.79
27	8.5	65	241	0.89
28	9.5	70	143	0.88
29	7.3	75	560	0.92
30	6.2	100	624	0.93

Task 3.11. Select the electric motor. Carry out kinematic (specify u_i, ω_i) and force calculations of the belt conveyor drive (specify T_i, F_t of pulleys), which consists of: electric motor, belt transmission, two-stage cylindrical reducer, coupling, drive drum, shaft supported by plain bearings. The data for the calculations are shown in Table 3.11.

Table 3.11. Initial data for Task 3.11

Var. no	Load of drums, kN	Drum diameter, mm	Belt speed, m/s	Pulley diameter, mm	Pulley transmition
	F_t	D_b	υ_l	D_2	$u_{p\,p}$
1	7	400	0.315	250	1.4
2	6	450	0.4	273	1.2
3	5	500	0.5	285	1.3
4	8	550	0.63	300	1.6
5	9	600	0.8	410	1.8
6	10	650	1	315	1.9
7	11	700	1.25	320	2
8	12	750	1.6	340	2.3
9	14	800	2	360	2.4
10	16	850	2.5	400	2.6
11	17	900	3.15	450	2.8
12	13	950	4	550	2.1
13	3	650	5	480	3
14	4	550	6.3	430	2.8
15	5	600	0.63	365	2.6
16	6	650	0.8	390	2.2
17	3.5	500	1	410	3.1
18	2.5	700	1.25	420	2.5
19	14	750	1.6	430	2.9
20	13	800	0.4	440	2.4
21	15	850	0.5	460	1.8
22	16	600	0.63	315	2
23	17	750	0.8	340	2.5
24	17.5	500	2.1	350	2.7
25	10.5	550	2.8	360	2.9
26	11.5	600	0.85	380	3.1
27	8.5	650	0.65	400	3.3
28	9.5	700	0.75	410	3.4
29	7.3	750	2.8	420	3.6
30	6.2	1000	3	480	3.8

3.5. Calculation of gears

General information

A **gearbox** is a mechanism that transmits or converts motion with a change in angular velocity and torque.

All concepts and definitions related to the geometry and kinematics of gear spans are standardised. The standards specify definitions, terms and designations and methods for calculating geometrical parameters.

A pinion gear consists of two wheels (Fig. 3.35) with teeth on the surface. The smaller of the gears is called a *pinion*, the larger a *wheel.* The definition of "pinion wheel" is generic. The parameters of the pinion are assigned index 1, those of the wheel are assigned index 2.

Fig. 3.35. Gear

There are the following types of gears: *helical, bevel, worm, planetary, wave, wave with Wildhaber-Novikov gearing*. Helical gears are the simplest, most reliable and most commonly used. Other gears are used when there is a need to transmit motion at an angle or when compactness of the drive is required.

Selection of material and allowable stresses

The choice of wheel material depends on the size, type, nature of the load, its operating conditions, dimensional and weight requirements, availability, price, means of obtaining semi-finished products and method of tooth processing.

The basic materials for gears are heat-treated steels. Cast iron and plastics are used less frequently, bronze and brass are used for worm gears.

Depending on the hardness of the working surfaces, steel gears can be divided into two basic groups:

(a) with hardness HB ≤ 350 – normalized and tempered;

(b) with hardness HB ≥ 350 – hardened, carburised, nitrided, nitrocarburised.

The mechanical properties of selected materials are shown in Tables D.19 and D.20.

Helical and bevel gears

The permissible contact stresses depending on the hardness of the tooth surfaces can be determined on the basis of gearbox application experience and research.

For steel wheels with hardness ≤ 350HB

$$k_k = 2.75 \cdot \mathrm{HB_{min}} K_{\mathrm{HL}}$$

The smallest value k_k

For steel wheels with hardness ≥ 350*HB*

$$k_k = 24.1 \cdot \mathrm{HRC_{min}} K_{\mathrm{HL}}$$

where $\mathrm{HB_{min}}$, $\mathrm{HRC_{min}}$ – minimum hardness of the material (Tables D.19 and D.20).

K_{HL} – durability factor, taking into account the service life and load mode of the gearbox.

For standardised and improved gears $1 \leq K_{HL} \leq 2.6$.

For 350 *HB* hardness and cast iron wheel is $0.585 \leq K_{HL} \leq 1.8$.

For slanted wheels with $HB_1 - HB_2 > 50$

$$k_k = 0.45(k_{k1} + k_{k2})$$

Whereby:

$k_k \leq 1.25 k_{k2}$- for helical wheels with angled teeth;

$k_k \leq 1.15 k_{k2}$- for tapered wheels with uneven teeth.

If $k_k \leq 1.25 k_{k2}$ then $k_k = 1.25 k_{k2}$

and if $k_k \leq 1.15 k_{k2}$ then $k_k = 1.15 k_{k2}$.

In other cases k_k take lower permissible stresses k_{k1} and k_{k2}.

Determination of the durability factor for tooth calculations based on contact stresses

$$K_{\mathrm{HL}} = \sqrt[6]{\frac{10^7}{N}}$$

where 10^7 – the basic number of cycles in determining the contact strength of steel;

N – number of stress-change cycles over the service life (working time), h.

Determination of the service life of the gearbox using formulas:

for sprocket $N_1 = 573\omega_1 L_h$;

for wheel $N_2 = 573\omega_2 L_h$,

where ω_1, ω_2 – angular velocity of the driving and propelling shafts, s^{-1};

L_h – specified gearbox life.

If the service life is not specified, the gearbox service life is assumed to be not less than 36 000 h (according to PN-M-88561:1987, the service life of a general-purpose gearbox should be not less than 36 000 h and the service life factor $K_{\mathrm{HL}} = 1$ is assume).

If the calculated K_{HL} value is greater or less than the specified range, the minimum or maximum value within the specified range is taken.

Determination of maximum permissible contact stresses

In the calculations, the maximum permissible contact stresses are determined to prevent either plastic deformation or brittle fracture of the tooth surfaces.

For steel

at HB ≤ 350 $\qquad k_{k\,max} = 2.5 \cdot k_k$

at HB ≥ 350 $\qquad k_{k\,max} = 2 \cdot k_k$

For cast iron

at HB ≤ 350 $\qquad k_{k\,max} = 1.8 \cdot \sigma_H$

at HB ≥ 350 $\qquad k_{k\,max} = 14 \cdot HRC$

Determination of allowable bending stresses

- if the teeth operate unilaterally (from a zero stress cycle, no backward movement)

$$k_g = \frac{(1.4 \div 1.6) \cdot R_{-1}}{K_F \cdot [n]} \cdot K_{FL}$$

- if the teeth work bilaterally (symmetrical stress cycle, backward movement)

$$k_{-1g} = \frac{R_{-1}}{K_F \cdot [n]} \cdot K_{FL}$$

where k_{0g} permissible bending stress at the zero cycle, MPa;

k_{-1g} – allowable bending stresses at symmetrical cycle, MPa;

R_{-1} – material strength limit at symmetric cycle, MPa;

for carbon steel $R_{-1} \approx 0.43 R_m$

for alloyed steel $R_{-1} \approx 0.45 R_m + (70 \div 120) \frac{\mathrm{N}}{\mathrm{mm}^2}$

for cast iron $R_{-1} \approx 0.45 R_m$

K_F – effective stress concentration factor at the base of the tooth.

For *design calculations* for Normalized and tempered steel wheels K_F = 1.8; for steel wheels after surface hardening and cast iron wheels K_F = 1.2;

[n] – permissible safety factor.

For forged normalized and tempered wheels [n] = 1.5; for forged hardened wheels [n] = 2.2; for cast Normalized and tempered wheels [n] = 1.8;

K_{FL} – durability factor when calculating the bending of teeth;

at $HB \leq 350$ $1 \leq K_{FL} \leq 2$

at $HB \geq 350$ $1 \leq K_{FL} \leq 1.65$

Determination of durability coefficient for bending tooth calculations

$$K_{FL} = \sqrt[9]{\frac{5 \cdot 10^6}{N}},$$

where $5 \cdot 10^6$ – number of stress cycles for all steel grades;

N – number of stress cycles over the service life (working time), h.

With continuously working gearbox (with an operating time of ≥ 36000 h) K_{FL} = 1.

If the project value of K_{FL} is less than or greater than the specified interval, the minimum or maximum value in the interval is taken..

The lower value k_k is used for further calculations.

Determination of maximum bending stresses

Maximum bending stresses are determined to prevent brittle fracture or plastic deformation of the teeth.

For steel wheels

at HB ≤ 350 $k_{g\,max} = 0.8 \cdot R_{e\,02}$

at HB ≥ 350 $k_{g\,max} = 0.36 \cdot R_m / K_F /$

Basic calculation formulae

Geometry of helical gears (Fig. 3.36).

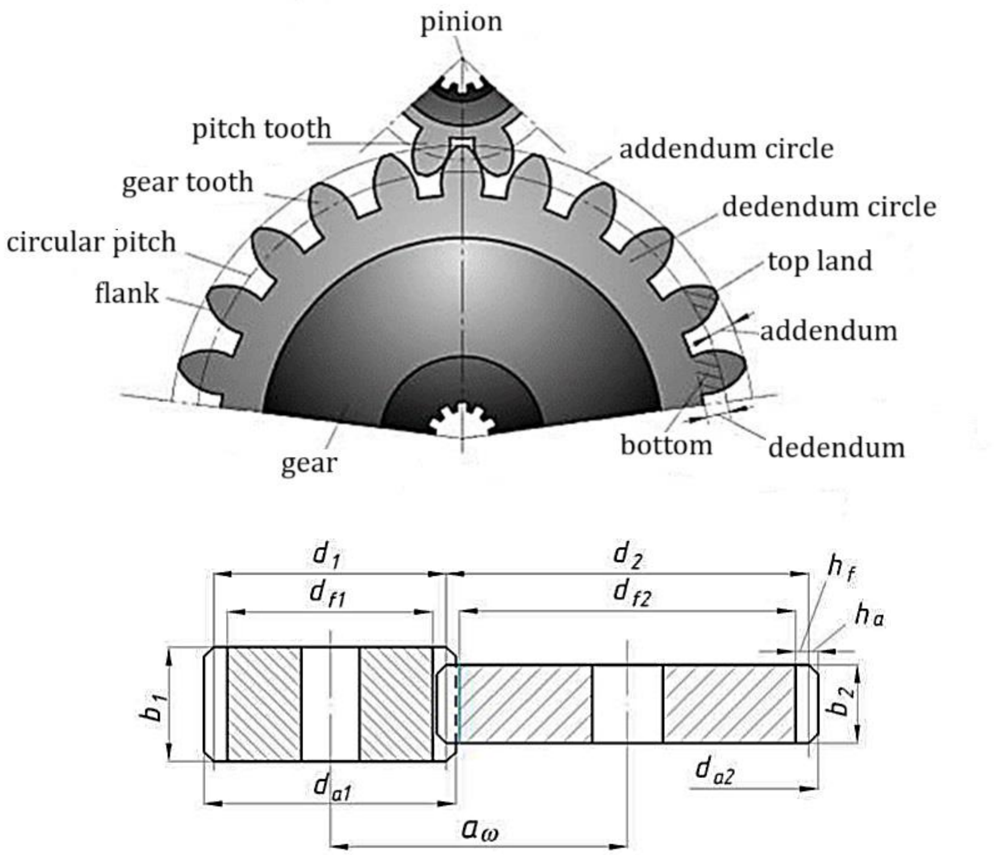


Fig. 3.36. The geometry of a helical gearbox

Gear ratios and transmission ratios

The parameter $u = z_2/z_1$ according to ISO/DIS 21771-2 is called the *gear ratio* and defines the ratio of the larger number of teeth to the smaller number.

The gear ratio is only considered about a pair of wheels, in other cases the ratio is considered, but due to its more frequent use, the ratio is called the gear ratio and the designation u is used:

$$i(u) = \frac{d_2}{d_1}; \frac{n_1}{n_2}; \frac{\omega_1}{\omega_2},$$

where d_1, n_1, ω_1 – pinion diameter, speed, pinion angular velocity, respectively;

d_2, n_2, ω_2 – wheel diameter, wheel speed, and wheel angular velocity respectively.

Reduction gearboxes are more commonly used, in which:

attachment module:

straight tooth, oblique gear tooth (Fig. 3.37)

$m = \frac{d}{z} = \frac{p}{\pi}$; $m_t = \frac{m_n}{\cos\beta}$,

where d – pinion diameter, mm;

z – number of teeth of the sprocket;

p – step of attachment, mm;

m_t – module, mm;

m_n – normal module, mm.

The straight-tooth gear is characterised by the fact that the end modulus is equal to the normal modulus;

β = 8 ÷ 16 ° – tooth angle of gearboxes β = 25 ÷ 40 ° – tooth angle of chevron gearboxes.

In a pair of related diagonal teeth with an external abutment, the angles β are equal in value but opposite in direction. One wheel is right-handed, the other left-handed.

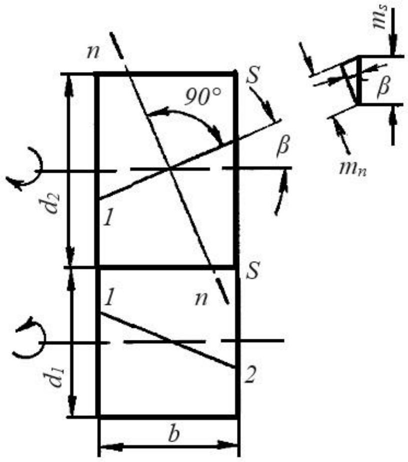


Fig. 3.37. Schematic diagram of a gearbox with oblique teeth

In practice, the module is often determined from the ratio:

$$m_n = (0.01 \div 0.02)a_w,$$

where a_w – the distance between axes from contact strength condition, mm.

Based on experience with gears, it is recommended to adopt a modulus $m_{\min} \geq 1.5$ mm.

The modules are standardised by PN-ISO 54:2001 in the range of 0.05 ÷ 00 mm (Table D.24).

Hitching step:

straight tooth	oblique tooth (Fig. 3.37)
$p_n = \pi \cdot m = \frac{\pi d}{z}$	$p_t = \frac{p_n}{\cos\beta}$
p_n – normal step, mm;	p_t – lateral step, mm

In a gearbox with a straight tooth, the normal steps are equal.

For a *pair of hitched wheels, the module should be the same.*

For non-corrected gears:

	gearbox with straight tooth	gearbox with oblique tooth
Height of tooth head, mm	$h_a = m$	$h_a = m_n$
Height of tooth base, mm	$h_f = 1.25m$	$h_f = 1.25m_n$
Tooth height, mm	$h = h_a + h_f = 2.25m$	$h = 2.25m_n$
Radial gap, mm	$c = 0.25m$	$c = 0.25m_n$
When cutting with a chisel	$c = 0{,}35m$	$c = 0{,}35m_n$

Non-corrected wheel diameter, mm:

	gearbox with straight tooth	gearbox with oblique tooth
(a) distribution wheel	$d = mz$	$d = m_t z = \frac{m_n z}{\cos\beta}$
(b) vertices	$d_a = d + 2m = d + 2h_a$	$d_a = d + 2m_n = d + 2h_a$
(c) tooth bases	$d_f = d - 2.5m = d - 2h_f$	$d_f = d - 2.5m_n = d - 2h_f$

Inter-axial distance, mm

$$a_w = 0.5(d_2 + d_1) = 0.5m(z_2 + z_1)$$

$$a_w = 0.5(d_2 + d_1) = 0.5m_s(z_2 + z_1)$$

$$a_w = \frac{m_n(z_2 + z_1)}{2\cos\beta}$$

Width of wheels, mm:

wheel	sprocket
$b_2 = \psi_a \cdot a_w$	$b_1 = b_2 + (5 \div 10)$ mm

where ψ_a – is the ratio of the width of the pinion rim to the inter-axial distance, determined from Table D.23.

Tooth length, mm:

gearbox with straight tooth	gearbox with oblique tooth
$l = b$	$l = \frac{b}{\cos\beta}$

Forces acting at the abutment

Straight tooth gearbox (Fig. 3.38):

peripheral force of sprocket and wheel $F_t = \frac{2T}{d}$;

radial force of pinion and wheel $F_r = F_t \cdot tg\alpha_W$

normal force $F_n = F_t/(cos\,\alpha_W \cdot cos\,\beta)$

where T – torque acting on the shaft, Nmm;

d – diameter of the distribution wheel, mm;

$\alpha_W = 20\,°$ – angle of engagement of non-corrected gear;

On the driven wheel, the direction of the peripheral force coincides with the direction of rotation; on the driven wheel, it is opposite.

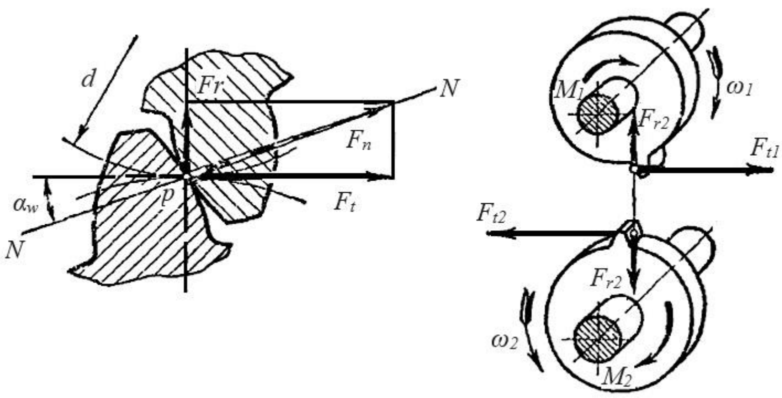


Fig. 3.38. Forces acting in a straight helical gear mesh

Gearbox with oblique tooth (Fig. 3.39):

peripheral force $F_t = \frac{2T}{d}$,

radial force $F_r = F_t \frac{tg\alpha_W}{cos\,\beta}$,

radial force $F_a = F_t \cdot tg\beta$,

normal force $F_n = \frac{F_t}{(cos\,\alpha_W \cdot cos\,\beta)}$.

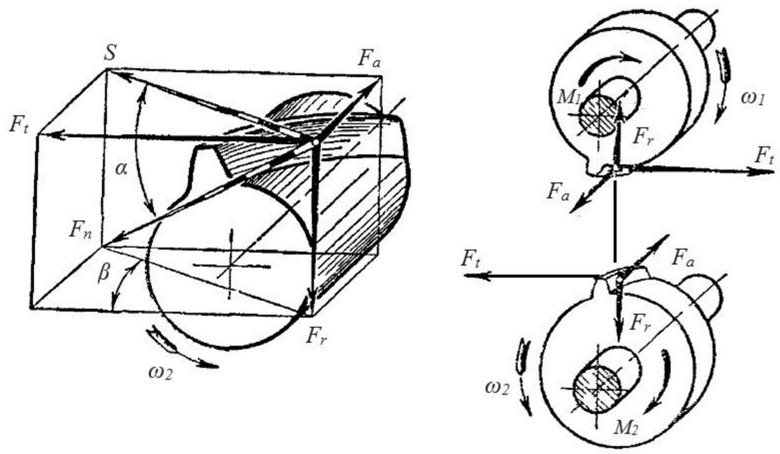


Fig. 3.39. Forces acting in a gear mesh with bevel teeth

Determination of the peripheral speed of the abutment

$$V = \frac{\omega d}{2 \cdot 1000} \text{ m/s}$$

where ω – wheel rotastional speed, s^{-1};

d – wheel diameter, mm.

Formulae for calculating spur gears

The basic performance criteria for helical gears are contact tooth strength and tooth bending strength.

For helical gears, calculations are carried out for contact strength, bending strength, calculation of maximum load to prevent plastic deformation or brittle fracture as a result of short-term peak loads (e.g. when starting an electric motor), and thermal calculations for heavily loaded high-speed gears.

When designing gears, a distinction is made between *design* and *verification* calculations.

In design calculations, the required gearbox dimensions are determined by the specified load and known allowable stresses.

In the verification calculations, the actual stresses in the teeth are determined using the specified load and dimensions and compared with the permissible ones. In addition, calculations at maximum load and, where necessary, thermal calculations are performed.

Verification calculations are generally carried out for the teeth of the less hard wheel. When using materials of the 1st hardness group < 350 HB, these are often the teeth of the wheel. For materials of the 2nd hardness group ≥ 350 HB, calculations are made for the teeth of the pinion and the wheel.

For open gears, it is fundamental to calculate the teeth for bending due to the high wear in this type of gear.

Straight tooth gearbox

For design calculations:

$$m = \sqrt[3]{\frac{2T_1 K_F Y_F}{k_g \psi_{bm} z_1}}$$

For verification:

$$\sigma_F = \frac{F_t K_F Y_F}{bm} \leq k_g,$$

where F_t – peripheral force, N;

K_F – load factor;

$$K_F = K_{F\beta} \cdot K_{F\alpha} \cdot K_{F\upsilon}$$

$K_{F\beta}$ – coefficient of unequal loading across the width of the rim (Table D.27);

$K_{F\alpha}$ – coefficient of uneven load between the teeth (Table D.27);

$K_{F\upsilon}$ – dynamic coefficient (Table D.28);

Y_F – tooth form factor (Table D.29);

b – wheel width, mm;

m – module, mm;

T_1 – torque on the sprocket, N·m;

z_1 – number of teeth on the sprocket;

$\psi_{bm} = b/m$ – wheel rim width ratio;

$[F_\sigma]$– allowable bending stresses, MPa.

Bevel tooth and chevron gearing

For design calculations:

$$m = \sqrt[3]{\frac{2T K_F Y_F Y_\beta K_{F\alpha} \cos\beta}{k_g \psi_{bm} z}}$$

For verification:

$$\sigma_F = \frac{F_t K_F Y_F Y_\beta K_{F\alpha}}{b m_n} \leq k_g,$$

where β – tine angle;

m_n – normal module, mm;

z – number of teeth of the pinion or wheel;

T – torque on the pinion or wheel, Nm;

$Y_\beta = 1 - \frac{\beta^\circ}{140}$ – tooth line inclination factor.

For closed gear bars, calculations for the contact strength of the teeth are basic, and calculations for bending are carried out on verification.

For design calculations:

$$a_w \geq K_a \cdot (u \pm 1) \cdot \sqrt[3]{\frac{T_2 \cdot 10^3 \cdot K_{H\beta}}{\Psi_a \cdot u^2 \cdot k_g^2}}.$$

where K_a – support factor. For gears with oblique teeth

K_a = 43, with straight teeth K_a = 49.5;

u – transmition ratio;

the "–" sign when calculating the internal abutment;

T_2 – torque on the free gear shaft, Nm;

10^3 – conversion factor m in mm;

K_H – coefficient of uneven load distribution along the length of the tooth;

Ψ_a – ratio of gear rim width to inter-axial distance;

k_g – permissible or average contact stresses, MPa.

The calculated value of the inter-axial distance is rounded to the nearest standardised one.

For verification calculations against contact stresses:

gearbox with straight teeth

$$\sigma_H = \frac{310}{a_w} \cdot \sqrt{\frac{T_2 . K \cdot (u+1)^3}{b_2 \cdot u^2}} \leq k_g,$$

gearbox with oblique tooth

$$\sigma_H = \frac{270}{a_w} \cdot \sqrt{\frac{T_2 . K_H \cdot (u+1)^3}{b_2 \cdot u^2}} \leq k_g,$$

where a_W – inter-axial distance, mm;

T_2 – torque on low-speed gear shaft, Nmm;

K_H – load factor:

$$K_H = K_{H\beta} K_{H\alpha} K_{H\upsilon}$$

$K_{H\beta}$ – coefficient of non-uniformity of load distribution over the width of the rim (Table D.25);

$K_{H\alpha}$ – coefficient of unevenness of stress distribution between teeth (Table D.27);

$K_{H\upsilon}$- dynamic factor (Table D.28);

u – transmission ratio;

b_2 – wheel width, mm;

k_g – permissible contact stresses, MPa.

For verification calculations against bending stresses:

$$\sigma_F = \frac{F_t K_F Y_F Y_\beta K_{F\alpha}}{b m_n} \leq k_g$$

For verification calculations against limit stresses for open, closed gears, straight tooth and helical oblique gears:

Relative to contact stresses:

$$k_{g_{max}} = \sigma_{\mathrm{H}} \sqrt{\frac{T_{1max}}{T_1}} \leq k_{g\ lim},$$

where σ_{H} – calculated contact stresses, MPa;

$k_{g\ lim}$– limit permissible contact stresses, MPa;

T_1 – pinion torque, Nmm;

$T_{1\max}$ – sprocket torque at maximum load, Nmm.

$$k_{g_{max}} = \sigma_F \frac{\mathrm{T}_{1max}}{\mathrm{T}_1} \leq k_{g\ lim}$$

where σ_{F} – calculated bending stress, MPa;

$k_{\ g\ lim}$– limit permissible wheel bending stress, MPa.

Thermal calculations for gearboxes

Thermal calculations for helical and bevel gears are performed as an additional calculation when operating at high loads and speeds. For worm gears, these are basic calculations, as they operate under conditions of strong heat generation that can lead to damage. Thermal calculations are performed for a fixed operating mode based on the heat balance, i.e. the equality of heat release and heat transfer:

$$Q_{\mathrm{rel}} = Q_{\mathrm{tran}}$$

In thermal calculations, the temperature of the lubricant is often specified, whereby the condition should be met:

$$t_S = t_0 + \frac{(1-\eta)N_1}{K_{\mathrm{T}} A (1+\psi)} \leq [t_S]$$

where η – the overall efficiency of the gearboxs;

N_1 – power at the pinion, W;

t_0 – heat transfer coefficient from the surface of the enclosure (higher value with good air circulation at the ambient temperature) t_0 = 20 °C;

K_{T} = 8 ÷ 17 W/(m^2 ·°C) – heat transfer coefficient from the surface of the enclosure (higher value with good air circulation at the ambient temperature);

A – cooling area of the gearbox housing (not including the bottom), m^2;

ψ – heat transfer coefficient through the bottom of the enclosure (0.3 if the bottom is against the enclosure, 0 if the bottom is against concrete or brick);

$[t_S]$ – permissible temperature of the lubricant in the housing.

For common grease $[t_S]$ = 70 ÷ 90 °C, for aviation grease $[t_S]$ = 20 ÷ 100 °C.

If the heat balance condition is not met $Q_B > Q_0$, , then additional heat dissipation must be taken into account. This is achieved by the following means; increasing the cooling surface area A using cooling fins; blowing air into the housing using a fan mounted on the worm shaft; placing water cavities or a coil with running water in the housing; using grease circulation systems with special coolers.

Examples of calculations

Example 3.26. For a helical gear with oblique teeth (Fig. 3.40), determine the pitch and inner diameter of the wheel, normal and final (circumferential) meshing modulus, tooth height, distance between axes and forces acting in the meshing. Take the necessary data from the drawing.

Data:
N_1 = 10 kW
n_1 = 1000 min^{-1}
z_1 = 20
z_2 = 80
m_n = 3 mm
β = 10°

Searched for:
d_1 - ? d_2 - ?
d_{a1} - ? d_{a2} - ?
p_t - ? h - ?
a_w - ? F_t - ?
F_r - ? F_a - ?

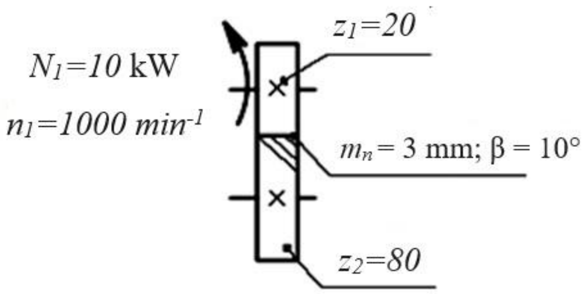


Fig. 3.40. Diagram of helical bevel gearbox

Solution

1. Define the unknowns.

1.1. We determine the diameters of the wheel bores:

sprocket

$$d_1 = \frac{m_n z_1}{cos\,\beta} = \frac{3 \cdot 20}{cos\,10°} = 61.2245 \text{ mm}$$

wheel

$$d_2 = \frac{m_n z_2}{cos\,\beta} = \frac{3 \cdot 80}{cos\,10°} = 244.8979 \text{ mm}$$

1.2. Determining the outside diameter of the wheels.

sprocket

$$d_{a1} = d_1 + 2m_n = 61.2245 + 2 \cdot 3 = 67.225 \text{ mm}$$

wheel

$$d_{a2} = d_2 + 2m_n = 244.8979 + 2 \cdot 3 = 250.898 \text{ mm}$$

1.3. Determining the pitch of the end gear.

$$p_t = \pi \frac{d_1}{z_1} = 3.14 \cdot \frac{61.2245}{20} = 9.612 \text{ mm}$$

1.4. Determining the height of the tooth.

$$h = 2.25 m_n = 2.25 \cdot 3 = 6.75 \text{ mm}$$

1.5. Determine the distance between the axes

$$a_w = 0.5(d_1 + d_2) = 0.5(61.2245 + 244.8979) = 153 \text{ mm}$$

From the the Table D.26 we assume a_w=160 mm.

1.6. Determine the forces acting in the mesh.

First determine the pinion torque

$$T_1 = 9.55 \frac{N_1}{n_1} = 9.55 \cdot \frac{10 \cdot 10^3}{1000} \approx 96 \text{ Nm, then}$$

Peripheral force

$$F_{t_1} = \frac{2T_1}{d_1} = \frac{2 \cdot 96 \cdot 10^3}{61.2245 = 3136\,N}$$

Radial force

$$F_{r1} = \frac{F_{t1} tg\alpha}{\cos\beta} = 3136 \cdot \frac{tg20°}{\cos 10°} = 1159\text{N}$$

where α – attachment angle, α = 20 °.

axial force

$$F_{a1} = F_{t1} tg\beta = 3136 \cdot tg10° = 552\ \text{N}$$

Answer: d_1 = 61.2245 mm; d_2 = 244.8979 mm; d_{a1} = 67.225 mm; d_{a2} = 250.898 mm; p_t = 9.612 mm; h =6.75 mm; a_w=160 mm; F_{t1} = 3136 N; F_{r1}=1159 N; F_{a1} =552 N.

Example 3.27. Check the contact tooth strength of a helical bevel gearbox if: transmitted power N_1 = 15 kW; rotational frequency of the high-speed shaft n_1 = 750 min^{-1}; transmission ratio u = 3.5; number of pinion teeth z_1 = 23; tooth inclination angle β = 12 °; mesh modulus m_n = 3 mm; wheel material - 40H steel Normalized, wheel width factor ψ_{ba} = 0.315; durability factor K_{HL} = 1; load factor K_H = 1.2.

Data:	*Searched for:*
N_1 = 15 kW	σ_H - ?
n_1 = 750 min^{-1}	
u = 3,5	
z_1 = 23	
β = 12 °	
m_n = 3 mm	
ψ_{ba} = 0.315	
K_{HL} = 1	
K_H = 1.2	
40H normalized steel	

Solution

1. Write the contact strength condition for a helical gear with bevel teeth:

$$\sigma_H = \frac{270}{a_w}\sqrt{\frac{T_2.K_H(u+1)^3}{b_2 u^2}} \le k_g$$

2. Define the unknowns.

2.1. Determine pinion and wheel parameters

$$d_1 = \frac{m_n z_1}{cos\,\beta} = \frac{3 \cdot 23}{cos\,12°} = 70.54 \text{ mm}$$

$$d_2 = d_1 u = 70.542 \cdot 3.5 = 246.895 \text{ mm}$$

2.2. Determine the distance between the axes

$$a_w = \frac{d_1 + d_2}{2} = \frac{70.542 + 246.895}{2} = 158.72 \text{ mm}$$

From the Table D.26 we assume a_w= 160 mm.

2.3. Determine the permissible contact stresses for the wheel.

Taking into account wheel diameter $d_2 \approx$ 245 mm, material steel 40H and type of heat treatment from the Table D.19, material hardness is HB_{min} = 220, for steel wheels with hardness $\leq$ 350 HB

$$k_g = 2.75 HB_{min} K_{HL} = 2.75 \cdot 220 \cdot 1 = 605 \text{ MPa}$$

2.4. Define torques.

pinion

$$T_1 = 9.55 \frac{N_m}{n_m} = 9.55 \cdot \frac{15 \cdot 10^3}{750} = 191 \text{ Nm}$$

wheel (because gear is helical η =0,98)

$$T_2 = T_1 u \eta = 191 \cdot 3.5 \cdot 0.98 = 655 \text{ Nm}$$

2.5. Determine wheel width.

$$b_2 = \psi_{ba} \cdot a_w = 0.315 \cdot 160 = 50{,}4 \text{ mm}$$

From the Table D.21 we assume b_2 = 50 mm.

3. We determine the contact stresses and assess the contact strength of the teeth

$$\sigma_\mathrm{H} = \frac{270}{\mathrm{a}_w}\sqrt{\frac{T_2 . K_\mathrm{H}(u+1)^3}{b_2 u^2}} = \frac{270}{160}\sqrt{\frac{655 \cdot 10^3 \cdot 1.2 \cdot (3.5+1)^3}{50 \cdot 3.5^2}} = 577 \frac{N}{\mathrm{mm}^2} = 577 \text{ MPa}$$

$$577 \text{ MPa} < k_g = 605 \text{ MPa}$$

Strength condition is met.

Example 3.28. Check the temperature of the worm gearbox (Fig. 3.41), if: transmitted power N_1 = 2.2 kW; worm speed z_1 = 1; ambient temperature t_o = 20 °C; the gearbox is installed on a metal frame; heat transfer coefficient K_T = 15 W/(m² · °C); permissible lubricant temperature range [t_S] = 70 ÷ 90 °C; case dimensions are shown in Fig. 3.41.

Data:

N_1 = 2.2 kW

z_1 =1

t_o = 20 °C

K_T = 15 W/(m²·°C)

$[t_s]$ = 70 ÷ 90 °C

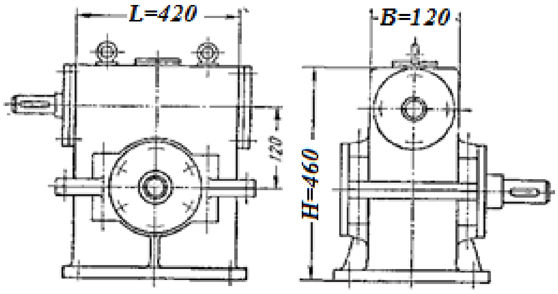


Fig. 3.41. The worm gearbox

Searched for:

t_s-?

Solution

1. Write down the heat balance condition for the worm gearbox.

$$t_s = t_o + \frac{(1-\eta)N_1}{K_T A(1+\psi)} \leq [t_s]$$

2. Define the unknowns.

2.1. Determine the efficiency of the worm gearbox.

With rough calculations, the efficiency of a worm gearbox can be determined from the number of worm revolutions.

z_1	η
1	0.72 ÷ 0.78
2	0.78 ÷ 0.82
3	0.82 ÷ 0.87
4	0.87 ÷ 0.92

For z_1 = 1 we take η = 0.75.

2.1. Determine the area of the gearbox housing through which heat penetrates.

$$A = 2(B \cdot H) + 2(L \cdot H) + B \cdot L = 2(0.12 \cdot 0.46) + 2(0.42 \cdot .46) + 0.12 \cdot 0.42 = 0.55 \text{ m}^2$$

2.1. Determine the coefficient, taking into account the heat dissipation through the lower part of the gearbox housing.

If the gearbox body is installed on a concrete, reinforced concrete or stone foundation ψ =0, if on an iron frame - ψ = 0.2 ÷0.3.

Assuming that the gears are installed on an iron frame, ψ = 0.2.

3. Determine the temperature of the lubricant and determine the temperature mode of the gearbox

$$t_s = t_o + \frac{(1-\eta)N_1}{K_T A(1+\psi)} = t_o + \frac{(1-0.75)2.2 \cdot 10^3}{0.15 \cdot 0.55(1+0.2)} = 65^{\circ}\text{C} < [t_s] = \; 70 \div 90\text{°C}$$

the temperature of the gearbox is normal.

Individual tasks
(calculation)

Task 3.12. For a helical bevel gearbox, determine the pitch and inside diameter of the wheel, the normal and end (circumferential) modulus of the mesh, the tooth height, the distance between axes and the forces acting in the mesh (Table 3.12).

Table 3.12. Initial data for Task 3.12

Var. no	N kW	n_1 min^{-1}	z_1	u	Normal step p_n, mm	β, °
1	4.5	250	18	2	6.28	9
2	5.0	300	19	3	7.85	10
3	5.5	250	20	4	9.42	11
4	6.0	320	21	5	10.99	12
5	6.5	300	22	6	12.56	13
6	7.0	340	25	4	14.13	14
7	7.5	180	26	2	15.7	15
8	8.0	160	28	2	6.28	16
9	8.5	150	30	3	7.85	9
10	9.5	240	22	4	9.42	10
11	10.0	260	20	5	6.28	11
12	10.5	220	18	6	7.85	12
13	11.0	200	18	4	9.42	13
14	11.5	280	19	2	10.99	14
15	12.0	300	20	2	12.56	15
16	16.5	260	21	3	14.13	16
17	12.5	240	22	4	15.7	9
18	13.0	230	25	5	6.28	10
19	13.5	260	26	6	7.85	11
20	14.0	220	28	4	9.42	12
21	14.5	250	30	2	6.28	13
22	15.0	180	22	2	7.85	14
23	16.0	240	20	3	9.42	15
24	17.0	240	18	4	10.99	16
25	17.5	150	18	5	12.56	9
26	18.0	170	19	6	14.13	10
27	19.0	180	20	4	15.7	11
28	20.0	140	21	2	6.28	12
29	21.0	300	22	2	7.85	13
30	22.0	320	25	3	9.42	14

Task 3.13. Check the contact strength of the teeth of a helical gearbox with bevelled teeth. The data for the calculationas is shown in Table 3.13.

Table 3.13. Initial data for Task 3.13

Var. no	N kW	n_1 min^{-1}	m_n, mm	z_1	u	β, °	ψ_{ba}	K_{HL}	K_H	Steel	Heat treatment
1	4.5	750	2	18	2	9	0.25	1.1	1.1	C35	Normalizing
2	5.0	800	2.5	19	3	10	0.315	1.0	1.05	C40	
3	5.5	900	3	20	4	11	0.4	1.12	1.0	C45	
4	6.0	950	3.5	21	5	12	0.25	1.14	1.12	C50	
5	6.5	1000	4	22	6	13	0.315	1.2	1.15	C55	
6	7.0	1050	4.5	25	4	14	0.4	1.4	1.11	30HGS	
7	7.5	1100	5	26	2	15	0.25	1.3	1.3	35H	Thermal upgrading
8	8.0	1200	1.5	28	2	16	0.315	1.5	1.4	40H	
9	8.5	1250	2	30	3	9	0.4	1.6	1.2	40HN	
10	9.5	1300	2.5	22	4	10	0.25	1.7	1.25	C35	
11	10.0	1400	3	20	5	11	0.315	1.8	1.35	C40	
12	10.5	1450	3.5	18	6	12	0.4	2.0	1.1	C45	
13	11.0	1500	4	18	4	13	0.25	2.1	1.05	C50	Normalizing
14	11.5	750	4.5	19	2	14	0.315	1.25	1.0	C55	
15	12.0	800	5	20	2	15	0.4	1.35	1.12	30HGS	
16	16.5	900	1.5	21	3	16	0.25	1.1	1.15	35H	
17	12.5	950	2	22	4	9	0.315	1.0	1.11	40H	
18	13.0	1000	2.5	25	5	10	0.4	1.12	1.3	40HN	
19	13.5	1050	3	26	6	11	0.25	1.14	1.4	C35	Thermal upgrading
20	14.0	1100	3.5	28	4	12	0.315	1.2	1.2	C40	
21	14.5	1200	4	30	2	13	0.4	1.4	1.25	C45	
22	15.0	1250	4.5	22	2	14	0.25	1.3	1.35	C50	
23	16.0	1300	5	20	3	15	0.315	1.5	1.1	C55	
24	17.0	1400	1.5	18	4	16	0.4	1.6	1.05	30HGS	
25	17.5	1450	2	18	5	9	0.25	1.7	1.0	35H	Normalizing
26	18.0	1500	2.5	19	6	10	0.315	1.8	1.12	40H	
27	19.0	750	3	20	4	11	0.4	2.0	1.15	40HN	
28	20.0	800	3.5	21	2	12	0.25	2.1	1.11	C35	
29	21.0	900	4	22	2	13	0.315	1.25	1.3	C40	
30	22.0	950	4.5	25	3	14	0.4	1.35	1.4	C45	

3.6. Selection of reducers

General information

Reducers - are devices that are manufactured as separate units and are designed to reduce angular velocities and increase torques.

Reducers are widely used in the drives of machines and mechanisms.

A general criterion for the *technical level* of reducers is the specific weight - the ratio of the weight of the reducer to the torque on its low-speed shaft:

$$\gamma = \frac{m}{T_2},$$

where m – weight, kg,

T_2 – torque, Nm.

Its value is highly dependent on the hardness of the gears. For high-tech reducers γ = 0.03 ÷ 0.05.

Gearboxes are very diverse in terms of kinematic schemes and design.

Classification of reducers. By *type of gearbox*: spur gearboxes, bevel gearboxes, worm gearboxes, planetary gearboxes, wave gearboxes, combination gearboxes (bevel and helical gearboxes, worm gearboxes);

- *according to the arrangement of the teeth on the wheel rim*: spur gears, helical gears, chevron gears, bow gears;
- *by several stages*: single-stage and multi-stage. The number of gear stages can be defined as the number of shafts minus one. Typically, the number of stages exceeding three is rarely used due to the large size and cost of such reducers;
- *by location of shafts and wheels in space*: horizontal, vertical, inclined;
- *according to the mounting method*: on feet or plate (with base) – for mounting on foundations, floors, and frames; with flanges for mounting on housings, frames of machines and mechanisms; slip-on - low-speed mounted directly on the working shaft of the machine; combined – for various mounting; according to mounting diagram;
- *according to climatic requirements*: the gearboxes are designed for operation in macroclimatic regions with temperate, tropical, moderately cold, cold climates, etc.
- *by location categories*, which are regulated by the relevant standards.

Selection and calculation of reducers

The selection of series reducers is made according to the transmitted torques from the manufacturers' catalogues. Preliminary data for gearbox selection are: the highest load value corresponding to normal gearbox operation; the operating mode; the speed of the high-speed shaft; the gear ratio; the location of the motor and the working body of the machine; and the operating conditions.

Condition for selection of reducer

$$T_{calc} = K_r T_n \leq T_t, u_f \approx u_t, n_{rz.p.o.} \leq n_{max},$$

where K_r – mode factor;

T_r, T_n, T_t – calculation torque, nominal on the input shaft and tabular torque, Nm;

u_f, u_t – factual and tabular translation;

$n_{rz.p.o}$ – actual speed of the high-speed reducer shaft, min^{-1};

n_{max} – maximum tabulated reducer shaft speed, min^{-1}.

The permissible overload is 10 %, and the permissible underload is 20 %.

The calculation of gear reducers includes the calculation of components such as gears, shafts, bearings, and bolts close to the bearings, checking of keyway connections and thermal calculations (for high-speed worm gears).

The design, calculation and selection of materials for gearbox components are described in design and technical manuals.

When solving gearbox selection tasks, it is advisable to pay attention to the position of the shafts in the task diagrams. If the shafts are aligned parallel in the gearbox diagram, the gearbox will be cylindrical; if they are angled, it may be a worm or bevel.

Examples of calculations

Example 3.29. Select from the catalogue a reducer for the electric drive of the winch (Fig. 3.42) indicating its type, number of stages, overall dimensions, weight and size of bolts for its attachment, if: motor shaft speed n_1 = 945 min^{-1}; winch lifting capacity Q = 1,5 t; load lifting speed v_l = 1.2 m/s; drive shaft diameter D_b = 500 mm; mode factor K_r = 1.3; permissible ratio deviation Δu ±4%.

Data:
n_1 = 945 min^{-1}
Q = 1.5 t
V_l = 1.2 m/s
D_b = 500 mm
K_r = 1.3
Δu ± 4 %

Searched for:
Reducer - ?

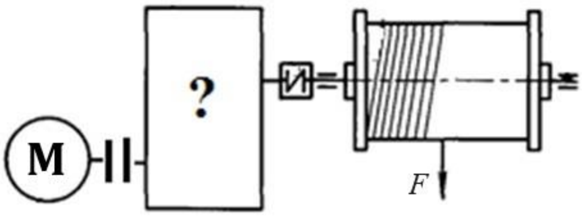


Fig. 3.42. Electric winch drive

Solution

1. Reducer selection condition.

$$T_{calc} = K_r T_n \le T_t, u_f \approx u_t, n_{rz.p.o.} \le n_{max}$$

2. Define the unknowns.

2.1. Reduce the load capacity to a single designation and units.

$$Q = F_t = 15000 \text{ N}$$

2.2. Determining the rated torque of the slow-speed shaft.

$$T_{z.w.w} = \frac{F_t D_b}{2} = \frac{15 \cdot 10^3 \cdot 0.5}{2} = 3750 \text{ Nm}$$

2.3. Determine torque.

$$T_{calc} = K_{calc} T_{z.w.w} = 1.3 \cdot 3750 = 4875 \text{ Nm}$$

2.4. Determine the rotational speed of the drum, which is equal to the speed of the slow-running shaft of the reducer.

$$n_b = n_{w.w} = \frac{60 v_l}{\pi D_b} = \frac{60 \cdot 1.2}{3.14 \cdot 0.5} = 46 \text{ rpm}$$

2.5. Determining the gear ratio of the reducer.

$$u_p = \frac{n_m}{n_b} = \frac{945}{46} = 20.5$$

3. Selecting a reducer.

Taking into account the condition for the selection of the gear reducer and the obtained values for the design torque and ratio, as well as the parallel alignment of the gear shafts in the drive diagram from the catalogue in Table D.33, a cylindrical two-stage gear reducer of size 1C2U-250 is selected, for which:

- rated torque T_z = 5000 Nm;
- ratio u_z = 20;

Permissible ratio deviations

$$\Delta u = \frac{u_f - u_z}{u_z} \cdot 100\% = 2.5\ \% < 4\ \%$$

Condition is met

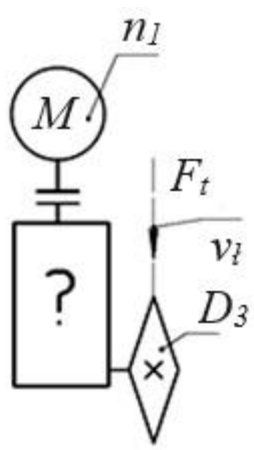
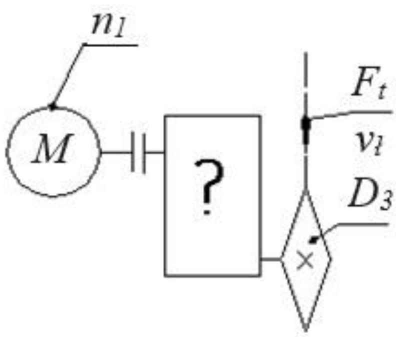
From Table D.33 we also extract the dimensions L = 825 mm; $B = L_4 + L_5$ =265+335= 600 mm; H = 515 m; weight of reducer m = 320 kg; diameter of the mounting screw holes d = 28 mm, we assume M24 screws.

Answer: Reducer 1C2U-250.

Individual task
(calculation)

Task 3.14. Select a reducer for an electric winch drive from a catalogue, indicating its type, number of steps, overall dimensions, weight and the size of the bolts to attach it. The initial data are shown in Table 3.14.

Table 3.14. Initial data for Task 3.14

Var. no	F_t, kN	n_1, min^{-1}	D_3, mm	V_l, m/s	K_r	Δu	Drive scheme
1	4.5	750	500	0.2	1.0	4	
2	5.0	800	600	0.25	1.1	5	
3	5.5	900	700	0.3	1.2	6	
4	6.0	950	750	0.35	1.15	7	
5	6.5	1000	800	0.5	1.2	9	
6	7.0	1050	850	0.4	1.3	8	
7	7.5	1100	900	0.45	1.4	10	
8	8.0	1200	950	0.6	1.5	4	
9	8.5	1250	1000	0.7	1.6	5	
10	9.5	1300	1100	0.8	1.05	6	
11	10.0	1400	1200	1.0	1.12	7	
12	10.5	1450	500	1.1	1.0	9	
13	11.0	1500	600	1.2	1.1	8	
14	11.5	750	700	1.3	1.2	10	
15	12.0	800	750	1.25	1.15	4	
16	16.5	900	800	1.5	1.2	5	
17	12.5	950	850	1.4	1.3	6	
18	13.0	1000	900	1.45	1.4	7	
19	13.5	1050	950	0.2	1.5	9	
20	14.0	1100	1000	0.25	1.6	8	
21	14.5	1200	1100	0.3	1.05	10	
22	15.0	1250	1200	0.35	1.12	4	
23	16.0	1300	500	0.5	1.0	5	
24	17.0	1400	600	0.4	1.1	6	
25	17.5	1450	700	0.45	1.2	7	
26	18.0	1500	750	0.6	1.15	9	
27	19.0	750	800	0.7	1.2	8	
28	20.0	800	850	0.8	1.3	10	
29	21.0	900	900	1.0	1.4	4	
30	22.0	950	950	1.1	1.5	5	

3.7. Graphical schemes of gearbox elements. Creation of kinematic diagrams of drives

General information

Conventional graphical representations of the elements of machines and mechanisms are special simplified images used for drawing up kinematic diagrams and showing the basic structure of the mechanism or machine and the interaction of the elements of machines and mechanisms. The conventional graphic symbols in kinematic diagrams and the rules for drawing up kinematic diagrams are governed by the relevant standards. Tables 3.2 and 3.3 below provide basic graphical representations of the kinematic elements of machines and machinery mechanisms.

Rules for plotting kinematic diagrams

As a rule, the kinematic diagram of the product is drawn up in the form of a summary drawing (Fig. 3.43).

The kinematic diagrams show:

- shafts, axles, connecting rods, cranks and others - basic continuous lines of thickness s;
- components shown in simplified outline, gears, worms, pulleys, couplings, bearings, etc. - in continuous lines of thickness $s/2$;
- the contour of the product (e.g. casing) into which the graph is inscribed - with continuous thin lines of thickness $s/3$.

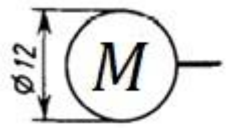
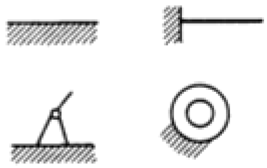
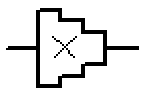
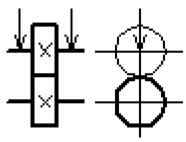
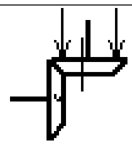
According to the standard, the thickness of the baselines on kinematic diagrams should be in the range of 0.5 ÷ 1.4 mm. When using computer graphics programs, it is recommended to set the following line thickness in the sheet: basic s = 0.6 mm, then s/2 = 0.3 mm and $s/3$ = 0.2 mm.

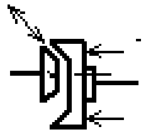
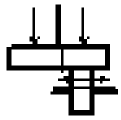
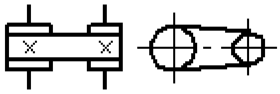
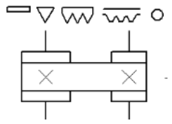
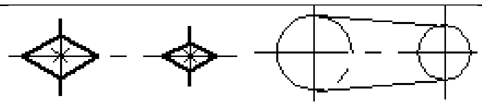
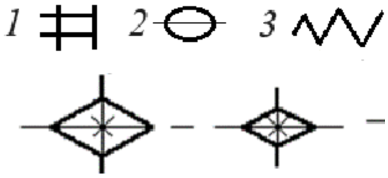
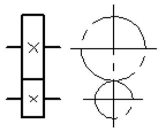
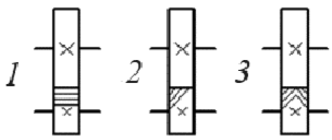
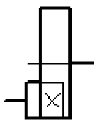
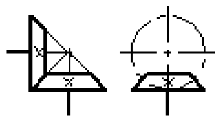
Each kinematic component shown in the diagram is usually assigned a serial number, starting with the motor. Shafts are numbered in Roman numerals, other components are numbered in Arabic numerals only (Fig. 3.43). The serial number of the component is placed on a line. Below the line are the main features and parameters of the kinematic element, a list of which is given in Table 3.15.

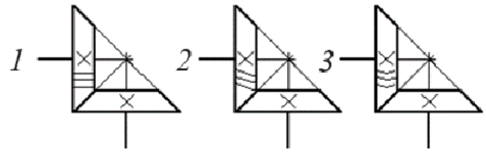
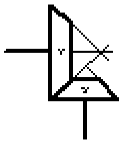
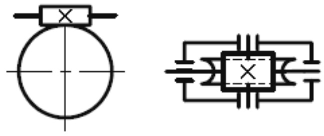
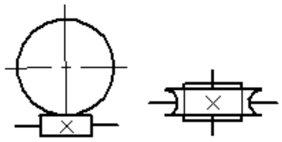
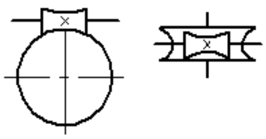
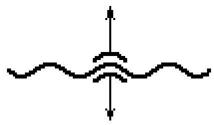
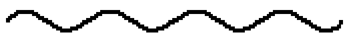
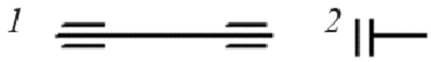
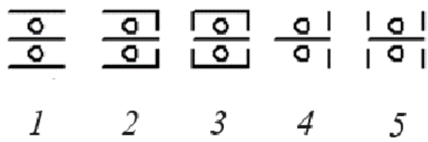
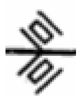
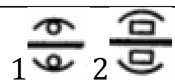
Table 3.15. List of basic parameters and characteristics of kinematic elements

Kinematic element	Parameter, characteristics
1. Motor	Type; power N, W; rotation n, rpm
2. Reducer, belt, chain and other gears	Transmission ration u
3. Pulleys	Diameter $\varnothing$, mm
4. Sprocket	Number of sprocked teeth z, chain step t, mm
5. Gears, worm wheels	Number of teeth z, module m, mm; tine angle β (for oblique teeth)
6. Snails	Screw type (if not Archimedean), number of screw coils z, module m, mm

Table 3.16. Graphical symbols for machine elements and mechanisms in kinematic diagrams (ISO 5127:2017)

Name	Symbol
Source of movement (motor)	
Shaft, axle, rod, connecting rod, etc.	
Fixed link (stand)	
Multistage pulley	
Friction gear:	
Helical	
oblique	

Name	Symbol
oblique adjustable	
front adjustable	
Belt transmission:	
without specifying the type of belt	
With the type of belt identified (the belt profile is drawn alongside). There are the profiles of flat, V-belts, multiple V-belts and round belts.	
Chain :	
Without specifying the type of chain	
With the identification of the type of chain. There are the conventional chain designations: 1) plate, roller, sleeve; 2) calibrated, anchor; 3) toothed	
Spur gear:	
without specifying the type of tooth	
with an indication of the type of tooth:	
1) straight tooth; 2) oblique tooth;	
3) chevron	
Internal pinion gears	

Name	Symbol
with an indication of the type of tooth:	
1) straight tooth; 2) oblique tooth;	
3) with round teeth	
Hypoid gearbox	
Worm gear with cylindrical worm:	
with top auger system	
with bottom auger system	
Worm gearbox	
Screw-nut gearbox:	
with one-piece guide	
integral with rolling elements	
sliding distributor	
Thread	
Plain and antifriction bearings without specification of type: 1) radial; 2) longitudinal	
Rolling bearings:	

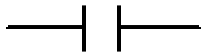
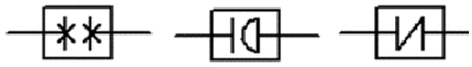
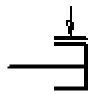
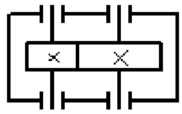
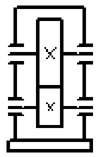
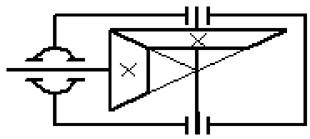
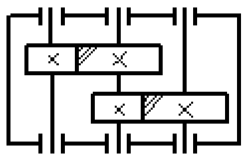
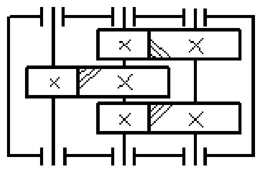
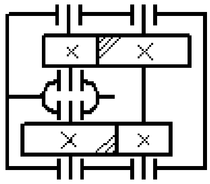
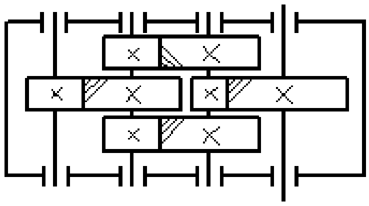
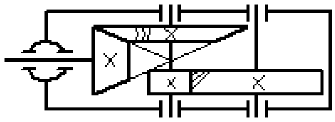
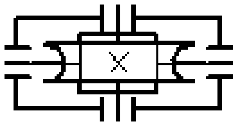
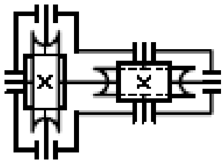
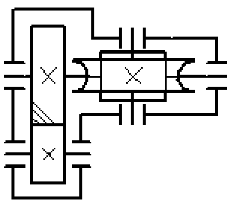
Name	Symbol
Plain bearings: 1) radial; 2) radial longitudinal unilateral; 3) radial longitudinal bilateral; 4) longitudinal unilateral; 5) longitudinal bilateral	 1 2 3 4 5
Coupling without type designation	
Fittings with type designation: 1) blind; 2) compensating; 3) flexible	 1 2 3
Brake (general designation)	

Table 3.17. Examples of kinematic diagrams some reducers

Reducer	Scheme
Single-stage roller	
Single-stage vertical roller	
Conical single-stage	
Cylindrical two-stage gearbox made according to an enlarged scheme with oblique teeth	
Cylindrical two-stage gearbox with split spur teeth	

Reducer	Scheme
Cylindrical two-stage coaxial	
Three-stage cylindrical gearbox with split intermediate shafts with bevel teeth	
Two-stage bevel and spur gear wheel with circular bevel stage teeth and spur teeth	
Single-stage auger with bottom auger system	
Two-stage worm gearbox	
Toothpick	

Examples

Example 3.30. Draw a kinematic diagram of the drive based on the structural diagram, indicating its components and main parameters.

Initial data

Motor → belt transmission cylindrical two-stage transmission made by the expanded diagram → compensating coupling → shaft of the working machine, supported on one side by a double-sided thrust bearing and on the other side by a single-sided thrust bearing.

Implementation

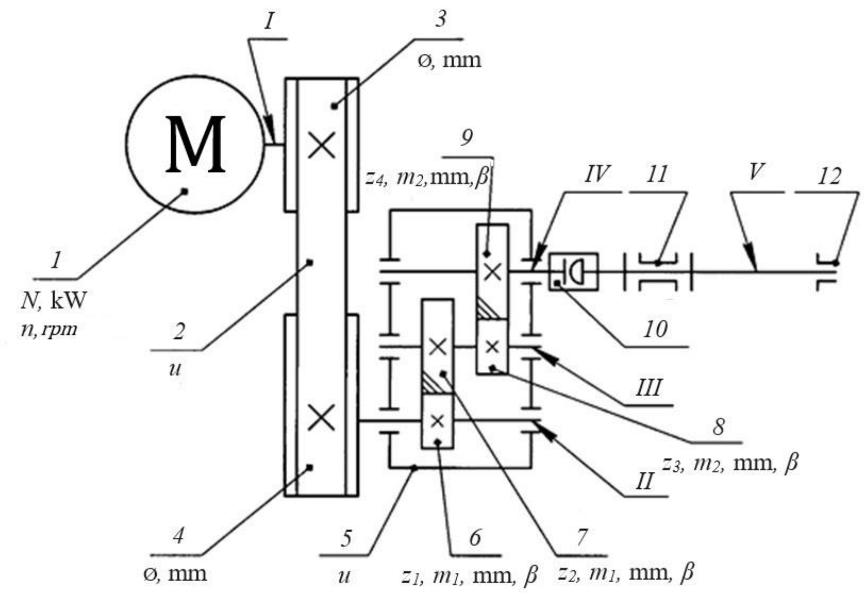


Fig. 3.43. Kinematic diagram of the drive:

1 - motor; 2 - belt transmission; 3 - driving pulley; 4 - driven pulley; 5 - cylindrical two-stage transmission made according to the extended scheme; 6 - first stage transmission; 7 - first stage wheel; 8 - second stage transmission; 9 - second stage wheel; 10 - compensating joint; 11 - double-sided thrust sleeve bearing; 12 - single-sided thrust sleeve bearing; I - motor shaft; III - low-speed reducer shaft; III - intermediate reducer shaft; IV - low-speed reducer shaft; V - working machine shaft.

Individual task
(graphical)

Task 3.15. Make a kinematic diagram of the drive based on the structural diagram (Table 3.18), indicating its components and main parameters.

Table 3.18. Initial data for Task 3.15

Var. no	Structural scheme
1	Motor → wedge gearbox → two-stage helical gearbox made by the extended diagram → compensating coupling → working shaft of machines, supported by single-sided plain bearings.
2	Motor → coupling → bevel gear → coupling → shaft of the working machine, supported by tapered roller bearings.
3	Motor → wedge-belt transmission → worm gearbox with upper worm position→ spring coupling → machine shaft supported by double-sided angular contact roller bearings.
4	Motor → coupling → helical bevel gearbox → chain gearbox → machine working shaft supported by thrust ball bearings.
5	Motor → coupling → two-stage helical gearbox made according to the extended scheme with oblique teeth → chain transmission → the working shaft of the machine is supported by radial ball bearings.
6	Motor → chain transmission → coaxial two-stage spur gear with oblique spur teeth → spring coupling → machine working shaft supported by roller bearings.
7	Motor → coupling → three-stage gearbox is made according to the split scheme with oblique teeth → compensating coupling → machine shaft supported by plain bearings.
8	Motor → chain transmission → two-stage worm gear → coupling → working shaft of the machine, supported by a roller bearing.
9	Motor → coupling→ two-stage worm gearbox → coupling → working shaft of the machine, supported by a roller bearing.
10	Motor → chain transmission → bevel gear with round teeth → coupling → machine working shaft supported by radial roller bearings.
11	Motor → chain transmission → bevel gear with round teeth → coupling → machine working shaft supported by radial roller bearings.
12	Motor → chain transmission → two-stage helical gearbox made according to the extended scheme on a low-speed shaft, on one side of which a brake is installed → coupling → machine working shaft based on sliding bearings.

Var. no	Structural scheme
13	Motor → spring coupling → vertical two-stage gearbox with spur gear → belt transmission with V-belt → machine working shaft supported by tapered roller bearings.
14	Motor → coupling → bevel gear → coupling → shaft → belt transmission of the machine's working shaft, supported by roller bearings.
15	Motor → coupling → two-stage helical gearbox made according to the extended scheme with bevel teeth → compensating coupling → working shaft of the machine, supported by plain bearings.
16	Motor → wedge gear → spur gear with chamfered teeth → coupling → machine working shaft supported by ball roller bearings.
17	Motor → chain transmission → worm gearbox with top worm arrangement → coupling → machine working shaft supported by angular contact roller bearings.
18	Motor → coupling → gearbox → coupling → the machine's working shaft is supported by roller bearings.
19	Motor → chain transmission → two-stage worm gearbox → coupling → working machine shaft supported by slipping bearings.
20	Motor → nipple → three-stage gearbox made by the division scheme of intermediate shafts with high- and low-speed helical teeth and chevron teeth → nipple → working shaft of the machine supported by roller bearings.
21	Motor → gearbox with V-belt and two-stage pulley → bevel gearbox → coupling → machine working shaft supported by roller bearings.
22	Motor → coupling → spur gear with chamfered teeth → open gear → machine working shaft supported by roller bearings.
23	Motor→ coupling → two-stage bevel gearbox → chain transmission → machine working shaft supported by roller bearings.
24	Moto r→ coupling → vertical single-stage gearbox with bevel teeth → chain transmission→ machine working shaft supported by angular contact roller bearings.
25	Motor → coupling → two-stage gearbox made according to an enlarged scheme with bevel teeth → chain transmission → working shaft of the machine supported by roller bearings.
26	Motor → chain transmission → spur gear with bevel teeth → open bevel gear → machine working shaft supported by roller bearings.
27	Motor → chain transmission → three-stage spur gear with bevel teeth → open bevel gear with machine working shaft supported by roller bearings.
28	Motor → chain gearbox → worm gearbox → open helical gearbox with the machine's working shaft supported by double-row self-aligning roller bearings.

Var. no	Structural scheme
29	Motor $\rightarrow$ wedge gear $\rightarrow$ bevel gear with bevel teeth $\rightarrow$ coupling $\rightarrow$ the machine's working shaft is supported on self-aligning roller bearings.
30	Motor $\rightarrow$ coupling $\rightarrow$ gearbox $\rightarrow$ coupling $\rightarrow$ the working shaft of the machine is supported by single-sided roller bearings.

3.8. Calculations of shafts and axles

General information

Shafts are the parts that are used to fix and support rotating parts (gears, pulleys, couplings, etc.) and transmit torque along their axes.

Some shafts do not support rotating parts (gimbal shafts, torsion bars, etc.) During operation, bending and torsional and, in some cases, tensile and compressive forces act on the shaft.

Axles are parts designed solely to hold and support the components on them. Unlike the shaft, the axle does not transmit torque and is only affected by bending forces.

The axles can either rotate with the parts fitted for better-bearing performance or be stationary if bearings are required to be housed in the rotating part.

Structural components of shafts and axles. The design, and surface quality of shafts and axles depend on their purpose, the nature and magnitude of the loads applied to them, the method of fixing the parts mounted on them, the assembly conditions of the assembly and their manufacturing technology.

Shaft and axle design includes: *bearing surfaces, seating surfaces, transition areas, shoulders, chamfers, bevels and other elements*.

Shaft and axle materials. Shaft and axle materials must be strong, rigid, easily machinable and have a high modulus of elasticity. Shafts and axles are mainly made from carbon and alloy steels, less frequently from cast iron. Steel grades used for shafts and axles without heat treatment S275, S315, C35, and C40; for shafts with heat treatment C45, 40H, 40HN, 40HN2MA, 30HG. High-speed shafts running in plain bearings are made from C20, 20H, and 12HN3A steel. The journals of these shafts are carburised or nitrided to increase wear resistance.

For steel shafts up to 150 mm in diameter, a round wire rod is usually used as the workpiece, while for larger diameter shafts and shaped shafts, forgings are used. The shafts are turned and the seating surfaces are further ground. Heavily loaded shafts are ground over the entire surface. The mechanical properties of some of the steels used for shafts and axles are given in Table D.41.

Basic calculation formulae

Rotating shafts and axles are subject to cyclically varying stresses during operation. The main performance criteria are *fatigue strength* (*durability*) and *stiffness*. The failure of shafts and axles is mostly related to fatigue, so the main calculations concern fatigue strength.

The main loads acting on the shafts and axles are those from the gears, the couplings and the working bodies of the machines or the mechanisms on them. As a result, shafts and axles undergo complex deformations: torsion, bending, tension, and compression. The effect of tensile and compressive forces is not significant and is not taken into account in most calculations. The dead weight of the shafts and axles, as well as the weight of parts on the shafts and axles, is only taken into account if their values are of the same order as the main loads.

Shafts are calculated in two stages: *design* (preliminary) and *verification* (final). Design calculations of shafts are carried out for static torsional strength in one direction only to determine approximate diameters. Unaccounted bending stresses, stress concentrations, load patterns and other factors are compensated for by reducing the allowable torsional stresses k_s. Design calculations usually specify the diameter of the output end of the shaft, which in most cases is only subjected to torsion. The intermediate shaft does not have an output end, so the diameter under the gear wheel is calculated for it.

Torsional strength condition

$$\tau_s = \frac{T}{0.2d^3} \leq k_s, \text{then } d \geq \sqrt[3]{\frac{T}{0.2k_s}}$$

where T – torque transmitted through the shaft, Nmm;

k_s – permissible torsional stresses, MPa. For output shaft parts;

k_s = 20 ÷ 30 MPa. For intermediate shafts when determining the diameter under the wheel k_s= 10 ÷ 20 MPa.

The resulting value is rounded off to the nearest standard size (Table D.43). Other shaft diameters are determined during the design process, taking into account the design and dimensions of the parts on the shaft, manufacturing and assembly technology.

When designing a gearbox, the diameter of the output end of the shaft can be assumed to be equal to the diameter of the shaft of the electric motor to which it will be connected by a coupling.

Shafts are checked for fatigue, static strength and stiffness, and in some cases for vibration (not discussed in the script). This is the final and basic calculation. It is carried out after the shaft and selected bearings have been designed when the diameters, lengths of shaft sections, roughness, fit, material, types of reinforcement, sizes of fillet passages and splines and keyways, etc. are known. Shaft verification calculations are carried out according to design diagrams.

Shaft design schemes

Based on a sketch of the shaft, a design scheme is developed in which the shafts are treated as beams fixed pivotally in rigid supports, one of which is movable. The loads acting on the shaft are reduced to two mutually perpendicular planes - horizontal and vertical. When selecting the type of support, it is assumed that if bearings transmit both radial and axial forces, they are considered as fixed supports, while bearings that transmit only radial forces are considered as mobile supports. In the calculation diagrams, acting continuous loads are replaced by concentrated loads for simplicity, and in approximate calculations, they are applied at the centre of the part on the shaft (Fig. 3.44). In more precise calculations, the points at which the loads are applied are determined as recommended, taking into account the structural features of the parts to be mounted on the shaft.

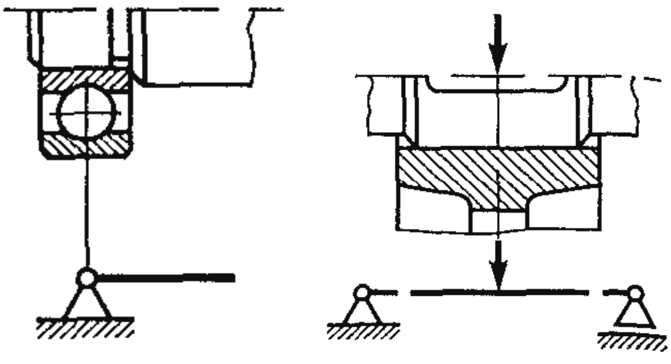


Fig. 3.44. Diagrams of shafts

Once the design diagram has been drawn up, the reactions of the supports are determined and transverse forces, bending, torsion and equivalent moments are plotted.

Fatigue strength calculations for shafts can be *simplified* and *exact*. **Precise** calculations are carried out for potentially dangerous cross-sections, pre-planned according to moment diagrams and the location of stress concentration zones.

Simplified calculations are carried out assuming that the normal and tangential stresses vary in a symmetrical (most unfavourable) cycle.

The strength condition takes the form of

$$\sigma_{red} = \frac{M_{red}}{0.1d^3} \leq k_{-1g} \text{ then } d = \sqrt[3]{\frac{M_{red}}{0.1k_{-1g}}}$$

where σ_{red} – reduced stresses in the calculated section, MPa;

M_{red} – reduced torque in section, Nmm;

d – diameter of the shaft in the calculated section, mm;

k_{-1g} - allowable bending stress under symmetrical load cycle, MPa.

The design values of the shaft diameter in the calculated cross-section are compared with the assumed design diameter. If the section to be calculated is weakened by a keyway, the design diameter is increased by 7 ÷ 10 %, if the fit is tight, by 5 %. The reduced moment is determined taking into account the simultaneous action of bending and torsional moments according to one of the strength hypotheses, for example, the highest shear stress hypothesis

$$M_{red} = \sqrt{M_g^2 + T^2},$$

where M, T – the respective bending and torsional moments, Nmm.

Precise calculations are carried out assuming that bending stresses vary according to a symmetrical alternating cycle and torsional stresses according to a zero (pulsating) cycle, and aim to determine design safety factors at potentially hazardous cross-sections, taking into account the nature of stress variation, dimensional influence, stress concentration, surface roughness and hardening.

The fatigue strength condition is of the form:

$$n = \frac{n_\sigma \cdot n_\tau}{\sqrt{{n_\sigma}^2 + {n_\tau}^2}} \geq [n]$$

where n_σ, n_τ - safety factor for normal and tangential stresses respectively;

[n] – permissible safety factor. Usually [n] = 1.2 ÷ 3 is adopted (a smaller value for accurate calculation schemes).

A section for which the safety factor is minimum is unsafe. If the strength reserve is below the permissible one, the shaft configuration is first changed to reduce the stress concentration. If these measures do not increase the strength to the required value, the diameter of the shaft is increased, the material is changed and the calculation is repeated.

Static strength calculations. Static strength checks on shafts are carried out to prevent the occurrence of plastic (residual) deformation during the application of maximum stresses (for example during start-up). Static strength calculations are generally carried out for the section with the smallest fatigue strength reserve, where the probability of failure under overload is greatest.

Using, for example, the stress-energy hypothesis, the reduced stress for a dangerous shaft section is given by the formula

$$\sigma_{red} = \sqrt{\sigma_{max}^2 + 3\tau_{max}^2} \le k_{max},$$

where σ_{max}, τ_{max} – the highest bending and torsional stresses in the section respectively, MPa.

$$k_{max} \approx 0.66R_e,$$

where R_e – the yield strength of the material, MPa.

Stiffness calculations. Verifying calculations for the stiffness of shafts are carried out in those cases where their deformations have a significant effect on the operation of the associated components. Thus, for example, increased deflection f of the shafts of gears (Fig. 3.45) causes divergence of the wheel axes, concentrating the load along the length of the teeth and causing premature wear and even destruction, and the angle of rotation θ - causing clamping in the rolling bearings, increased friction and their overheating.

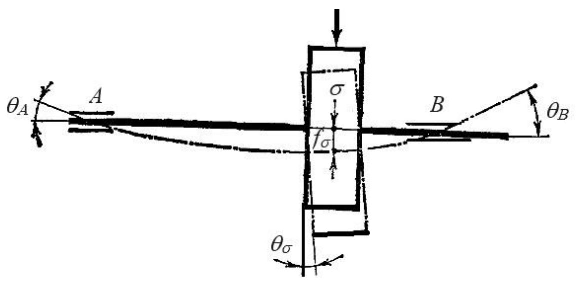


Fig.3.45. Deflection and rotation angles of shaft sections

A distinction is made between *flexural* and *torsional* stiffness.

The bending stiffness is assessed by the deflection f (y - another designation for bending used in the technical literature) and the angle of rotation θ, which are determined by material strength methods. Then the bending stiffness condition will take the form:

$$F \leq [f];\ \theta \leq [\theta],$$

where [f] and [θ] – permissible deflection [mm] and angle of rotation [rad] respectively, depending on the purpose of the shaft, determined at the design stage. Where gears are installed, [f] ≤ 0.01m, where m – the abutment modulus. For plain bearings recommended [θ] = 0.001 rad, in ball bearings [θ] ≈ 0.01 rad.

Methods for determining deflections and angles are discussed in the chapter "Strength of materials". For typical shaft loading schemes, the unit force method or the independent force principle is widely used, which allows calculations to be made using the ready-made formulas shown in Table 3.19.

In most cases, gear shafts are not checked for stiffness because the safety factors are overestimated. The exception is worm shafts, which are always checked for bending stiffness due to the large distance between supports.

With symmetrical support positions, the maximum deflection is

$$f = \frac{l^3\sqrt{F_{t1}^2+F_{r1}^2}}{48EJ} \leq [f],$$

where l – is the distance between the auger support axes, mm;

F_{t1} and F_{r1} – peripheral and radial force of the worm, N;

E – longitudinal modulus of elasticity (Young's modulus), MPa; for steel $E = 2{,}1 \cdot 10^5$ MPa;

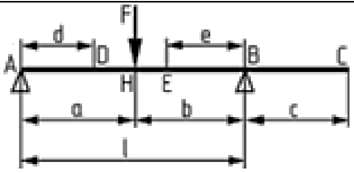
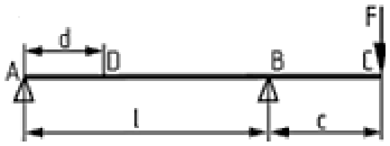
J – reduced moment of inertia of the worm shaft section with regard to the thread profile, mm^4.

$$J = \frac{\pi D_{f1}^4}{64}\left(0.375 + 0.625\frac{D_{a1}}{D_{f1}}\right),$$

where D_{a1} and D_{f1} – outer and inner diameter of the worm, mm.

If the calculated shaft deflection $f > [f]$, the worm diameter factor q is increased and the calculation is repeated.

Table 3.19. Formulae for the determination of deflection f and cross-sectional angles θ *of* shafts of constant cross-section

Θ or f		
θ_A	$\frac{Fab(l+b)}{6EJl}$	$-\frac{F_1cl}{6EJ}$
θ_B	$-\frac{Fab(l+a)}{6EJl}$	$-\frac{F_1cl}{3EJ}$
θ_C	θ_B	$\frac{F_1c(2l+3c)}{6EJ}$
θ_D	$\frac{Fb(l^2-b^2-3d^2)}{6EJl}$	$\frac{F_1c(3d^2-l^2)}{6EJl}$
θ_E	$\frac{Fa(l^2-a^2-3e^2)}{6EJl}$	-
θ_H	$\frac{Fab(b-a)}{3EJl}$	-
f_D	$\frac{Fbd(l^2-b^2-d^2)}{6EJl}$	$-\frac{F_1cd(l^2-d_2)}{6EJl}$
f_E	$\frac{Fae(l^2-a^2-e^2)}{6EJl}$	-
f_H	$\frac{Fa^2b^2}{3EJl}$	-
f_C	θ_{BC}	$\frac{F_1c^2(l+c)}{3EJ}$

Comment: $E = 2{,}1 \cdot 10^5$ MPa – Young's modulus for steel; $J = \frac{\pi d^4}{64}$ – axial moment of inertia of the circular section; l – length of the section between the transitions.

To increase the bending stiffness of shafts and axles, it is recommended that components be placed closer to the supports.

The torsional stiffness of shafts is assessed by the torsion angle φ_0 per unit length of shaft:

$$\varphi_0 = \frac{T}{GJ_b} \le [\varphi_0],$$

where T – section torsional moment, Nm;

G – Kirchhoff modulus, MPa; for steel $G = 8 \cdot 10^4$ MPa;

J_p – polar moment of inertia in the section, m^4. For a fulll round section $J_p = \frac{\pi d^4}{32}$;

$[\varphi_0]$ – permissible torsion angle of the shaft [rad] per 1 m length. The value depends on the purpose of the shaft and falls within a wide range

$[\varphi_0] = (5.0 \div 22) \cdot 10^{-3}$ rad/m.

For many gear shafts, torsional stiffness is not relevant and such calculations are not carried out.

Axles calculations

Axles only support components and are therefore subject to bending stresses. For axles, as for shafts, *design* and *verification* calculations are carried out. **Design** (preliminary) calculations of the axles for static strength are carried out, as for beams with pinned supports, using conventional material strength methods, determining the lengths of the sections depending on the design of the node. Fixed axis calculations are based on the assumption that bending stresses vary according to a zipper cycle, the most unfavourable of all known fixed cycles, and moving axis calculations are based on the assumption that stresses vary according to a symmetrical cycle. The diameter in the calculated section is determined from the bending strength condition:

$$\sigma_g = \frac{M_g}{0.1d^3} \le k_{-1(0)g} \text{ from where } d \ge \sqrt[3]{\frac{M_g}{0.1k_{-1(0)g}}},$$

where M_g – bending moment, Nmm;

d – axle diameter, mm;

$k_{-1(0)g}$ – permissible bending stresses for symmetric and zero cycle, respectively, MPa.

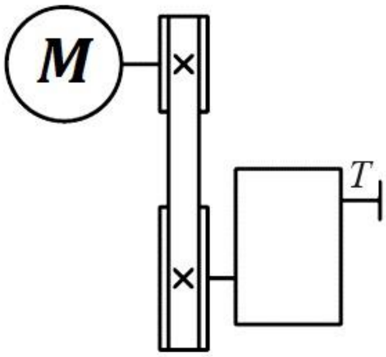
For axles made of medium-carbon steels, allowable bending stresses are k_{0g} = 100 ÷ 160 MPa. Lower values are recommended for sharp stress concentrators. The stresses in axles rotating in a symmetrical cycle assume $k_{-1g} = (0.5 \div 0.6)k_{0g}$. If the axle in the calculated section has a groove or veneer in the structural section, the resulting diameter is increased by approximately 10% and rounded to the nearest standard diameter.

Verifying (final) axle calculations for fatigue strength and stiffness are carried out, as for shaft calculations, at T = 0.

Examples of calculations

Example 3.31. The drive (Fig. 3.46) contains a motor, a belt transmission, and a reducer. From the strength condition, determine the diameter of the output end of the low-speed shaft if the motor power N_m = 10 W; motor shaft speed n_m = 1000 min^{-1}, pulley diameters D_1 = 160 mm, D_2 = 320 mm, transmission ratio u_p = 5, permissible torsional stress for the shaft material k_s = 45 MPa.

Data:
N_m = 10 W
N_m = 1000 min^{-1}
D_1 = 160 mm
D_2 = 320 mm
u_p = 5
k_s = 45 MPa.



Searched for:
$d_{w.c.}$ -?

Fig. 3.46. Drive to the calculations

Solution

1. Write down the torsional strength condition for the low-speed shaft.

$$\tau_s = \frac{T_{w.c}}{0.2d_{w.c.}^3} \le k_s$$

2. Define unknowns.

2.1. Determining the motor shaft torque.

$$T_m = 9.55\frac{N_m}{n_m} = 9.55\frac{10 \cdot 10^3}{1000} \approx 96\text{Nm}$$

2.2. Determining the transmition ratio.

$$u_{prz} = \frac{D_2}{D_1}u_p = \frac{320}{160} \cdot 5 = 10$$

2.3. Determining the efficiency of the drive.

The drive consists of a belt drive and a gearbox, given that the ratio is 5, it will be a single-stage gearbox and the parallel arrangement of the shafts in the diagram indicates that it is a cylindrical gearbox. From Table D.17, we take the efficiency of the belt drive $\eta_{p.p}$ = 0.96, the efficiency of the pinion gear η_p = 0.98, and then the efficiency of the drive.

$$\eta_n = \eta_{prz} \cdot \eta_{red} = 0.94$$

2.4. Determining the torsional moment on the low-speed shaft.

$$T_{w.c} = T_m \cdot u_{prz} \cdot \eta_{prz} = 96 \cdot 10 \cdot 0.94 \approx 902 \text{ Nm}$$

2.5. From the torsional strength condition we determine the diameter of the low-speed shaft

$$d_{w.c} \geq \sqrt[3]{\frac{T_{w.c} \cdot 10^3}{0.2k_s}} = \sqrt[3]{\frac{902 \cdot 10^3}{0,2 \cdot 45}} \geq 47 \text{ mm}$$

From Table D.43 we assume $d_{w.c}$= 48 mm.

Answer: $d_{w.c.}$ = 48 mm.

Example 3.32. Determine the at the point of application of the load F_1, if: transmitted power N = 10 kW; rotation shaft speed n = 500 min^{-1}; F_1 = 3 kN; shaft material C45 steel; a = 300 mm; b = 200 mm; section weakened by keyway.

Data:
N = 10 kW
n = 500 min^{-1}
F_1 = 3 kN
a = 300 mm
b = 200 mm
material – C45 steel

Searched for:
d - ?

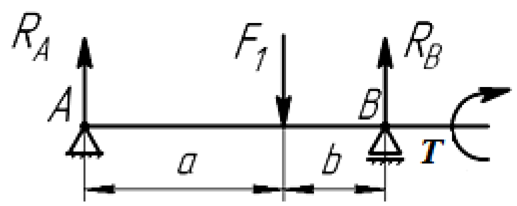


Fig. 3.47. Diagram of forces acting on the shaft

Solution

1. Write down the strength condition for the shaft keep in mind the bending and torsional moments:

$$\sigma_{red} = \frac{M_{red}}{0.1d^3} \leq k_{-1g}$$

2. Define unknowns.

2.1. Determine the permissible stresses for the shaft material.

For C45 steel from Table D.41 we take σ = 560 MPa (since the diameter of the shaft is unknown), for which we take the permissible stresses from the Table D.42 $k_{-1g} = 50$ MPa.

2.2. We determine the torsional moment which the shaft transmits.

$$T = 9.55\frac{N}{n} = 9.55\frac{10 \cdot 10^3}{500} = 191 \text{ Nm}$$

2.3. Determine reactions in the supports.

We adopt the sign rule. We take the counterclockwise moment as positive.

$$\Sigma \mathrm{M_A} = 0;\ R_B(a+b) - F_1 a = 0; R_B = \frac{F_1 a}{a+b} = \frac{3000 \cdot 0.3}{0.3 + 0.2} = 1800 \text{ N}$$

$$\Sigma M_B = 0;\ -R_A(a+b) + F_1 b = 0; R_A = \frac{F_1 b}{a+b} = \frac{3000 \cdot 0.2}{0.3 + 0.2} = 1200\ \text{N}$$

Verification

$$\Sigma F_y = 0. R_A - F_1 + R_B = 0; 1200 - 3000 + 1800 = 0; 0 = 0$$

Reactions were determined correctly.

2.4. Determining the bending moment

$$M = R_A a = 1200 \cdot 0.3 = 360\ \text{Nm}$$

2.5. Determining the reduced moment

$$M_{red} = \sqrt{M_g^2 + T^2} = \sqrt{360^2 + 191^2} = 408\ \text{Nm}$$

2.6. From the strength condition we determine the diameter of the shaft

$$d = \sqrt[3]{\frac{M_{red}}{0.1[\sigma_{-1}]_g}} = \sqrt[3]{\frac{408 \cdot 10^3}{0.1 \cdot 50}} = 43\ \text{mm}$$

As the cross-section is weakened by the keyway, we increase the diameter by 10 %.

$$d \cdot 1.1 = 43 \cdot 1.1 = 47.3\ \text{mm}$$

The resulting value is rounded up to the value in the Table D.43 d = 50 mm.

Answer: d = 50 mm.

Example 3.33. Check the strength of the axle at the point of application of the force F_1, if the load value F_1 = 10 kN, axle diameter d = 40 mm; a = 300 mm; b = 200 mm, fixed axle, section weakened by a keyway, permissible stress of the axle material k_{0g} = 70 MPa.

Data:
F_1 = 10 kN
d = 40 mm
a = 300 mm
b = 200 mm
k_{0g} =70 MPa

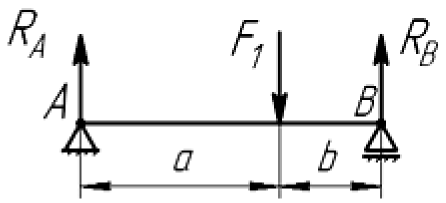


Fig. 3.48. Diagram of forces acting on the axle

Searched for:
σ_g - ?

Solution

1. Plot a calculation scheme (Fig. 3.48).

2. Write the strength condition for the axis

$$\sigma_g = \frac{M_g}{0.1 d^3} \leq k_{0g}$$

3. Define unknowns.

3.1. Determining the value of the reaction in the supports.

Adopt the sign rule. We take the counterclockwise moment as positive.

$$\Sigma M_A = 0;\ R_B(a+b) - F_1 a = 0; R_B = \frac{F_1 a}{a+b} = \frac{10000 \cdot 0.3}{0.3+0.2} = 6000\ \mathrm{N}$$

$$\Sigma M_B = 0;\ -R_\mathrm{A}(a+b) + F_1 b = 0; R_A = \frac{F_1 b}{a+b} = \frac{10000 \cdot 0.2}{0.3+0.2} = 4000\ \mathrm{N}$$

Verification

$$\Sigma F_y = 0$$

$$R_A - F_1 + R_B = 0; 4000 - 10000 + 6000 = 0; 0 = 0$$

Reactions were determined correctly.

3.2. Reactions were determined correctly.

$$M = R_A a = 4000 \cdot 0.3 = 1200\ \mathrm{Nm}$$

3.3. Determine the bending stresses and assess the strength of the shaft.

$$\sigma_g = \frac{M_g}{0.1 d^3} = \frac{1200 \cdot 10^3}{0.1 \cdot 40^3} = 188\ \frac{\mathrm{N}}{\mathrm{mm}^2} = 188\ \mathrm{MPa}$$

As the axle section is weakened by the keyway, we increase the calculated stresses by 10 %:

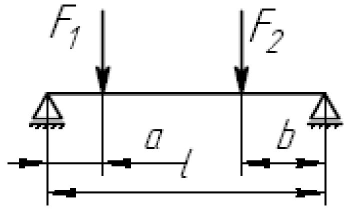
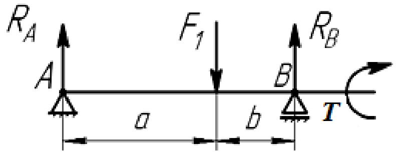
$$\sigma_g \cdot 1.1 = 188 \cdot 1.1 = 207\ \mathrm{MPa} > k_{0g} = 70\ \mathrm{MPa}$$

The strength condition is not met. In order to meet the strength condition, either the diameter has to be increased or a stronger material has to be chosen.

Individual tasks
(calculation)

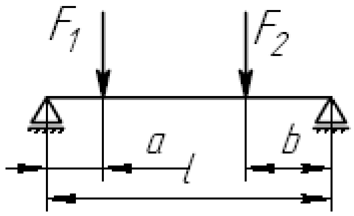
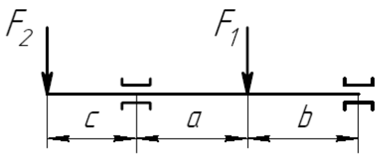
Task 3.16. Determine the diameter of the shaft at the point of application of the load F_1. The initial input data is shown in Table 3.20.

Table 3.20. Initial data for Task 3.16

Var. no	F_1, kN	F_2, kN	N kW	n_1, min^{-1}	a, mm	b, mm	l, mm	k_{-1g} MPa	Scheme
1	4.5	3.0	–	–	50	100	300	30	 Section weakened by keyway
2	5.0	4.0	–	–	100	150	400	35	
3	5.5	5.0	–	–	150	200	500	40	
4	6.0	2.0	–	–	200	250	600	45	
5	6.5	4.0	–	–	300	300	700	50	
6	7.0	5.0	–	–	250	120	800	55	
7	7.5	2.5	–	–	350	140	600	60	
8	8.0	3.0	–	–	180	160	700	65	
9	8.5	5.0	–	–	220	180	800	70	
10	9.5	6.0	–	–	300	150	1000	75	
11	10.0	4.0	–	–	180	240	600	80	
12	10.5	3.0	–	–	150	125	700	85	
13	11.0	5.0	–	–	100	140	500	90	
14	11.5	2.0	–	–	200	160	650	30	
15	12.0	1.0	–	–	250	180	750	35	
16	16.5	–	16.5	900	100	50	–	40	 Section non-weakened by keyway
17	12.5	–	12.5	950	150	100	–	45	
18	13.0	–	13.0	1000	200	150	–	50	
19	13.5	–	13.5	1050	250	200	–	55	
20	14.0	–	14.0	1100	300	250	–	60	
21	14.5	–	14.5	1200	120	50	–	65	
22	15.0	–	15.0	1250	140	100	–	70	
23	16.0	–	16.0	1300	160	120	–	75	
24	17.0	–	17.0	1400	180	140	–	80	
25	17.5	–	17.5	1450	150	100	–	85	
26	18.0	–	18.0	1500	240	200	–	90	
27	19.0	–	19.0	750	125	50	–	30	
28	20.0	–	20.0	800	140	100		35	
29	21.0	–	21.0	750	160	120		40	
30	22.0	–	22.0	800	180	150		45	

Task 3.14. Determine the diameter of the axis at the point of application of force *F.* The initial input data is shown in Table 3.21.

Table 3.21. Initial data for Task 3.14

Var. no	F_1, kN	F_2, kN	*a*, mm	*b*, mm	*c*, mm	*l*, mm	k_{-1g} MPa	Scheme
1	4.5	3.0	50	100	–	300	30	 Section non-weakened by keyway
2	5.0	4.0	100	150	–	400	35	
3	5.5	5.0	150	200	–	500	40	
4	6.0	2.0	200	250	–	600	45	
5	6.5	4.0	300	300	–	700	50	
6	7.0	5.0	250	120	–	800	55	
7	7.5	2.5	350	140	–	600	60	
8	8.0	3.0	180	160	–	700	65	
9	8.5	5.0	220	180	–	800	70	
10	9.5	6.0	300	150	–	1000	75	
11	10.0	4.0	180	240	–	600	80	
12	10.5	3.0	150	125	–	700	85	
13	11.0	5.0	100	140	–	500	90	
14	11.5	2.0	200	160	–	650	30	
15	12.0	1.0	250	180	–	750	35	
16	16.5	5.0	100	50	50	–	40	 Section weakened by keyway
17	12.5	2.5	150	100	100	–	45	
18	13.0	3.0	200	150	150	–	50	
19	13.5	5.0	250	200	200	–	55	
20	14.0	6.0	300	250	300	–	60	
21	14.5	4.0	120	50	250	–	65	
22	15.0	3.0	140	100	350	–	70	
23	16.0	5.0	160	120	180	–	75	
24	17.0	2.0	180	140	220	–	80	
25	17.5	1.0	150	100	300	–	85	
26	18.0	3.0	240	200	180	–	90	
27	19.0	4.0	125	50	150	–	30	
28	20.0	5.0	140	100	100	–	35	
29	21.0	2.0	160	120	200	–	40	
30	22.0	5.0	180	150	250	–	45	

3.9. Calculation of plain bearings

General information

Plain bearings are supports for rotating shafts and axles that ensure their positioning in space, their ability to rotate or sway and to carry all the loads acting on them.

Plain bearing system

In the simplified version (Fig. 3.48), the plain bearing consists of insert 1 installed in housing 2.

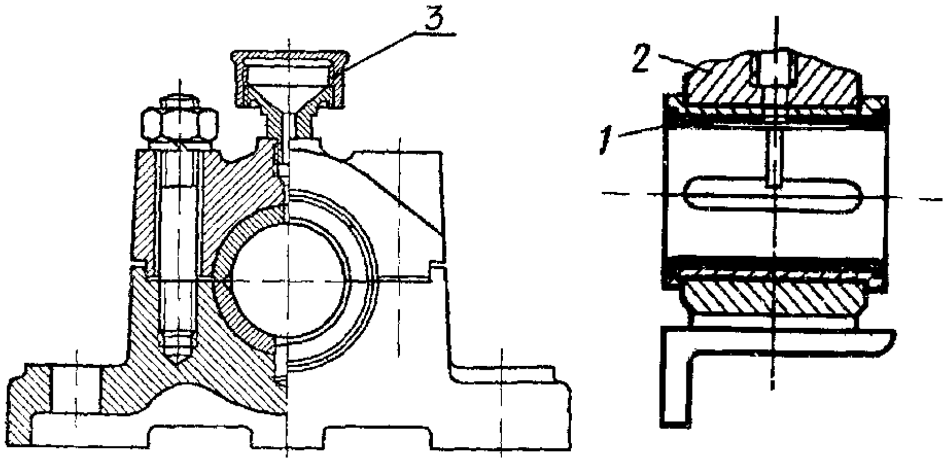


Fig. 3.48. Slide bearing:
1- insert; 2 - housing; 3 - lubricator

The insert, housing, lubricator and seal form a **bearing node**, which is often referred to as a **plain bearing**.

The operation of a plain bearing is associated with different **modes of friction**.

Depending on the mode of operation of the bearing, the friction in the bearing can be *dry, boundary, semi-dry, semi-fluid and fluid, transitioning from one mode to another when the angular velocity of the shaft is increased from zero to a certain value*. The most favourable friction conditions for a plain bearing is the fluid friction mode, where the friction surfaces are completely separated by lubricant (Fig. 3.49), a thickness h which is greater than the sum of $\delta_1 + \delta_2$.

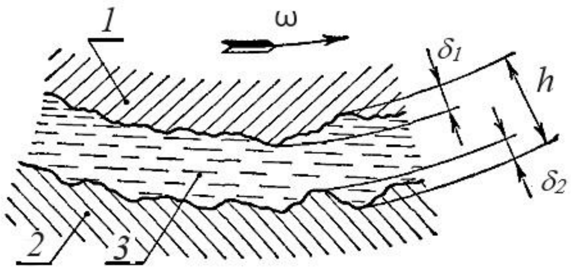


Fig. 3.49. For calculation of the fluid friction mode:
1 – insert; 2 – thrust surface of the shaft; 3 - layer of grease

With fluid friction, there is no surface wear, minimal rotational resistance, heat release and high efficiency. Fluid friction only occurs in special bearings under certain conditions. With other friction modes, wear of the friction surface, significant heat release and reduced efficiency are observed.

Most plain bearings operate under conditions of semi-fluid friction, and when starting and stopping, under conditions of semi-dry and boundary friction.

Boundary, semi-dry and semi-fluid friction share a concept - **friction with imperfect lubrication**.

Insert - is the plain bearing itself. They are used to avoid the need for housing made of expensive wear-resistant material so that they can be replaced after start-up. Inserts are *non-detachable, detachable* and, in the case of large-diameter shafts, take the form of a *set of washers* that form a bearing surface.

The materials for the contributions are:

a) metals and metal alloys - babbite, bronze, zinc-based alloys, aluminium-based alloys, anti-friction cast irons;

b) bimetallic materials;

c) non-metallic materials (plastics, wood, rubber, graphite materials);

d) composite materials;

e) metalloceramics.

The choice of insert material depends on the load, speed and operating conditions. The most common insert materials and their properties are shown in Table D.44.

Basic calculation formulae

The main performance criterion for plain bearings is wear resistance - resistance to wear and jamming.

Calculation of bearings operating in imperfect lubrication mode

As mentioned above, most plain bearings operate under imperfect lubrication conditions (semi-dry, boundary and semi-fluid friction). Due to the lack of calculation theory in the imperfect lubrication mode, bearings are calculated conventionally based on the average pressure p and the specific work of friction forces $p\upsilon$. *The calculation based on the average pressure p* guarantees the absence of grease extrusion, while the calculation of $p\upsilon$ - guarantees the normal thermal mode and the absence of jamming.

1. Transverse bearings (Fig. 3.50, *a*)

average pressure

$$p = \frac{F_r}{dl} \leq [p]$$

proper work of friction forces

$$pv \leq [pv]$$

Angular velocity of the opposing shaft surface (sliding speed)

$$v = \frac{\omega d}{2 \cdot 1000} = \frac{\pi n d}{60 \cdot 1000} \leq v_{max},$$

where F_r – radial bearing force, N;

υ - peripheral speed of the journal surface (sliding speed);

d and l – diameter and length of the bearing surface of the shaft, which are determined during the calculation and design of the shaft, mm. For most bearings $l = (0.5 \div 1.3)d$;

$[p]$ and $[p\upsilon]$ – permissible pressure and specific work of friction forces, MPa.

$\upsilon_{\max}$ – maximum sliding speed;

1000 – conversion factor of millimetres into metres;

n –rotation speed of the supporting surface, min^{-1};

ω – angular velocity of the bearing surface, s^{-1}.

The permissible values $[p]$, $[p\upsilon]$, $\upsilon_{\max}$ depend on the material of the friction surface and are determined based on the operating experience of similar structures, selected from Table D.44.

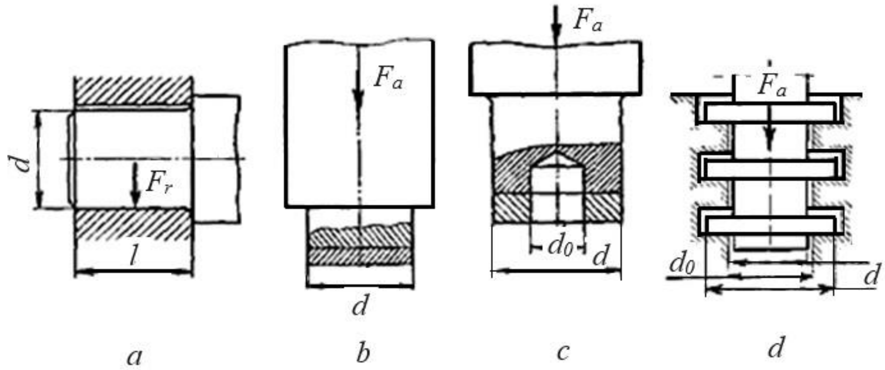


Fig. 3.50. Diagram of occurrence of forces in sliding bearings with imperfect lubrication:
a - thrust bearing; *b* - flange bearing, *c* - thrust ring bearing, *d* - thrust bearing with comb

2. Flat thrust bearings (thrust bearings):

(a) thrust bearing with solid foot (Fig. 3.50, *b*)

average pressure

$$p = \frac{4F_{\mathrm{a}}}{\pi d^2 \varphi} \leq [p],$$

where φ - coefficient taking into account the reduction in bearing area by lubrication grooves, 0.8 ÷ 0.9.

The specific work of the frictional forces and the sliding speed are determined for radial bearings.

(b) bearing with thrust ring (Fig. 3.50, *b*)

average pressure

$$p = \frac{4F_{\mathrm{a}}}{\pi(d^2 - d_0^2)\varphi} \leq [p]$$

proper work of friction forces

$$pv \leq [pv]$$

average glide speed

$$v_{av} = \frac{\omega \sigma_{red}}{1000} = \frac{\pi n \sigma_{red}}{30 \cdot 1000} \leq v_{max},$$

where d_0 – internal diameter, is assumed (0.6 … 0.8)d, mm;

$\sigma_{red} = 0{,}33 \frac{d^3 - d_0^3}{d^2 - d_0^2}$ – reduced foot radius, mm;

(c) comb thrust bearing (Fig. 3.50, *d*)

average pressure

$$p = \frac{4F_{\mathrm{a}}}{z\pi(d^2 - d_0^2)\varphi} \leq [p],$$

where z – the numer of combs.

The specific work of frictional forces p and the average sliding speed υ_{av} are determined as for ring bearings.

The values of $[p]$ and $[p\upsilon]$ will decrease by 20 ÷ 40 % compared to $[p]$ and $[p\upsilon]$ for other bearings due to uneven axial load distribution F_a between the supporting surfaces of the combs.

Dimensions of the flanged bearing insert (Fig. 3.51)

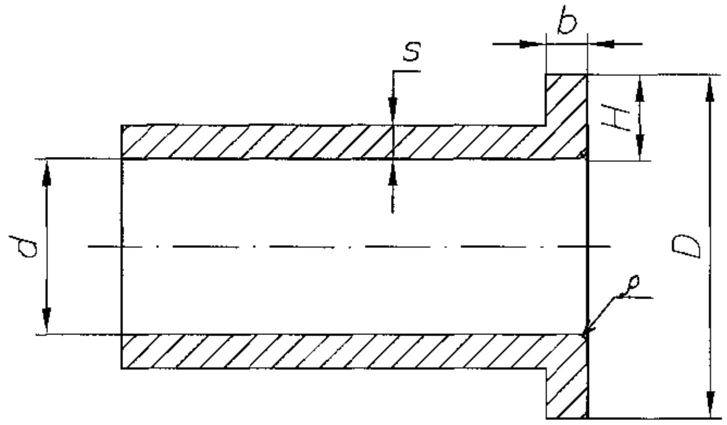


Fig. 3.51. Diagram for determining the dimensions of a flanged insert

Insert wall thickness

Cast iron, bronze

$$s = 0.03d + (2 \div 5)\ \text{mm}$$

Flange feight

$$H = 1.2s + (3 \div 5)\ \text{mm}$$

Flange width

$$b = 1.2s$$

Flange outer diameter

$$D = d + 2H$$

Radius of rounding·

$$\rho = (0.03 \div 0.05)d$$

Examples of calculation

Example 3.34. Check the bogie axle bearing (Fig. 3.52) if the neck dimensions are d = 60 mm and l = 70 mm. Radial bearing load F_r = 16 kN at maximum angular velocity ω = 30 s^{-1}. Insert material – CuSn6Zn6Pb3. Axle material – normalized C45 steel.

Data:
F_r = 16 kN
d = 60 mm
l = 70 mm
ω = 30 s^{-1}
insert
CuSn6Zn6Pb3
axle -
normalized C45 steel

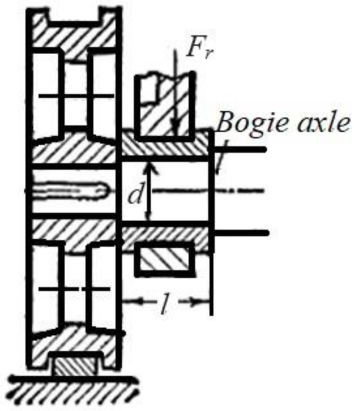


Fig. 3.52. Bearing to Example 3.34

Searched for:
υ– ?
p – ?
pυ – ?

Solution

1. For a given input material from Table D.44 we take
$[p]$ = 4 ÷ 6 MPa; $[p\upsilon]$ = 4 ÷ 6 MPa·m/s; υ_{max} = 8 m/s.

2. Determine the angular velocity (glide speed) and compare with the maximum:

$$\upsilon = \frac{\omega d}{2 \cdot 1000} = \frac{30 \cdot 60}{2 \cdot 1000} = 0.9 \text{ m/s} < \upsilon_{\max} = 8 \text{ m/s}$$

the condition is met.

3. Check the average bearing pressure:

$$p = \frac{F_r}{dl} = \frac{16 \cdot 10^3}{60 \cdot 70} = 3.8 \frac{\text{N}}{\text{mm}^2} = 3.8 \text{ MPa} < [p] = 4 \div 6 \text{ MPa}$$

the condition is met.

4. Check the bearing for heat and no jamming:

$$p\upsilon = 3.8 \cdot 0.9 = 3.42 \text{ MPa} \cdot \frac{\text{m}}{\text{s}} < [p\upsilon] = 4 \div 6 \text{ MPa} \cdot \frac{\text{m}}{\text{s}}$$

the condition is met.

Conclusion: the bearing is suitable for the specified operating conditions.

Example 3.35. Select the material for the plain bearing if: acting radial load Q_r = 7 T; pivot material d = 80 mm; rotation shaft speed n = 100 min^{-1}; shaft material – hardened C45 steel.

Data:
Q_r = 7 T
d = 80 mm
n = 100 min^{-1}
shaft
hardened C45 steel

Szukane:
l - ?
υ - ?
p - ?
$p\upsilon$ - ?
Insert material - ?

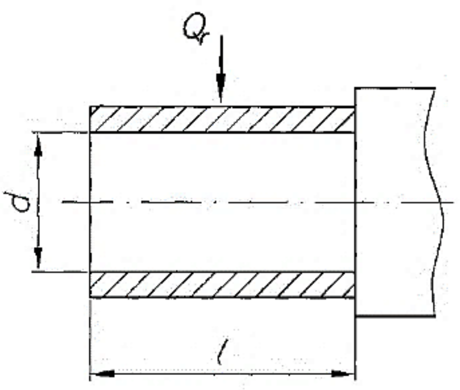


Fig. 3.53. Slide bearing scheme
to Example 3.35

Solution

1. Plot a scheme (Fig. 3.53).
2. Convert units:

$$Q_r = F_r = 7\ \mathrm{T} = 70 \cdot 10^3 \mathrm{N}$$

3. Determine the insole lenght:

$$l = (0.5 \div 1.3)d = (0.5 \div 1.3) \cdot 80 = 40 \div 104\ \mathrm{mm}$$

For design reasons, we assume l = 70 mm.

4. Determine the pivot speed (sliding speed):

$$\upsilon = \frac{\pi n d}{60 \cdot 1000} = \frac{3{,}14 \cdot 100 \cdot 80}{60 \cdot 1000} = 0.42\ \mathrm{m/s}$$

5. Determine the average bearing pressure:

$$p = \frac{F_r}{dl} = \frac{70 \cdot 10^3}{80 \cdot 70} = 12{,}5 \frac{\mathrm{N}}{\mathrm{mm}^2} = 12.5\ \mathrm{MPa}$$

6. Determine the proper work of frictional forces:

$$p\upsilon = 12.5 \cdot 0.42 = 5.25\ \mathrm{MPa} \cdot \frac{\mathrm{m}}{\mathrm{s}}$$

7. From Table D.44 taking into account the calculated values, we select the material of the plain bearing. We adopt CuAl9Fe4, for which:

υ_{max} = 8 m/s > υ = 0.42 m/s – condition is met;

[p] = 15 MPa > 12.5 MPa – condition is met;

[$p\upsilon$] = 12 MPa·m/s > $p\upsilon$ – 5.25 MPa·m/s – condition is met.

Answer: υ =0.42 m/s; p = 12.5 MPa; $p\upsilon$ = 5.25 MPa·m/s; insert material - CuAl9Fe4.

Example 3.36. Calculate the sliding bearing of a worm gear shaft (Fig. 3.54) if: bearing radial load F_r = 11 kN, axial load F_a = 4.4 kN, pivot shaft diameter d = 80 mm, rotational speed n = 115 rpm.

Data:
F_r = 11 kN
F_a = 4.4 kN
d = 80 mm
n = 115 rpm

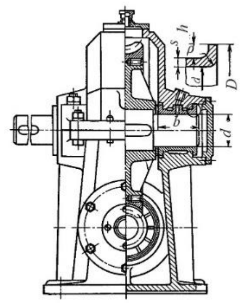


Fig. 3.54. Worm gear to Example 3.36

Searched for:
l – ?
υ – ?
p – ?
$p\upsilon$ – ?
insert material – ?

Solution

1. Determine the length of the insert:

$$l = (0.5 \div 1.3)d = (0.5 \div 1.3) \cdot 80 = 40 \div 104 \text{ mm}$$

For design reasons, taking into account the chamfer in Table D.21, we assume a working insert length of l = 60 mm.

2. Determine the speed of the pivot (sliding speed):

$$\upsilon = \frac{\pi n d}{60 \cdot 1000} = \frac{3.14 \cdot 115 \cdot 80}{60 \cdot 1000} = 0.48 \text{ m/s}$$

3. Determine the average bearing pressure due to the radial load F_r:

$$p = \frac{F_r}{dl} = \frac{11 \cdot 10^3}{80 \cdot 60} = 2.3 \frac{\text{N}}{\text{mm}^2} = 2.3 \text{ MPa}$$

4. Determine the proper work of frictional forces due to the radial load F_r:

$$p\upsilon = 2.3 \cdot 0.48 = 1.1 \text{ MPa} \cdot \text{m/s}$$

5. The material of the insert is selected from Table D.44 - wear-resistant cast iron EN-GJL-HB200, for which:

at υ = 2 m/s we have $[p]$ = 0.05 MPa; $[p\upsilon]$ =0.1 MPa·m/s;

at υ = 0.2 m/s we have $[p]$ = 9 MPa; $[p\upsilon]$ = 1.8 MPa·m/s.

6. For the calculated sliding speed υ = 0.48 m/s by interpolation we determine the values $[p\upsilon]$ = 1.54 MPa·m/s > 1.1 MPa·m/s, then

$$[p] = \frac{[p\upsilon]}{\upsilon} = \frac{1{,}54}{0.48} = 3.2 \text{ MPa} > 2.3\text{MPa}$$

the condition is met.

7. Determine the dimensions of the cast iron insert:

We assume the length of the insert

$$s = 0.03d + (1 \div 3) \text{ mm} = 0.03 \cdot 80 + 2.6 \text{ mm} = 5 \text{ mm}$$

Flange height

$$H = 1.2s + (3 \div 5)\text{mm} = 1.2 \cdot 5 + 4 = 10 \text{ mm}$$

Flange thickness

$$b = 1.2s = 1.2 \cdot 5 = 6 \text{ mm}$$

Diameter

$$\rho = 0.03d = 0.03 \cdot 5 = 0.15 \text{ mm}$$

We assume ρ = 2 mm.

Outer flange diameter $D = d + 2H = 80 + 2 \cdot 10 = 100$ mm.

The inner diameter of the annular surface of the insert is determined taking into account the radius ρ = 2 mm

$$d_0 = d + 2\rho = 80 + 2 \cdot 2 = 84 \text{ mm}$$

8. Determine the reduced diameter of the lateral surface of the insert:

$$\sigma_{red} = 0.33 \frac{D^3 - d_0^3}{D^2 - d_0^2} = 0.33 \frac{100^3 - 84^3}{100^2 - 84^2} = 45 \text{ mm}$$

9. Determine the average sliding speed of the lateral surface of the insert:

$$\upsilon_{av} = \frac{\pi \, n\sigma_{red}}{30 \cdot 1000} = 0.54 \text{ m/s}$$

10. Determine the average pressure on the lateral surface of the insert under the action of the axial force F_a, assuming a coefficient that takes into account the reduction in the area of resistance by lubrication grooves φ = 0,9:

$$p_{\mathrm{T}} = \frac{4F_{\mathrm{a}}}{\pi(D^2 - d_0^2)\varphi} = \frac{4 \cdot 4.4 \cdot 10^3}{3.14(100^2 - 80^2) \cdot 0.9} = 2.1 \text{ N/mm}^2 = 2.1 \text{ MPa}$$

11. Determine the specific work of the frictional forces on the lateral surface of the cartridge when the axial force F_a:

$$p\upsilon_{av} = 2.1 \cdot 0.54 = 1.1 \text{ MPa} \cdot \text{m/s}$$

12. Determine the permissible values of [p] and [$p\upsilon$] by linear interpolation at υ_{av} = 0.54 values of [pυ] =1.48 MPa·m/s, so

$$[p] = \frac{[\mathrm{p}\upsilon]}{\upsilon} = \frac{1.48}{0.54} = 2.74 \text{ MPa}$$

Comparing the permissible values with the design values for the lateral contribution surface p = 2.1 MPa < [p] =2.74 MPa and

$p\upsilon_{av}$ = 1.1 MPa·m/s < [$p\upsilon$]=1.48 MPa ·m/s- condition is met.

Answer: l = 60 mm; υ = 0.48 m/s; υ_{av} = 0.54 m/s; p = 2.3 MPa; p_{T} = 2.1 MPa; $p\upsilon_{av}$ = 1.1 MPa·m/s; insert material – wear-resistant grey cast iron EN-GJL-HB200.

Individual tasks
(calculation)

Task 3.18. Check the bogie axle bearing (Fig. 3.52). Axle material - normalized C45 steel, the data for the calculations are given in Table 3.22.

Table 3.22. Initial data for Task 3.18

Var. no	Neck dimension, mm		Load	Angular velocity	Insert material
	d	l	F_r, kN	ω, s^{-1}	
1	40	20	7	30	CuSn6Zn6Pb3
2	45	30	6	40	
3	50	35	5	25	
4	55	45	8	20	
5	60	55	9	35	
6	65	35	10	52	
7	70	40	11	22	CuSn10F1
8	75	43	12	33	
9	80	52	20	38	
10	85	62	16	51	
11	90	72	17	65	
12	95	60	19	30	
13	100	50	3	15	EN-GJS-400-15
14	55	23	4	18	
15	60	28	5	16	
16	65	33	6	14	
17	50	41	3.5	13	
18	70	35	2.5	44	
19	75	38	14	32	CuAl9Fe4
20	80	42	13	44	
21	85	44	15	48	
22	40	46	16	54	
23	45	60	17	62	
24	50	80	17.5	70	
25	55	64	10.5	35	Babbit B16
26	60	56	11.5	38	
27	65	46	8.5	41	
28	70	44	9.5	43	
29	75	34	7.3	60	
30	105	72	6.2	24	

Task 3.19. Calculate the sliding bearing of the worm gear shaft (Fig. 3.54). The data for the calculations are given in Table 3.23.

Table 3.23. Initial data for Task 3.19

Var. no	Load, kN		Shaft diameter, mm	Rotational speed
	F_r	F_a	d	n, min^{-1}
1	7	3.6	40	100
2	6	4	45	115
3	5	2.5	50	125
4	8	4.4	55	120
5	9	5.6	60	135
6	10	5.9	65	152
7	11	6	70	122
8	12	4.6	75	133
9	14	7	80	138
10	16	7.3	85	151
11	17	8	90	165
12	13	5.5	95	130
13	3	2	35	115
14	4	2.4	55	118
15	5	2.8	60	116
16	6	3.3	65	114
17	3.5	2.3	50	113
18	2.5	1.7	70	144
19	14	7.4	75	132
20	13	6.9	80	144
21	15	7.5	85	148
22	16	8	40	154
23	17	8.3	45	162
24	17.5	6.2	50	170
25	10.5	5.8	55	135
26	11.5	7.3	60	138
27	8.5	6.4	65	141
28	9.5	5.9	70	143
29	7.3	4.7	75	160
30	6.2	3.8	35	124

3.10. Selection of rolling bearings

General information

Rolling bearings are supports for rotating shafts and axles that ensure their specific position in space, their ability to rotate or sway and to absorb all loads acting on them.

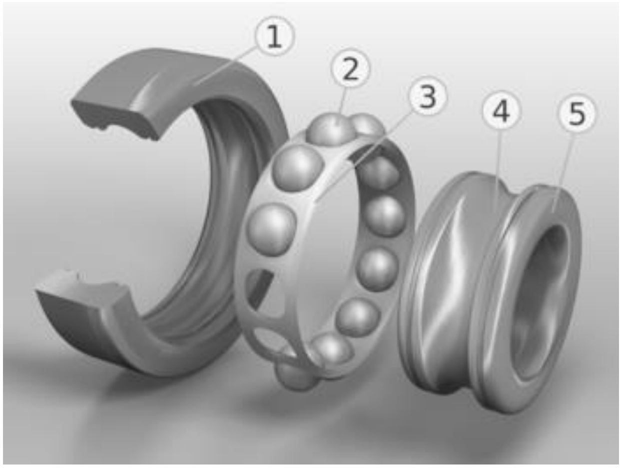


Fig. 3.55. Rolling bearing

A rolling bearing (Fig. 3.55) is a finished assembly consisting of an outer (a) and an inner (5) ring, between which the rolling elements (2) - balls, and rollers - are placed. To prevent the rolling elements from moving against each other and colliding, they are separated from each other by a separator (3). During operation, the rolling elements roll on the raceways (4) of the rings, one of which, in most cases, is stationary. The load distribution between the load-bearing rolling elements is uneven and depends on the amount of radial clearance in the bearing and the accuracy of the geometrical shape of its elements.

In some cases, to reduce the geometrical dimensions of the bearing, the rings are omitted and the rolling elements move directly on the journal and housing.

In addition to the rolling bearings themselves, bearing assemblies include a housing with covers, bearing ring mounting devices and protective and lubricating devices.

Basic parameters of rolling bearings

The basic force parameters of rolling bearings are their static load capacity C_0 [N] and dynamic load capacity C [N].

The basic geometrical parameters are outer ring diameter, D [mm] inner ring diameter, d [mm], bearing width B [mm] or height H [mm].

The basic kinematic parameter is the limiting speed n_g.

The values of these parameters are given in the tables for rolling bearings (Tables D.50 ÷ D.55).

Basic calculation formulae

The correct selection of the bearing with correct installation and handling, determines its reliable operation and the functioning of the mechanism and device as a whole.

When selecting a rolling bearing, the following factors must be taken into account: the value and direction of the load; the nature of the additional load; the diameter of the shaft; the speed of one or both rings; the operating conditions (temperature) and other requirements arising from the design of the device.

Bearings are selected for their dynamic load-carrying capacity to prevent fatigue fracture and their static load-carrying capacity to prevent plastic deformation.

Selection of bearings based on dynamic load-carrying capacity

The calculation method for the dynamic load capacity C (for the specified service life or durability) is performed at an assumed rotational speed n >1 min^{-1}.

If n = 1 ÷ 10 min^{-1} n =10 min^{-1} is used for calculations.

Bearing selection condition:

$$C_{S_a} \leq C; n \leq n_{lim},$$

where C_{Sa} – adjusted design dynamic bearing load capacity, N;

C – assumed dynamic bearing load capacity, N;

n – shaft or housing speed, min^{-1};

$n_{\lim}$ – limiting bearing speed, min^{-1} (selected from catalogue).

Dynamic load-bearing capacity and durability (service life) are linked by an empirical relationship.

The adjusted calculated bearing life (service life) in millions of revolutions or the calculated adjusted dynamic load carrying capacity are determined from the formulae:

$$L_{sa} = a_1 a_{23}\left(\frac{C}{P}\right)^p \text{ or } C_{S_a} = P\sqrt[p]{\frac{L_{S_a}}{(a_1 a_{23})}}$$

then **the bearing selection condition**

$$L_{S_a} \geq L'_{S_a} \text{ or } C_{S_a} \leq C; n \leq n_{sk}$$

The adjusted calculated bearing life (service life) [h] or the adjusted calculated dynamic load carrying capacity are determined from the formulae:

$$L_{sah} = a_1 a_{23}\frac{10^6}{60 \cdot n}\left(\frac{C}{P}\right)^p \text{ or } C_{sa} = P \cdot \sqrt[p]{\frac{L_{sah} \cdot 60 \cdot n}{10^6(a_1 a_{23})}}$$

then the bearing selection condition

$$L_{sah} \geq L'_{sah} \text{ or } C_{S_a} \leq C; n \leq n_{sk}$$

where C – specified dynamic bearing load capacity, N;

P – reduced dynamic load, N;

p – step index; for ball bearings p = 3; for roller bearings p = 10/3;

n – rotation speed of outer or inner ring, min $^{-1}$;

L'_{sa} – basic durability, mln rpm;

L'_{sah} – basic bearing life (life expectancy), h; (this is either given or taken from tables);

$n_{\lim}$ – limiting bearing speed, min^{-1};

a_1 – reliability correction factor;

a_{23} – material and lubricant correction factor.

Instead of the index s in the designation of service life and dynamic load carrying capacity, s = 100-S is written, where S - bearing reliability (given in tables). Most bearings are made with a reliability of 90 % then s = 10 or L_{10ah}.

For the generally accepted reliability of 90 % at ordinary steel quality and lubrication conditions which condition the separation of working contact surfaces, the correction factors are a_1 = 1; a_{23} = 1. For other reliability requirements, steel quality and lubrication modes, the values of the correction factors a_1 ,a_{23} are selected from the bearing catalogues.

Determination of the reduced dynamic load capacity

Reduced dynamic load for radial and angular contact bearings

$$P = (XVF_r + YF_a) \cdot K_b \cdot K_{\mathrm{T}}$$

Reduced dynamic load for radial thrust bearings

$$P = (XF_r + YF_a) \cdot K_b \cdot K_{\mathrm{T}}$$

Reduced dynamic load for thrust bearings

$$P = F_a \cdot K_b \cdot K_T$$

where F_r – the highest radial load, N;

F_a – the highest axial load, N;

X, Y – radial and axial load factors (indicated in the catalogues according to the ratio F_a/VF_r);

V – ring rotation factor (with inner ring rotation V=1, with outer ring rotation V =1.2);

K_b – safety factor, taking into account the nature of the load (selected from tables);

K_T – temperature coefficient selected from tables. For t ≤ 100 °C K_T = 1.

The above formulae are applied at continuous load and speed.

Variable-mode bearings are selected on the basis of reduced load and conditional speed. If the load varies linearly from P_{max} and P_{min} (for example, supports with single-sided winding), the reduced load:

$$P = \frac{P_{min} + 2P_{max}}{3}$$

If the change in load and speed follows a more complicated law, the load is reduced:

$$P = \sqrt[3]{\frac{P_1^3 \cdot L_1 + P_2^3 \cdot L_2 + \ldots + P_n^3 \cdot L_n}{L}}$$

where P_1, P_2, ..., P_n – constant loads, acting within L_1, L_2, L_n – milions of rpm;

L – the total number of revolutions in millions during which the indicated loads operate.

Characteristics of angular contact bearing selection

In angular contact bearings, when radial loads are applied to them, there are axial components S, which are calculated from the formulae:

$S = 0{,}83eF_r$ – for tapered roller bearings

$S = eF_r$ – for radial ball bearings

where e – axial load influence factor (Tables D.48, D.51, D.52, D.55).

The axial components are designed to reduce the external axial forces and spread the bearing rings in the axial direction. This is prevented by the thrust arms of the shaft and housing with reactions F_{a1} i F_{a2}. For normal bearing operation, the axial force loading the bearing mustn't be less than the axial component of the radial force:

$$F_{a_1} \geq S_1 \text{ and } F_{a_2} \geq S_2$$

In addition to this, the equilibrium condition of the shaft should be met - the sum of all axial forces should be zero. For the scheme in Fig. 3.56, *a*:

$$F_{a_1} + F_a - F_{a_2} = 0$$

The positive directions of the axial forces are those coinciding with the direction of the force F_a. The number 2 indicates the bearing which takes the axial load *Fa*.

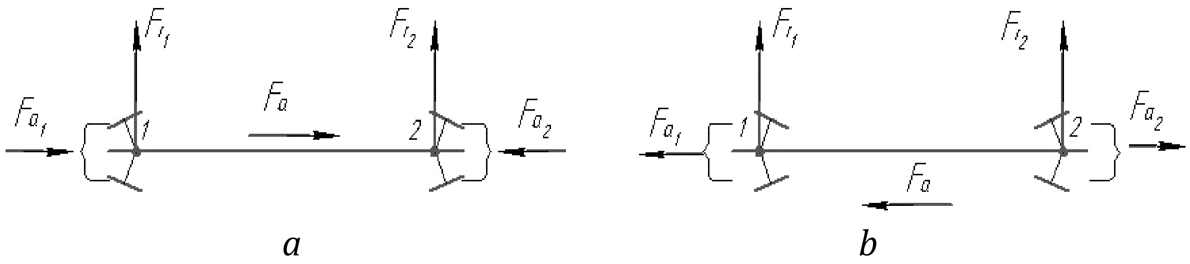


Fig. 3.56. Load patterns for angular contact bearings:
a - "striping" scheme; *b* - "extending" scheme

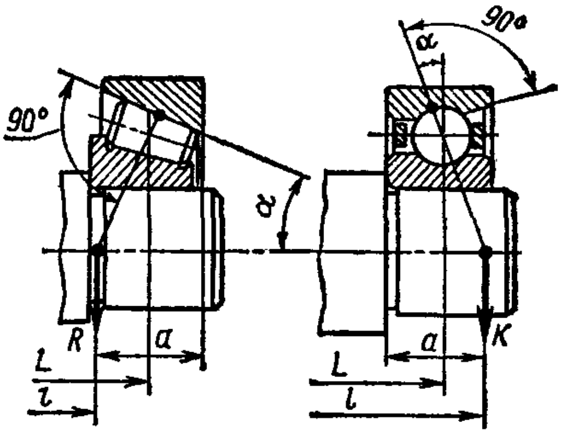
Table 3.24 shows the formulae for calculating the axial forces.

Table 3.24. Formulae for calculating axial loads on angular contact bearings

Load conditions	Design axial loads	
	support I	support II
$S_1 \geq S_2;\ F_a \geq 0$	$F_{a_1} = S_1$	$F_{a2} = F_a + S_1$
$S_1 < S_2;\ F_a \geq S_2 - S_1$	$F_{a_1} = S_2 - F_a$	$F_{a_2} = S_2$

Reduced dynamic loads *P* are determined for each support, only instead of the axial load F_a the corresponding axial load F_{a1} or F_{a2} is used. The design life is determined by the more heavily loaded support.

When determining the radial reactions of angular contact bearings, it is worth remembering that the point of addition of this reaction is located at the intersection of the normal to the centre of the contact surface of the rolling body with the outer ring and the shaft axis, i.e. at a distance a from the lateral surface of the bearing ring (Fig. 3.57).



a *b*

Fig. 3.57. Diagram for calculating the addition points of the angular contact bearing support reactions: a - "striping" scheme; *b* - "extending" scheme

The distance a can be determined either by a graphical method or by one of the following formulae:

for single angular contact ball bearings

$$a = 0.5 \cdot [B + 0.5 \cdot (d + D) \cdot tg\alpha]$$

for single-row tapered roller bearings

$$a = 0.5H + (d + D)e/6$$

where a – the distance from the lateral surface to the point of addition of the radial reaction;

B, d, D, H – bearing dimentions;

α - contact angle;

e – axial load affect factor.

Bearing selection based on static load-carrying capacity

Bearings accepting loads at a standstill or n < 1 rpm (bearings for cranes, transport equipment and other equipment, for example, thrust bearings for slewing cranes, load hooks, elevators, rolling presses, bearings for rotating propeller blades of aircraft and helicopters, etc.) are selected by the static load rating C_0. Bearings with increased requirements are selected based on dynamic load-carrying capacity and are additionally checked for static load-carrying capacity.

Condition for checking and selecting bearings

$$P_0 \leq C_0$$

where P_0 – reduced static load, N;

C_0 – permissible static load rating of a rolling bearing, N (for each bearing type from Tables D.55 ÷ D.61).

Determination of the reduced static load

For radial and angular contact ball and roller bearings, the reduced static load is defined as the greater of the two formulae:

$$P_0 = X_0 F_r + Y_0 F_a$$

If $P_0 < F_r$ then $P_0 = F_r$,

where X_0, Y_0 – radial and axial load factors selected from Tables D.55 ÷ D.61.

For radial roller bearings with short cylindrical rollers, the reduced static load is:

$$P_0 = F_r$$

Most rolling bearings are selected and calculated in terms of dynamic load-carrying capacity.

If bearings of the same type are installed on the same shaft, but carry different loads, it is advisable to select the most heavily loaded bearing to reduce the range of bearings used in the product, and the second bearing should be the same size.

Sequence of bearing selection based on dynamic load-carrying capacity

1. Pre-determine the type and mounting scheme of bearings, taking into account loads, operating and mounting conditions.

2. From the catalogue, taking into account the diameter of the bearing seat for the type of bearing envisaged, list its power, geometrical, kinematic parameters, e, Y, Y_0 factors (for tapered, spherical).

3. Make a conditional or full scheme of the shaft including the bearings on it and the approximate distance between supports. In this case, you need to know in advance which parts are on the shaft and what their dimensions are.

4. Make a design load scheme for bearing supports.

5. Determine the total reactions of each support and select the most heavily loaded bearing for which further calculations will be carried out. For radial thrust bearings, depending on the type of bearing and mounting scheme ("striping" or "extensile" Fig.s 3.56 and 3.57), determine:

- points of application of radial reactions (dimension a) of each support;
- determine the total reactions of each support;
- determine the axial components S of the radial loads for each support;
- determine the calculated axial loads using the formulae given in Table 3.5.

6. Determine the reduced dynamic loads, whereby:

a) taking into account ring mobility, temperature conditions and the nature of the load on the bearing unit, the coefficients V, K_b, and K_T should be selected (for thrust and radial thrust bearings the coefficient V- is not taken into account).

b) taking into account the type of the planned bearing, determine the ratio F/C_{a0}, by which, using linear interpolation from the tables, determine the coefficient e (for angular contact and spherical bearings, the values of e are indicated in the tables of their main parameters), determine the ratio F_a/V_{Fr} and compare it with the coefficient e, based on the results of the comparison F_a/V_{Fr} < or > e, determine the coefficients X, Y according to the tables;

For angular contact bearings, the equivalent load is determined for each bearing (see specific features of angular contact bearing selection).

7. Determine the design life L_{sah} adjusted based on the reliability level and operating conditions or the adjusted design dynamic load rating C_{Sa} for the most heavily loaded bearing.

8. Evaluate the suitability of the intended bearing size under the following conditions:

$$L_{sah} \geq L'_{sah} \text{ or } C_{S_a} \leq c; n \leq n_g$$

Examples of calculations

Example 3.37. Determine the life of the radial single row bearing NUP 412 M ZVL, which is subjected to the highest radial load F_r = 7 kN, loads with significant shocks, temperature mode t < 100 °C, rotational shaft speed n = 500 min^{-1}, , inner ring rotates, no ring warping, lubrication conditions good.

Data:	*Searched for:*
Bearing NUP 412 M ZVL	L_{10ah} – ?
F_r = 7 kN	
t < 100 °C	
n = 500 min^{-1}	
significant shocks	

Solutions

1. From Table D.50 we take the basic force and geometric parameters for the ball bearing NUP 412 M ZVL:

C = 108 kN; C_0 = 70 kN; d = 60 mm; D = 150 mm; B = 35 mm.

2. Determine the reduced dynamic load.

From Table D.46, we take K_T =1; from Table D.45 K_b = 1.8 and, taking into account the mobility of the inner ring, V = 1.

Since only the radial load acts on the bearing, the formula for reduced loads will take the form:

$$P = VF_rK_bK_T = 1 \cdot 7000 \cdot 1 \cdot 1.8 = 12600 \text{ N}$$

3. Determine the adjusted design life of the bearing.

Taking into account the probability of continuous operation of 90 % (most bearings), the absence of ring warping and good lubrication conditions, from Tables D.53 and D.54 we take the corrective factors a_1 =1, a_{23} = 1, the grade index p for ball bearings *p=3.*

$$L_{10ah} = a_1 a_{23} \frac{10^6}{60 \cdot n}\left(\frac{C}{P}\right)^p = 1 \cdot 1 \frac{10^6}{60 \cdot 500}\left(\frac{108 \cdot 10^3}{12600}\right)^3 = 2100 \text{ h}$$

Answer: The service life of NUP 412 M ZVL bearing in the specified operating mode is h.

Example 3.38. Check the service life of a single-row SKF 210-2Z ball bearing placed on the low-noise shaft of a single-stage helical bevel gear reducer (Fig. 3.58) if: axial load F_a = 1.2 kN, highest radial load F_r =3.3 kN; rotational shaft speed n = 200 min^{-1}. Gear operation mode - moderate shocks; required bearing life 95% reliability L'_{5ah} = 20000 h, bearing junction temperature mode t < 100 °C, possible bearing warping, lubrication conditions poor.

Data:
Bearing 210-2Z SKF
F_a = 1.2 kN
F_r = 3.3 kN
n = 200 min^{-1}
L'_{5ah} = 20000 h
t < 100 °C
moderate shocks
reliability 95%

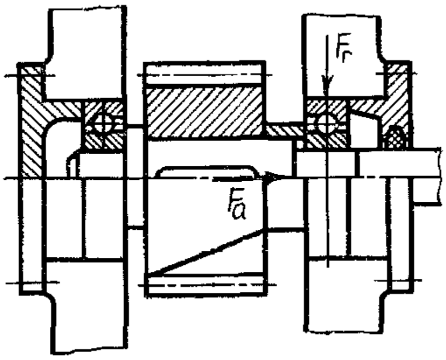


Fig. 3.58. Bevel gear reduction gearbox

Searched for:
L_{5ah} – ?

Solution

1. From Table D.50, we take the basic force and geometric parameters of the SKF 210-2Z ball bearing 210-2Z SKF:

C =35.1 kN; C_0 = 19.8 kN; d = 50 mm; D = 90 mm; B = 20 mm

2. Determine the reduced dynamic load according to the formula:

$$P = (XVF_r + YF_a) \cdot K_b \cdot K_T$$

Define:

a) ratio

$$\frac{F_a}{VF_r} = \frac{1.2 \cdot 10^3}{1 \cdot 3.3 \cdot 10^3} = 0.3654$$

b) ratio

$$\frac{F_a}{C_0} = \frac{1.2 \cdot 10^3}{19.8 \cdot 10^3} = 0.061$$

From Table D.48 we determine the axial load ratio e by linear interpolation. In Table D.48, the value of this ratio is in the range of 0.056 and 0.084, for values of the coefficient e of 0.26 and 0.28. Let us denote the values of the ratios F_a/C_0 and the coefficients e by any symbols, then mathematically:

$0.056 - a \qquad 0.26 - e_1$

$0.061 - b \qquad x - e$

$0.084 - c \qquad 0.28 - e_2$

Then

$$e = e_2 - (c-b)\frac{e_2 - e_1}{c-a} = 0{,}28 - (0.084 - 0.061)\frac{0.28 - 0.26}{0.084 - 0.056} = 0.264$$

For e= 0.264 from Table D.48, in the same way, determine the axial load factor Y = 1.58. Compare F_a/VF_r with e.

Since the ratio F_a/VF_r =0.3654 > e = 0.263, from Table D.48 we take

$$X = 0.56 \cdot Y = 1.58$$

c) we assume V=1 because the inner ring rotates; from the Table D.46 we assume K_T = 1; from the Table D.45 taking into account the bearing mode K_b = 1.3. From the formula determining the reduced load we obtain:

$$P = (0.56 \cdot 3.3 \cdot 10^3 + 1.58 \cdot 1.2 \cdot 10^3) \cdot 1 \cdot 1.3 = 4867\ N$$

3. Determine the adjusted design life of the bearing.

We calculate a reliability of 95%, taking into account the possibility of ring warping and poor lubrication conditions, from Tables D.53 and D.54 take the correction factorsa_1=0.62, a_{23}=0.75, degree index p for ball bearings p = 3.

$$L_{5ah} = a_1 a_{23}\frac{10^6}{60 \cdot n}\left(\frac{C}{P}\right)^p = 0.62 \cdot 0.75\frac{10^6}{60 \cdot 200}\left(\frac{31.5 \cdot 10^3}{4867}\right)^3 = 14531\ \mathrm{h}$$

4. Assess the suitability of the bearing.

Taking into account the bearing selection condition, we have

$$L_{5ah} = 14531\ h < L'_{5ah} = 20000\ h$$

The bearing given is not useful.

Example 3.39. Carry out a bearing selection for a high-speed gear shaft (Fig. 3.58) if: acting radial forces are F_{r1} = 4.2 kN and F_{r2} = 5 kN; axial force F_a = 2 kN, direction - right support; diameter of the shaft under bearing d = 50 mm; shaft speed n= 975 min^{-1}; gearbox operation mode - moderate shocks; operating temperature t < 100^0 C; required bearing life L'_{10ah} = 18000 h, ring warping does not occur; lubrication conditions good.

Data:	*Searched for:*
F_{r1} = 4.2 kN	*Bearing* – ?
F_{r2} = 5 kN	L_{10ah} – ?
F_a = 2 kN	
D = 50 mm	

n = 975 min^{-1}
t < 100 °C
L'_{10ah} = 18000 h

Solution

1. Determine the type of bearing.

When determining the type of bearing, it is important to consider the factors influencing the choice of bearing and to familiarise yourself with the characteristics of bearings.

It is recommended to start with a deep groove ball bearing, which is not expensive, is not scarce and can carry axial loads. It is also possible to use recommendations which, derived from the ratio F_a/F_{rmax} make it possible to tentatively determine the type of bearing from Table D.49. We tentatively assume a radial ball bearing NJ 310 E ZVL, for which we extract the force, kinematic and geometric parameters from Table D.50.

$$C = 65.8 \text{ kN};\ C_0 = 36 \text{ kN}; d = 50 \text{ mm}; D = 110 \text{ mm};$$
$$B = 27 \text{ mm};\ n_{lim} = 6{,}3 \text{ thousand min}^{-1}$$

2. Determine the reduced dynamic load:

$$P = (XVF_r + YF_a) \cdot K_b \cdot K_T$$

The calculation is carried out for the most heavily loaded bearing.

Determine the ratio

$$F_a/_{F_{max}} = \frac{2000}{5000} = 0.4$$
$$F_a/_{C_0} = \frac{2000}{36000} = 0.055$$

From the Table D.48, we determine the axial load influence factor e = 0.255 Y=1.75 by linear interpolation.

Since F_a/VF_{rmax} =0.4 > e = 0.255, from the Table D.48 the radial and axial load factors X= 0.56 and Y = 1.75.

We assume V = 1, because the inner ring rotates. From the Table D.46 we take K_T = 1; from the Table D.45, taking into account the gear mode, we take K_b= 1.3. From the reduced load formula:

$$P = (0.56 \cdot 1 \cdot 5000 + 1.75 \cdot 2000) \cdot 1 \cdot 1.3 = 8190 \text{ N}$$

3. Determine the design-adjusted dynamic bearing capacity.

Taking into account the probability of reliable operation of 90 % (most bearings), the absence of ring warping and good lubrication conditions, from Tables D.53 and D.54 we take the correction factors a_1 = 1, a_{23} = 1, the degree index p for ball bearings $p = 3$.

$$C_{10a} = P \cdot \sqrt[p]{\frac{L_{10ah} \cdot 60 \cdot n}{10^6(a_1 a_{23})}} = 8190 \cdot \sqrt[3]{\frac{18000 \cdot 60 \cdot 975}{10^6 \cdot 1 \cdot 1}} = 3303 \text{ N}$$

4. Assess the suitability of the selected bearing.

Considering the bearing selection condition based on dynamic load carrying capacity:

$C_{Sa} \leq$ C; $n \leq n_{lim}$, we get C_{10a} = 83 303 N > C = 65 800 N – the selected bearing does not suit.

We accept the radial ball bearing of the NU-410 series, for which:

$$C = 87.\text{kN};\ C_0 = 52 \text{ kN}; d = 50 \text{ mm}; D = 130 \text{ mm};$$
$$B = 31 \text{ mm};\ n_{lim} = 5 \text{ thousand min}^{-1}$$

Repeat calculations:

$$F_a/C_0 = \frac{2000}{52000} = 0.038$$

From Table D.43, we determine the axial force influence coefficient e = 0.234 and Y = 1.81 by linear interpolation.

Since the ratio F_a/VF_{rmax} = 0.4 > e = 0.234, from Table D.48 the radial and axial load factors X = 0.56 and Y = 1.81; V = 1; from Table D.45 and D.46 K_T = 1; K_b = 1.3.

Reduced dynamic loads:

$$P = (0.56 \cdot 1 \cdot 5000 + 1.81 \cdot 2000) \cdot 1 \cdot 1.3 = 8346 \text{ N}$$

Design adjusted dynamic bearing load capacity:

$$C_{10a} = P \cdot \sqrt[p]{\frac{L_{10ah} \cdot 60 \cdot n}{10^6(a_1 a_{23})}} = 8346 \cdot \sqrt[3]{\frac{18000 \cdot 60 \cdot 975}{10^6 \cdot 1 \cdot 1}} = 4489 \text{ N}$$

We assess the suitability of the bearing

$$C_{10a} = 84\ 889 \text{ N} < C = 87\ 100 \text{ N}$$
$$n = 975 \text{ min}^{-1} < n_{lim} = 5000 \text{ min}^{-1}$$

5. Determine the adjusted life of the selected bearing:

$$L_{10ah} = a_1 a_{23} \frac{10^6}{60 \cdot n}\left(\frac{C}{P}\right)^p = 1 \cdot 1 \frac{10^6}{60 \cdot 975}\left(\frac{87.1 \cdot 10^3}{8346}\right)^3 = 9340 \text{ h} > L'_{10ah} = 18000 \text{ h}$$

A selected bearing with a spare will provide the required durability.

Answer: Bearing NU-410; L_{10ah} = 19430 h.

Example 3.40. Carry out a bearing selection for a high-speed gear shaft (Fig. 3.59). Forces acting in the system: rotational force F_t = 3 kN; radial force F_r = 1 kN; axial force F_a = 0.5 kN; d = 40 mm, d_1 = 100 mm, b_1 = 45 mm, c_1 = 85 mm, the working temperature of bearings 60 °C, load with moderate run-out, required life of bearings L'_{10ah} = 25000 h, rotational shaft speed n = 1475 min^{-1}; normal work conditions.

Data:
F_t = 3 kN
F_r = 1 kN
F_a = 0.5 kN
d = 40 mm
d_1 = 100 mm
b_1 = 45 mm
c_1 = 85 mm
n = 1475 min^{-1}
T = 60 °C
L'_{10ah} = 25000 h
moderate beating

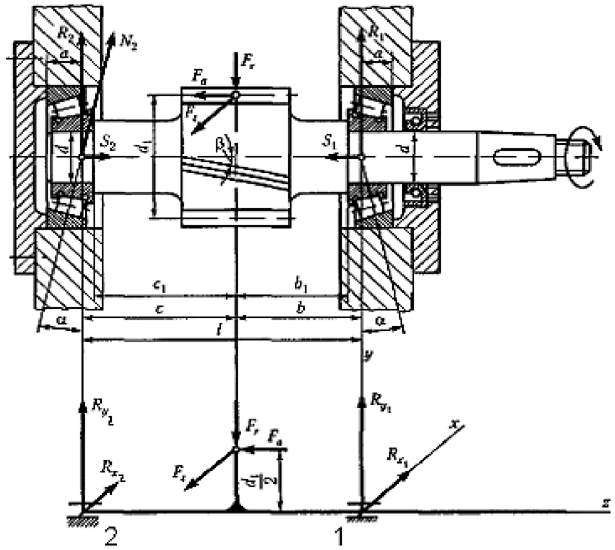


Fig. 3.59. Bearing to Example 3.40

Searched for:
Bearing – ?
L_{10ah} – ?

Solution

1. Determine the type and dimensions of the bearing.

Firstly a medium series tapered roller bearing with an angle α = 12 °. Designation of the angular contact tapered roller bearing 7308 B ZVL, for which from Table D.55:

C = 66 kN, C_0 = 47.5 kN; d = 40 mm; D = 90 mm;
T = 25.25 mm; with liquid grease n_{lim} = 4000 min^{-1};
e = 0.28; Y = 2.16; Y_0 = 1.18.

Placement diagram for the bearing - striping.

2. Plot a diagram of the shaft loading and determine the reactions of the supports (Fig. 3.59).

The distance from the lateral bearing surface to the point of addition of the radial reaction a is determined from the formula:

$$a = 0.5T + [(d + D)/6] \cdot e = 0.5{\cdot}25.25 + \frac{(40 + 90)6}{0.28} \approx 19 \text{ mm}$$

Determine the dimensions c and b, which determine the position of the points of addition of the radial bearing reactions (Fig. 3.59)

$$c = c_1 + T - a = 85 + 25.25 - 19 = 91 \text{ mm}$$
$$b = b_1 + T - a = 45 + 25.25 - 19 = 51 \text{ mm}$$

Determine the reactions of the supports in two mutually perpendicular planes:

Vertical plane (YZ)

$$\sum M_1 = 0;\ R_{y2}(c+b) - F_r b - F_a \frac{d_1}{2} = 0;$$

$$R_{y2} = \frac{F_r b + F_a \frac{d_1}{2}}{c+b} = \frac{1000 \cdot 51 + 500 \cdot \frac{100}{2}}{91+51} = 535 \text{ N}$$

$$\sum M_2 = 0;\ -R_1(c+b) + F_r c - F_a \frac{d_1}{2} = 0;$$

$$R_{y1} = \frac{F_r c - F_a \frac{d_1}{2}}{c+b} = \frac{1000 \cdot 91 - 500 \cdot \frac{100}{2}}{91+51} = 465 \text{ N}$$

Verification

$$\sum Y = 0;\ R_{y2} - F_r + R_{y1} = 0;$$

$$535 - 1000 + 465 = 0;$$

0 = 0 – reactions determined correctly.

Horizontal plane (XY)

$$\sum M_1 = 0;\ R_{x2}(c+b) - F_t b = 0;$$

$$R_{x2} = \frac{F_t b}{c+b} = \frac{3000 \cdot 51}{91+51} = 1078 \text{ N}$$

$$\sum M_2 = 0;\ -R_{x1}(c+b) + F_t c = 0;$$

$$R_{x1} = \frac{F_t c}{c+b} = \frac{3000 \cdot 91}{91+51} = 1922 \text{ N}$$

Verification

$$\sum Y = 0;\ R_{x2} - F_t + R_{x1} = 0$$

$$1078 - 3000 + 1922 = 0$$

0=0 – reactions determined correctly.

Determine the summed reactions of the supports:

$$R_1 = F_{r1} = \sqrt{R_{x1}^2 + R_{y1}^2} = \sqrt{1922^2 + 465^2} = 1978 \text{ N}$$

$$R_2 = F_{21} = \sqrt{R_{x2}^2 + R_{y2}^2} = \sqrt{1078^2 + 532^2} = 1202 \text{ N}$$

3. Determine the axial components of the radial forces and the design axial forces acting on the bearings.

Axial components (e = 0.28 – from Table D.55)

$$S_1 = eF_{r1} = 0.28 \cdot 1978 = 554 \text{ N}$$

$$S_2 = eF_{r2} = 0.28 \cdot 1202 = 337 \text{ N}$$

Design axial forces

Using the calculation scheme and formulae in the Table 3.5 we get:

$$S_1 = 554 \text{ N} > S_2 = 337 \text{ N}, F_a = 500 \text{ N} > 0,$$

then

$$F_{a1} = S_1 = 554 \text{ N}; F_{a2} = F_a + S_1 = 500 + 554 = 1051 \text{ N}$$

4. Determine the reduced dynamic loads P_1 and P_2 of bearings.

We assume $V = 1$ because only the inner ring rotates. From Table D.46, we assume $K_T = 1$, from Table D.45 taking into account the gear mode we assume $K_b = 1.3$.

Right-hand bearing

Determing the ratio

$$\frac{F_{a1}}{VF_{r1}} = \frac{554}{1 \cdot 1978} = 0.88$$

which is greater than $e = 0.28$; so from Table D.55 $X = 0.4$, $Y = 2.16$.

Reduced dynamic radial load:

$$P_1 = (XVF_{r1} + YF_{\mathrm{a}1}) \cdot K_b \cdot K_T = (0.4 \cdot 1 \cdot 1978 + 2.18 \cdot 554) \cdot 1 \cdot 1{,}3 \approx 2599\ \mathrm{N}$$

Left-hand bearing

Determine the ratio

$$\frac{F_{a2}}{VF_{r2}} = \frac{1054}{1 \cdot 1978} = 0.28$$

Which is equal to $e = 0.28$; therefore from Table D.55 $X= 0.4$, $Y =2.16$.

Reduced radial dynamic forces

$$P_2 = (XVF_{r2} + YF_{\mathrm{a}2}) \cdot K_b \cdot K_T = (0.4 \cdot 1 \cdot 1202 + 2.18 \cdot 1054) \cdot 1 \cdot 1.3 \approx 3612\ \mathrm{N}$$

5. We determine the calculated corrected life for the more heavily loaded bearing (the left).

Considering 90% bearing life and normal operating conditions and grease, from Tables D.53 and D.54 we assume $a_1 = 1$, $a_{23} = 0.65$. Degree index p for roller bearings *$p = 10/3$.*

$$L_{10ah} = a_1 a_{23} \frac{10^6}{60 \cdot n}\left(\frac{C}{P}\right)^p = 1 \cdot 0.65 \frac{10^6}{60 \cdot 1475}\left(\frac{66 \cdot 10^3}{3612}\right)^{\frac{10}{3}} = 116883\ \mathrm{h}\ > $$

$$L_{10ah}\ =\ 25000\ \mathrm{h}$$

The selected bearing significantly exceeds the specified life. It is recommended to change the series and type of bearing and repeat the calculation (by yourself).

6. Check the selected bearing based on the static load carrying capacity including an overload of 1.5 (input data and Table D.55).

Reduced static loads:

From the Table D.55 $X_0 = 0{,}5$,$Y_0 =1.18$.

$$P_{01} = \left(X_0F_{r1} + Y_0F_{a_1}\right)1.5 = (0.5 \cdot 1978 + 1.18 \cdot 554) \cdot 1.5 = 2464\mathrm{N} < C_0\ = 47500\ \mathrm{N}$$

condition $P_0 \leq C_0$ is met.

$$\mathrm{P}_{02} = \left(\mathrm{X}_0F_{r2} + Y_0F_{a_2}\right)1.5 = (0.5 \cdot 1202 + 1.18 \cdot 1054) \cdot 1.5 = 2767\mathrm{N}\ <\ C_0\ = 47500\ \mathrm{N}$$

condition $P_0 \leq C_0$ is met.

Asnwer: L_{10ah} = 116883 h; bearing 7308 B ZVL.

Individual task

(calculation)

Task 3.20. Select a bearing for the given operating conditions (Table 3.25).

Table 3.25. Initial data for Task 3.20

Var. no	d, mm	Load			Direction of axial load	n, min^{-1}	Target lifetime L_{0h}, th. h	Ring movement	Temperature conditions	Character of load
		Radial, kN		Axial, kN						
		F_{r1}	F_{r2}	F_a						
1	40	3.0	3.2	1.0	→	750	12.0	Internal	80 °C	Moderate beat
2	45	2.5	2.0	0.8	←	930	16.0			
3	50	3.56	3.6	1.3	→	975	18.0			
4	55	7.0	4.2	1.5	←	1200	20.0			
5	60	4.5	3.62	1.6	→	860	10.0			
6	70	2.0	2.25	0.85	←	730	8.0			Gentle beat
7	75	2.5	2.2	0.9	→	600	12.0			
8	80	3.2	4.0	1.6	←	650	18.0			
9	85	3.6	4.2	1.5	→	700	20.0			
10	90	3.8	4.5	1.43	←	800	5.0			
11	100	4.2	4.6	1.6	→	960	2.5			Moderate beat
12	40	4.6	5.0	1.8	←	950	10.0			
13	60	5.0	5.5	2.0	→	1110	12.			
14	45	5.6	4.8	1.65	←	1255	8.0			
15	55	5.8	6.2	1.	→	1300	6.0			
16	75	6.0	6.2	1.2	←	500	10.0			Gentle beat
17	85	6.25	6.5	1.8	→	400	8.0			
18	90	6.6	5.68	2.4	←	300	20.0			
19	100	1.2	1.0	0.5	→	200	25.0			
20	50	6.8	7.0	2.6	←	450	12.0			
21	55	7.0	6.85	2.55	→	620	8.0			Moderate beat
22	65	7.25	7.86	2.65	←	750	12.0			
23	75	7.5	8.0	3.0	→	550	16.0			
24	85	8.5	8.65	3.2	←	950	12.0			
25	45	8.67	8.5	3.15	→	1050	16.0			
26	40	9.0	8.57	3.3	←	1000	10.0			Gentle beat
27	60	9.28	8.77	3.4	→	1250	12.0			
28	80	9.53	8.64	2.8	←	1100	8.0			
29	50	10.0	8.67	1.2	→	735	22.0			
30	70	12.5	10.0	4.0	←	620	8.0			
	Additional data for variants									
	Bearing arrangement diagram striping									

3.11. Selection of connectors

General information

In the modern engineering industry, most machines consist of assemblies and mechanisms. To ensure kinematic and force coupling, the shafts of the components are connected by couplings.

Connectors are mounting units that, depending on their purpose and design, can perform several primary and secondary functions.a

The main purpose of connectors is to connect the ends of composite shafts or the shaft ends of individual machines and mechanisms; to transmit torque without changing its value along the axis.

Additional functions of the connectors are compensation of minor shaft misalignments that may occur as a result of inaccurate workmanship, installation, thermal deformation, loads and design features of the machine or mechanism; reduction of dynamic and vibration loads; protection of machines and mechanisms from overloads; switching on and off of machines and mechanisms.

Modern machines use a large number of connectors, many of which are standardised.

Connectors selection

Standard and standardised connectors are not calculated. As a rule, they are selected, like rolling bearings, according to tables in reference books or catalogues.

The selection of standard and standardised connectors consists of selecting the required connector size from the catalogue according to the torque.

The main characteristic of the connectors is the transmitted torque T. The specific size of the connector is selected according to the intended use, the design features of the drive and the operating conditions as follows

$$T_{calc} = K_r \cdot T_{zn} \le T_{tab},$$

where T_{calc} is the design torque transmitted through the fitting, Nm;

K_r – operating mode factor, assumed K_r = 1.0 ÷ 1.5 – for machines of small weight and insignificant load (conveyors, machine tools), K_r = 1.5 ÷ 2.0 – for machines of medium weight and medium load (compressors, pumps, woodworking machines), K_r = 2.0 ÷ 3.0 –

for heavy and significantly loaded machinery (breakers, crushers, rollers, cranes);

T_{zn} – rated torque at constant mode of operation on the corresponding shaft, Nm;

T_{tab} – nameplate torque, for this type of connector.

When selecting a connector, it is also necessary to consider the diameters, lengths and shapes of the ends of the shafts to be connected, the mode of operation, the application of the machine or mechanism, the nature of the movement (reversible, non-reversible), speed, weight requirements, dimensions, design features of the drive and the characteristics of the connector.

The standard provides for the production of two types of connectors, with cylindrical and conical holes, in two versions: for long and short shaft ends, and also allows couplings with different *d-hole* diameters and shapes to be combined.

Before selecting a fitting, it is important to familiarise yourself with its design and characteristics.

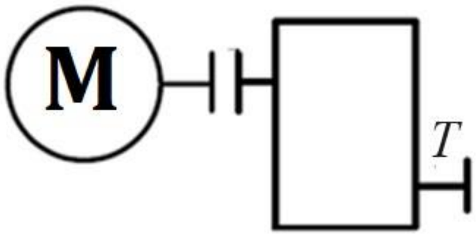
All connectors used in machinery are subject to strength, stiffness and wear resistance calculations.

The selection of connectors is usually a complex task, as it is often necessary to determine the shaft diameter using shaft calculations, as well as determining the kinematic and force parameters.

Examples of calculations

Example 3.41. Write down the selection condition and select the connector that connects the low-speed machine shaft (Fig. 3.60) if: the drive and the working body of the machine are mounted on a common rigid frame; motor power N_m = 15 kW, motor shaft angular velocity ω_m = 102 s^{-1}; reducer ratio u_{red} = 10; permissible torsional stress of the shaft material k_s = 70 MPa; mode factor K_r = 1.5.

Data:
N_m = 15 kW
Ω_m = 102 s^{-1}
u_{red} = 10
k_s = 70 MPa
K_r = 1.5



Searched for:
Connector – ?

Fig. 3.60. Scheme to Example 3.41

Solution

1. Write down the condition for the selection of the fitting including the shaft

$$T_{calc,l-s} = K_r \cdot T_{zn,l-s} \leq T_{tab}$$

2. Define unknowns.

2.1. Determining the moment on the high-speed shaft

$$T_{h-s} = \frac{N_m}{\omega_m} = \frac{15000}{102} = 147 \text{ Nm}$$

2.2. Determining the rated torque on the low-speed shaft

$$l - s = T_{h-s} \cdot u_{red} = 147 \cdot 10 = 1470 \text{ Nm}$$

2.3. Determining the design moment on the pulley shaft

$$T_{calc,l-s} = K_r \cdot T_{zn.,l-s} = 1.5 \cdot 1470 = 2205 \text{ Nm}$$

In addition to the calculated torque, it is important to know the diameter of the shaft for which the connector is selected when selecting the connector.

2.4. From the torsional strength condition, we determine the diameter of the low-speed shaft.

Torsional strength condition

$$\tau_s = \frac{T \cdot 10^3}{0.2d^3} \leq k_s \text{ then}$$

$$d_{l-s} \geq \sqrt[3]{\frac{\mathrm{T}_{calc.l-s} \cdot 10^3}{0.2k_s}} = \sqrt[3]{\frac{2205 \cdot 10^3}{0.2 \cdot 70}} = 54 \text{ mm}$$

3. Select the connector.

Given that the drive and the working body of the machine are on a common frame, it is possible first to try to select a sleeve and plug coupling according to Table D.56, taking into account the selection condition, the calculated torque on the low-speed shaft and its diameter. From Table D.56 it can be seen that for the closed higher torque value T_{tab} = 4000 Nm, the shaft diameter should be in the range d = 80 ÷ 95 mm, which is significantly larger than the design diameter of the low-speed shaft $d_{l\text{-}s}$ = 54 mm.

We will try to select a connector from Table D.57. From the data in the table, you can see that the most suitable connector is a 3 mm, the parameters of which

$$T_{tab} = 3150 \text{ Nm} > T_{calc.l-s} = 2205 \text{ Nm}$$

The shaft diameter can be in the range d = 40 ÷ 60 mm, within which the calculated diameter of the low-speed shaft falls.

Answer: Connector 3 mm with T_{tab} = 3150 Nm; d = 40 ÷ 60 mm.

Example 3.42. Evaluate the feasibility of using a sleeve-and-plug fitting to connect the output shaft of a drive reducer to the working shaft of a machine (Fig. 3.61) if: motor power N_{m} = 5.5 kW; n_{m} = 960 min^{-1}; permissible torsional stress on shaft material k_s = 50 MPa; transmission ratio of belt reductor $u_{\text{p.p}}$ = 2; transmition ratio u_{red} = 3; dynamic coefficient K_r = 1.8; torque T_{tab} = 500 Nm; diameter d_{tab} = 40 mm.

Data:
N_{m} = 5.5 kW
n_{m} = 960 min^{-1}
k_s = 50 MPa
$u_{\text{p.p}}$ = 2
u_{red} = 3
K_r = 1.8
T_{tab} = 500 Nm
d_{tab} = 40 mm

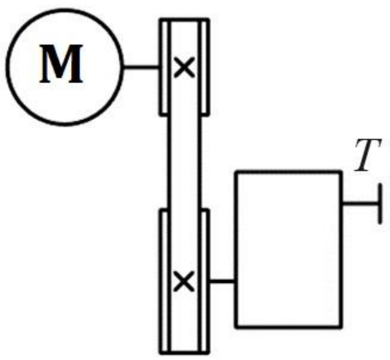


Fig. 3.61. Scheme to Example 3.42

Searched for:
Connector – ?

Solution

1. Write down the condition for the selection of the connector including the shaft

$$T_{calc,l-s} = K_r \cdot T_{zn,l-s} \leq T_{tab}$$

2. Define the unknowns.

2.1. Determining the motor shaft torque

$$T_m = 9.55\frac{N_m}{n_m} = 9.55\frac{5500}{960} \approx 55 \text{ Nm}$$

2.2. Determining the rated torque on the low-speed shaft of the reducer

$$T_{zn,l-s} = T_m u_{p.p}\, u_{red} = 55 \cdot 2 \cdot 3 = 330 \text{ Nm}$$

2.3. Determining the design moment on the low-speed shaft

$$\mathrm{T}_{calc,l-s} = K_r \cdot T_{zn.,l-s} = 1.8 \cdot 330 = 594 \text{ Nm}$$

2.4. From the torsional strength condition, we determine the design diameter of the low-speed shaft.

Shaft torsion strength condition

$$\tau_s = \frac{T \cdot 10^3}{0.2d^3} \leq k_s\,, then$$

$$d_{l-s.} \geq \sqrt[3]{\frac{T_{calc.l-s} \cdot 10^3}{0.2k_s}} = \sqrt[3]{\frac{594 \cdot 10^3}{0{,}2 \cdot 50}} = 39 \text{ mm}$$

Assume $d_{l\text{-}s}$ = 40 mm.

3. Assess the feasibility of a connector.

Comparing the calculated values and the values in the table, we obtain:

$$T_{calc,l-s} = 594 \text{ Nm} > T_{tab} = 500 \text{ Nm} \text{ – condition is not met}$$

$$d_{w.c} = 40 \text{ mm} = d_{tab} = 40 \text{ mm} \text{ – condition is met}$$

Conclusion: the connector does not fit because the computational moment is greater than the moment in the Table.

Individual tasks
(calculation)

Tasks 3.21. Select the connector that connects the low-speed coupling shaft to the working shaft of the machine (Fig. 3.62). The data for the calculations is shown in Table 3.26.

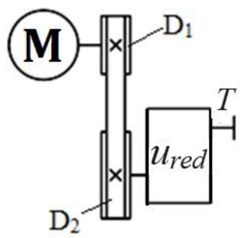


Fig. 3.62

Table 3.26. Initial data for Task 3.21

Var. no	N_{m}, kW	n_{m}, min^{-1};	D_1, mm	D_2, mm	u_{red}	k_S, MPa	K_r
1	10	750	100	300	3	35	2.0
2	11	850	120	360	4	40	1.5
3	15	950	140	420	5	45	1.2
4	17	1000	160	480	6	50	1.3
5	20	1100	180	360	8	60	1.4
6	5	1200	200	600	10	75	1.6
7	7	1500	250	500	3	80	1.8
8	10	750	100	300	3	35	2.2
9	11	850	120	360	4	40	2.0
10	15	950	140	420	5	45	1.5
11	10	1000	160	480	6	50	1.2
12	5	1100	180	360	8	60	1.3
13	8	1200	200	600	10	75	1.4
14	5	1500	250	500	3	80	1.6
15	11	750	100	300	3	35	1.8
16	10	850	120	360	4	40	2.2
17	15	950	140	420	5	45	2.0
18	18	1000	160	480	6	50	1.5
19	20	1100	180	360	8	60	1.2
20	10	1200	200	600	10	75	1.3
21	5	1500	250	500	3	80	1.4
22	15	750	100	300	3	35	1.6
23	10	750	100	300	3	35	2.0
24	11	850	120	360	4	40	1.5
25	5	950	140	420	5	45	1.2
26	10	1000	160	480	6	50	1.3
27	15	1100	180	360	8	60	1.4
28	5	1200	200	600	10	75	1.6
29	20	1500	250	500	3	80	1.8
30	15	750	100	300	5	35	2.2
Supplementary data							
Variants 1 ÷ 5, 11 ÷ 15, 21 ÷ 25 The drive and accessories are mounted on the same frame.							
Variants 6 ÷ 10, 16 ÷ 20, 26 ÷ 30 The drive and accessories are mounted on different frames.							

Task 3.22. Evaluate the feasibility of using a connector to connect the gearbox output shaft to the working shaft of the machine (Table 3.27).

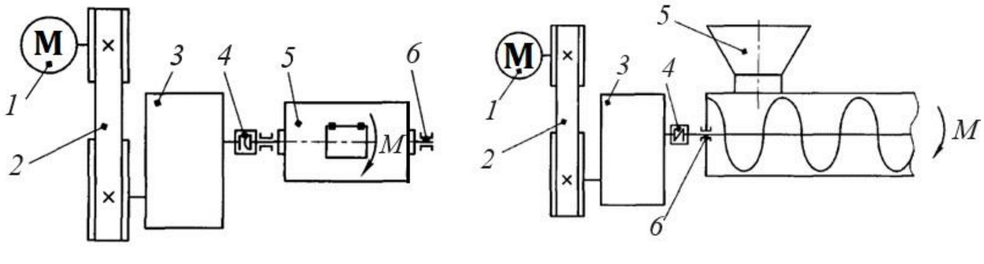


Fig. 3.63. Machine gearbox schemes:
a – impact drum; *b* – worm mixer; 1– motor; 2 – belt transmission; 3 – reducer; 4, a – compensating connector – gear; 4, *b* – elastic connector; 5, a – impact drum; 5, *b* – mixer; 6 – plain

Table 3.27. Initial data for Task 3.22

Var. no	N_m, kW	n_m, min^{-1}	$u_{p.p}$	u_{red}	k_s, MPa	K_r	T_{tab}, Nm	d_{tab} mm	Fig. 3.63
1	10	750	2	3	35	1.4	710	40	*a*
2	11	850	3	4	40	1.6	1400	40÷50	
3	15	950	2.5	5	45	1.8	3150	40÷60	
4	17	1000	4	6	50	2.2	5600	45÷75	
5	20	1100	3	8	60	2.0	8000	50÷90	
6	5	1200	2.5	10	75	1.5	500	40÷45	*b*
7	7	1500	3	3	80	1.2	710	45÷55	
8	10	750	3.5	3	35	1.3	1000	50÷70	
9	11	850	2	4	40	1.4	2000	63÷85	
10	15	950	3	5	45	1.4	4000	80÷95	
11	10	1000	2.5	6	50	1.6	710	40	*a*
12	5	1100	4	8	60	1.8	1400	40÷50	
13	8	1200	3	10	75	2.2	3150	40÷60	
14	5	1500	2.5	3	80	1.4	5600	45÷75	
15	11	750	3	3	35	1.6	8000	50÷90	
16	10	850	3.5	4	40	1.8	500	40÷45	*b*
17	15	950	2	5	45	2.2	710	45÷55	
18	18	1000	3	6	50	2.0	1000	50÷70	
19	20	1100	2.5	8	60	1.5	2000	63÷85	
20	10	1200	4	10	75	1.2	4000	80÷95	
21	5	1500	3	3	80	1.3	710	40	*a*
22	15	750	2.5	3	35	1.4	1400	40÷50	
23	10	750	3	3	35	1.4	3150	40÷60	
24	11	850	3.5	4	40	1.6	5600	45÷75	
25	5	950	2	5	45	1.8	8000	50÷90	
26	10	1000	3	6	50	2.2	500	40÷45	*b*
27	15	1100	2.5	8	60	2.0	710	45÷55	
28	5	1200	4	10	75	1.5	1000	50÷70	
29	20	1500	3	3	80	1.2	2000	63÷85	
30	15	750	2.5	5	35	1.3	4000	80÷95	

APPENDIX

Appendix A (recommended)

Order of performance and requirements for the practical task

1. Select the task variant based on the number on the group list. If the table is missing a number, then the task number is equal to the sum of the numbers in the number on the list (e.g. for number "15" 1 + 5 = 6).
2. Write down the task conditions in the short form "Data". On the right-hand side, write down the values you are looking for "Searched for". When writing down the task conditions, the given and sought values are converted to letters (Appendix B).
3. Plot a drawing or a schematic diagram (if not attached to the task), with the necessary and sufficient number of types, sections, markings, dimensions, acting forces and other parameters. Drawings and diagrams are made by hand. When determining the dimensions, forces and other parameters on a drawing or diagram, the letter designations of the relevant values must be used. Dimensions are measured in millimetres without units at the end.
4. A centred "Solution" is written underneath the drawing or diagram, under which the formulae are written and the calculations are performed. The order in which the solution is performed can vary, for example, all calculation formulas are written down first and then the unknowns are determined, or a formula is written down and the desired value is determined immediately. The procedure for performing the calculation is as follows: first, the formula is written in alphabetical notation, then their numerical values are replaced by letters without specifying the units of measurement, and the result of the calculation is written with SI units of measurement. The letter designation of the values must be the same within the calculation. Intermediate calculations are not performed. The calculations must be accompanied by brief explanations of the adopted coefficients, values, design decisions, references, etc.
5. At the end of the task, the answer is written if it is a design calculation or the conclusion if it is a verification calculation. It is permitted to write the conclusion immediately after the calculation while solving the task. The sequence of practical tasks is shown in the script examples.

Appendix B (recommended)

Designation of certain values

Q	–	concentrated force;
F	–	Force (general designation); load; area;
F_t	–	peripheral force; tangential force;
F_r	–	radial (strut) force;
F_{n}	–	normal force;
P	–	force; load;
M	–	moment of force (general designation);
T	–	torque;
M_{g}	–	bending moment in the beam cross-section;
M_{x}, M_y	–	bending moment in the cross-section of the beam about the x or y axis;
M_{red}	–	the reduced moment according to the adopted strength hypothesis;
N	–	power; longitudinal force in the transverse section of the beam;
σ	–	normal stresses (general designation);
σ_r;σ_{c};σ_{g}	–	stresses normally in tension, compression, and bending respectively;
σ_{red}	–	reduced stresses according to the strength hypothesis;
R_m	–	strength limit (general designations);
R_e	–	yield point;
R_{-1}	–	bending strength limit under symmetric stress cycling;
R_0	–	bending strength limit under pulsating stress cycling;
σ_k, σ_H	–	contact stresses;
k	–	permissible normal stresses (general designation);
k_r; k_c; k_g;	–	allowable stresses in tension, compression, and bending respectively;
k_k	–	permissible contact stresses;
τ	–	tangential stresses (general designations);
τ_{c}	–	shear stress;
τ_p	–	torsional yield strength;
τ_{-1}	–	torsional strength limit under a variable stress cycle;
τ_0	–	torsional strength limit under pulsating stress cycle;

k_s	–	allowable tangential stresses (general designations);
k_t	–	allowable shear stresses;
p	–	pressure; thread pitch;
q	–	continuous force intensity, load per unit length of contact line;
$[q]$	–	distributed permissible stresses;
$[n]$	–	permissible strength reserve factor;
E	–	longitudinal modulus of elasticity;
G	–	shear modulus; mass; gravity;
υ	–	linear speed, rotational speed;
ω	–	speed;
n	–	speed (ob./min); strength factor; number of products;
g	–	acceleration of free fall;
L, l	–	length;
H, h	–	altitude;
B, b	–	width;
D, d	–	diameter;
r	–	radius; cycle asymmetry factor;
δ	–	thickness;
S, s	–	thickness; thread step;
A, a	–	distance between gear axes (toothed, belt, etc.); area;
m	–	the teeth hooking module;
m_n	–	normal tooth attachment modulus;
$m_s\ (m)_t$	–	spur gear module; worm gear axial module;
J_x, J_y	–	centrifugal moment of inertia of the cross-section about the x or y axis respectively;
J_p	–	polar moment of inertia of the beam cross-section;
W_x, W_y	–	centrifugal strength index relative to the x or y axis, respectively;
W_p	–	polar strength index;
z	–	number of products; number of teeth;
i	–	gear transmision ratio; the number of products; the number of spring coils;
u	–	the gear ratio of a pair of gears;
$з$	–	index of cylindrical threaded spring;
HB	–	Brinell hardness;

HRC_e – Rockwell hardness (C scale);

HV – Vickers hardness;

HSh – Shore hardness;

α – coefficient of linear expansion; angle of attachment, angle of thread profile, tooth; angle of belt pulley circumference;

β – tine angle;

ε – linear deformation; slip factor; ratio overlap factor; scale factor;

η – efficiency;

f – coefficient of sliding friction;

μ – Poisson's ratio; dynamic viscosity;

ρ – friction angle; tightness; radius of curvature in gears;

φ – angle of ascent of the thread line; angle of turn.

Comments:

1. The designations of the values within the calculation should be the same.
2. Unpresented designations you can find while discussing the topics of the theoretical and practical parts of the disciplines.

Appendix C (informative)

Table C1. Greek alphabet

Symbol	Pronunciation	Symbol	Pronunciation	Symbol	Pronunciation
$A\ \alpha$	alpha	$I\ \iota$	jota	$P\ \rho$	R(h)o
$B\ \beta$	beta	$K\ \kappa$	kappa	$\Sigma\ \sigma$	sigma
$\Gamma\ \gamma$	gamma	$\Lambda\ \lambda$	lambda	T	tau
$\Delta\ \delta$	delta	$M\ \mu$	mi	$\Upsilon\ \upsilon$	ipsilon
$E\ \varepsilon$	epsilon	$N\ \nu$	ni	$\Phi\ \varphi$	fi
$Z\ \zeta$	dzeta	$\Xi\ \xi$	ksi	$X\ \chi$	chi
$H\ \eta$	eta	$O\ o$	omicron	$\Psi\ \psi$	dog
$\Theta\ \theta$	theta	$\Pi\ \pi$	pi	$\Omega\ \omega$	omega

Nowadays, the International System of Units (SI) is used in all fields of science, technology, economy and education.

Table C2. Selected SI units

Size name	Unit	Designation
Core values		
Length	Metre	m
Mass	Kilogram	kg
Time	Seconds	S
Thermodynamic temperature	Kelvin	K
Additional values		
Obtuse angle	Radian	rad
Derived values		
Surface area	Square metre	m^2
Volume	Cubic metre	m^3
Static moment and moment of resistance of a plane section	Cubic metre	m^3
Moment of inertia of a plane section	One metre to the power of four	m^4
Density	Kilogram per cubic metre	kg/m^3
Speed	Metre per second	m/s
Speed	Radian per second	rad/s, s^{-1}
Power	Newton	N
Stress (mechanical pressure)	Pascal (Niuton per square metre)	Pa (N/m)^2
Power	Watt	W
Specific gravity	Niuton per cubic metre	N/m^3
Moment of inertia (dynamic)	Kilogram per square metre	kg·m^2

Comment: In addition to Kelvin temperature, it is acceptable to use Celsius temperature (t), expressed in degrees Celsius (°C).

Table C3. Multipliers and their names and symbols currently used to form multiple and sub-multiple units

Name	Symbol	Multiplier	Name	Symbol	Multiplier
Mega	M	10^6	Decy	d	10^{-1}
Kilo	k	10^3	Centy	c	10^{-2}
Hekto	h	10^2	Nice	m	10^{-3}
Deka	da	10	Micro	μ	10^{-6}

Comment: 1. It is not permitted to use two prefixes for a simple unit name, e. g., mega-kiloton;

Comment 2. Prefixes may not be used for the names of the following units, which denote a multiple or a unit of value, for example, in tonnes to centres.

Table C4. Conversion of some old and non-SI units to SI units

Units outside the SI system	SI units
Units of length	
1 millimetre, mm	10^{-3} m
1 centimetre, cm	10 m^{-2}
1 micrometre, μm	10 m^{-6}
1 m =1000 mm = 100 cm; 1 cm = 10 mm; 1 μm = 0.001 mm (10^{-3} mm)*	
Units of area	
1 mm^2	10^{-6} m^2
1 cm^2	10 m^{-4} = 100 mm^2
Units of static moment and moment of resistance of a plane section	
1 cm^3	10^{-6} m^3 =1000 mm^3
Units of the moment of inertia of a flat section	
1 cm^4	10^{-8} m^4 = 10000 mm^4
Mass units	
1 tonne, t	1000 kg
1 kg = 1000 g*	
Speed units	
1 rpm (min)$^{-1}$	π/180 rad/s
1 rpm	2π rad/s
Units of force, load, mass	
1 kG (kgf)	9.80665 ≈ 9.81 ≈ 10 N
1 T (tf)	9806.65 ≈ 9810 ≈ 104 N
Force moment units, force pairs	
1 kGm (kgfm)	9.80665 ≈ 9.81 ≈ 10 Nm
1 kGcm (kgfcm)	0.0980665 ≈ 0.0981 ≈ 0.1 Nm
1 Nm = 1000 Nmm*	

Units outside the SI system	SI units
Unity of labour, Energy	
1 kGm (kgfm)	9.80665 ≈ 9.81 ≈ 10 J
1 kWh = 3.6 106 J*	
Power units	
1 kGm· m/s; kG· cm/s	9.80665 W ≈ 9.81 W ≈ 10 W
1 HP	735, 499 W ≈ 735.5 W ≈ 736 W
Mechanical stress units, pressure	
1 kG/cm^2 $(kgf/cm)^2$	98066.5 ≈ 9.81· 10^4 Pa ≈ 0.1 MPa
1 kG/mm^2 $(kgf/mm)^2$	9806650 ≈ 9.81· 10^6 Pa ≈ 10 MPa
1 at	98066.5 ≈ 9.81· 10^4 ≈ Pa ≈ 0.1 MPa
1 at = 1 kgf/cm^2 ; 1 N/mm^2 = 1 MPa*	
Dynamic viscosity units	
1 pause P	0.1 Pas
1 centipauz cP	0.001 Pas
Kinetic viscosity units	
1 stokes, St	$1.0 \cdot 10^{-4}$ m^2/s
1 centistokes, cSt	$1.0 \cdot 10^{-6}$ m^2/s
1 cSt = 1 mm^2 /s = $1.0 \cdot 10^{-6}$ m^2/s*	
Heat transfer units and heat transfer coefficients	
1 $kcal/m^2$ · h · step	1.163 W/m^2 · °C
Units of thermal conductivity coefficient	
1 kcal/m · degree	1.163 W/m · °C
Heat rate unit	
1 kcal	≈ 4.187103 ≈ 4103 J
Units of volume	
1 l	10^{-3} m^3
1000 l = 1 m $*^3$	

Comment: * - additional coefficients are often used in calculations

Table D.1. E, G, μ, ρ values for selected materials at 20 °C.

Material	E-modulus of elasticity, MPa	Kirchhoff modulus $G \cdot 10^4$, MPa	Poisson's ratio μ	Temperature coefficient of linear expansion $\alpha \cdot 10^{-6}$ $^oC^{-1}$	Density ρ_m, kg/m^3
Steel	$(1.90 \div 2.15) \cdot 10^5$	7.8 ÷ 8.30	0.25 ÷ 0.3 0.33	10 ÷ 13	7.7 ÷ 7.8
Grey cast iron	$(0.78 \div 1.47) \cdot 10^5$	4.42	0.23 ÷ 0.27	8.7 ÷ 11	7.0 ÷ 7.1
Tin bronze	$(0.74 \div 1.22) \cdot 10^5$	4.2	0.32 ÷ 0.35	17 ÷ 22	8.6 ÷ 8.8
Tinless bronze	$(1.03 \div 1.18) \cdot 10^5$	4.0 ÷ 4.2	0.36	17 ÷ 22	8.6 ÷ 8.8
Aluminium brass	$(0.98 \div 1.08) \cdot 10^5$	3.63 ÷ 3.92	0.32 ÷ 0.34	17 ÷ 22	8.2 ÷ 8.5
Aluminium alloys	$(6.87 \div 7.07) \cdot 10^4$	2.65	0.33	22 ÷ 24	2.6 ÷ 2.7

Table D.2. Mechanical properties of selected steel grades

Steel	R_e, MPa	k, MPa	R_{-1}, MPa	E, MPa
C10, S195	210	140	160	$2 \cdot 10^5$
C20, S215	240	160	170	
S235	260	175	180	
C25	280	190	210	
C30, S275	300	200	225	
C35, S315	320	210	240	
C45	360	240	275	
C50	380	250	290	
09G2S	310	205	240	

Comment. R_e - yield point, k - allowable stress,
E - elastic modulus, R_{-1} - flexural strength limit

Table D.3. Permissible stresses for welded joints

Welding	Permissible stresses for welded joints		
	stretching k_r'	bending k_g'	shearing k_t'
Automatic, semi-automatic with self-consuming wire (flux), manual, with E42A and E50A electrodes, in the protective gas environment, contact welding	k_r	k_r	0.65 k_r
Manual welding with electrodes E42, and E50 (usual quality), gas welding	0.9 k_r	k_r	0.6 k_r
Manual with E34 electrodes	0.75 k_r	0.6 k_r	0.6 k_r
Contact point	–	–	0.5 k_r

Comment 1. The accepted stress standards apply to low and medium carbon and low alloy steels (types 14GS, 09G2S, 09G2, 15GS, 15HSND and others).

Comment 2. $k_r = R_e/n$ - permissible stresses for the material of the elements to be joined under static loading. For metal structures, the safety factor is n = 1.4 ÷ 1.6. The higher value applies to heavy loading modes.

Under variable loads, the strength of welded joints decreases (influence of thermal zones, technological defects). The calculation of joints under variable loads is carried out using the formulae for the calculation of static loads, the permissible weld stress under static load is multiplied by the variable load factor γ . For variable loads, it is recommended to calculate the strength not only of the weld but also of the components to be joined in the weld zone. The permissible stresses in the weld zone are multiplied by the factorγ calculated from the formula:

$$\gamma = \frac{1}{(0{,}6K_{ef} + 0{,}2) - (0{,}6K_{ef} - 0{,}2)R}$$

where $R = \sigma_{min}/\sigma_{max}$ or τ_{min}/τ_{max} - stress cycle asymmetry factor;

K_{ef} - effective stress concentration factor (from Tables D.4 and D.5).

Table D.4. Effective stress concentration factor (for welds and welded components)

Calculation element	K_{ef} - electric arc welding	
	Low-headed steel	Low-alloy steel
Element at the transition to the butt joint	1.5	1.9
Element at the transition to the butt weld	2.7	3.3
Element at the transition to the side weld	3.5	4.5
Fully remelted butt welds	1.2	1.4
Angle butt welds	2.0	2.0
Angled side welds	3.5	4.5

Table D.5. Effective stress concentration factor (for welds and welded pieces) in contact welding

Steel brand	Sample condition	Thickness, mm	K_{ef} in points	
			Links	Working
C10 steel	Normalization	3 + 3	1.4 (1.25)	7.5 (5)
30HGSA steel	Stress relieving	1.5 + 1.5	1.35	12
Titanium alloy Grade2	Delivery	1.5 + 1.5	2.0 (1.3)	10 (5)
Aluminium alloy 2024 (PA7)	Delivery	1.5 + 1.5	2.0 (1.3)	5 (2.25)

Comment. In brackets, the factor for butt welding and welding

Table D.6. Permissible safety factors for threaded connections

[n] in case of uncontrolled screwing on						
Steel	Fixed loads			Variable loads		
	Thread diameter d, mm			Thread diameter d, mm		
	6 to 16	from 16 to 30	from 30 up to 60	6 to 16	from 16 to 30	30 to 60
Carbon	5 ÷ 4	4 ÷ 2.5	2.5 ÷ 1.6	10 ÷ 6.5	6.5	6.5 ÷ 5
Alloy	6.6 ÷ 5	5 ÷ 3.3	3.3	7.5 ÷ 5	5	5 ÷ 4
[n] = 1.5 ÷ 2.5 for controlled turning						

Table D.7. Permissible stresses for threaded connections, MPa

Type of load	Recommended value
External tensile forces:	
Without tightening the screw	k_r = 0,6R_e
External lateral forces:	
Bolts without backlash	k_t = 0.4R_m (static)
	k_t = 0,2 ÷ 0,3R_m (variable)
Element strength at the joint	k_c = 0,8R_m - steel
	k_c = (0.4 ÷ 0.5)R_e - cast iron
	k_c = 1.2 R_e - concrete
	k_c = 2.4 R_e - wood

Wear-resistant propellers and load screws					
steel+bronze	steel+cast iron	steel+steel	nut-steel	screw-bronze	nut-iron
k_c = 10 ÷ 13	k_c = 4.5 ÷ 8	k_c = 7.5 ÷ 13	k_t = 0.2R_m	k_t = 20 ÷ 25	k_t = 20 ÷ 30

Table D.8. Metric threads (PN-ISO 261:2001), mm

Nominal thread diameter d	Coarse thread			Fine thread		
	p	d_1	d_2	p	d_1	d_2
M6	1	4.918	5.350	0.75	5.188	5.513
M8	1.25	6.647	7.188	1	6.918	7.350
M10	1.5	8.376	9.026	1.25	8.647	9.188
M12	1.75	10.106	10.863	1.25	10.647	11.188
(M14)	2	11.835	12.701	1.5	12.376	13.026
M16	2	13.835	14.701	1.5	14.376	15.026
(M18)	2.5	15.294	16.376	1.5	16.376	17.026
M20	2.5	17.294	18.376	1.5	18.376	19.026
(M22)	2.5	19.294	20.376	1.5	20.376	21.026
M24	3	20.752	22.051	2	21.835	22.701
(M27)	3	23.752	25.051	2	24.835	25.701
M30	3.5	26.211	27.727	2	27.835	28.701
(M33)	3.5	29.211	30.727	2	30.835	31.701
M36	4	31.670	33.402	3	32.752	34.051
M(39)	4	34.670	36.402	3	35.752	37.051
M42	4.5	37.129	39.077	3	38.752	40.051
M(45)	4.5	40.129	42.077	3	41.752	43.051
M48	5	42.587	44.752	3	44.752	46.051
(M52)	5	46.587	48.752	3	48.752	50.051
M56	5.5	50.046	52.428	3	52.752	54.051
(M60)	5.5	54.046	56.428	3	56.752	58.051
M64	6	57.505	60.103	3	60.752	62.051
(M68)	6	61.505	64.103	3	64.752	66.051

Warning. Values in brackets should not be used if possible.

Table D.9. Permissible stresses k_t of keyway connections, MPa

Type of connection	Hub material	Nature of the load	
		Fixed	Variables
Immobile	Steel	150	100
	Cast iron	90	60
Mobile	Steel	50	30
$k_t = 70 \div 100$ The higher value is taken at a constant load			

Table D.10. Permissible stresses k_c of spline connections, MPa.

Type of connection	Operating conditions	Tooth surface	
		Without heat treatment	After heat treatment
Immobile	a	35 ÷ 50	40 ÷ 70
	b	60 ÷ 100	100 ÷ 140
	c	80 ÷ 120	120 ÷ 200
Moving without load (e.g. gearbox)	a	15 ÷ 20	20 ÷ 35
	b	20 ÷ 30	30 ÷ 60
	c	25 ÷ 40	40 ÷ 70
Moving with load (e.g. cardan shaft in cars)	a	–	3 ÷ 10
	b	–	5 ÷ 15
	c	–	10 ÷ 20

Comment: a – heavy operating conditions - loads with run-out, high-frequency and amplitude vibration, poor lubrication conditions in moving joints, low manufacturing accuracy; b -– medium operating conditions; c – good operating conditions. Smaller values for light load modes.

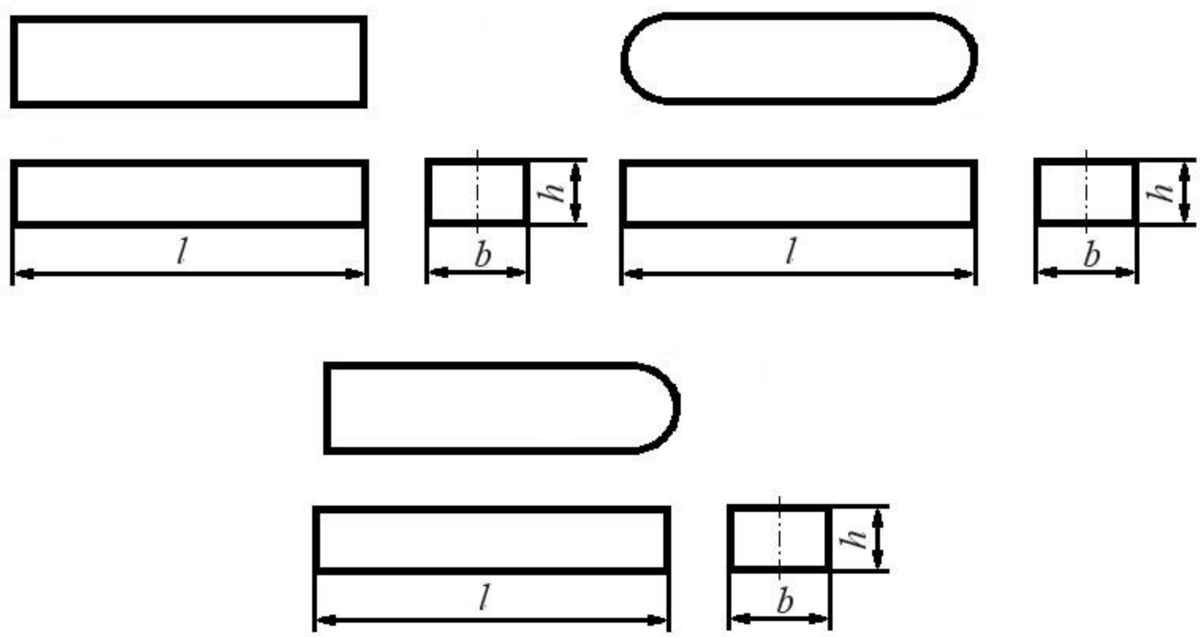


Fig. D.1. Prismatic inlets

Table D.11. Prismatic inlets (PN 85005), mm

Shaft diameter		Dimensions		Groove depth		Length L		Radius r		Chamfering c	
from	to	b	h	shaft t	hub t_1	from	to	from	to	from	to
8	10	3	3	1.8	1.4	6	36	0.08	0.16	0.16	0.25
10	12	4	4	2.5	1.8	8	45				
12	17	5	5	3	2.3	10	56	0.16	0.25	0.25	0.4
17	22	6	6	3.5	2.8	14	70				
22	30	8	7	4	3.3	18	90				
30	38	10	8	5	3.3	22	110	0.25	0.4	0.4	0.6
38	44	12	8	5	3.3	28	140				
44	50	14	9	5.5	3.8	36	160				
50	58	16	10	6	4.3	45	180				
58	65	18	11	7	4.4	50	200				
65	75	20	12	7.5	4.9	56	220	0.4	0.6	0.6	0.8
75	85	22	14	9	5.4	63	250				
85	95	25	14	9	5.4	70	280				
95	110	28	16	10	6.4	80	320				
110	130	32	18	11	7.4	90	360				
130	150	36	20	12	8.4	100	400	0.7	1.0	1.0	1.2
150	170	40	22	13	9,4	100	400				
170	200	45	25	15	10,4	110	450				

Comment: An example of the designation and a standard series of values are shown in the Comments to Table D.12.

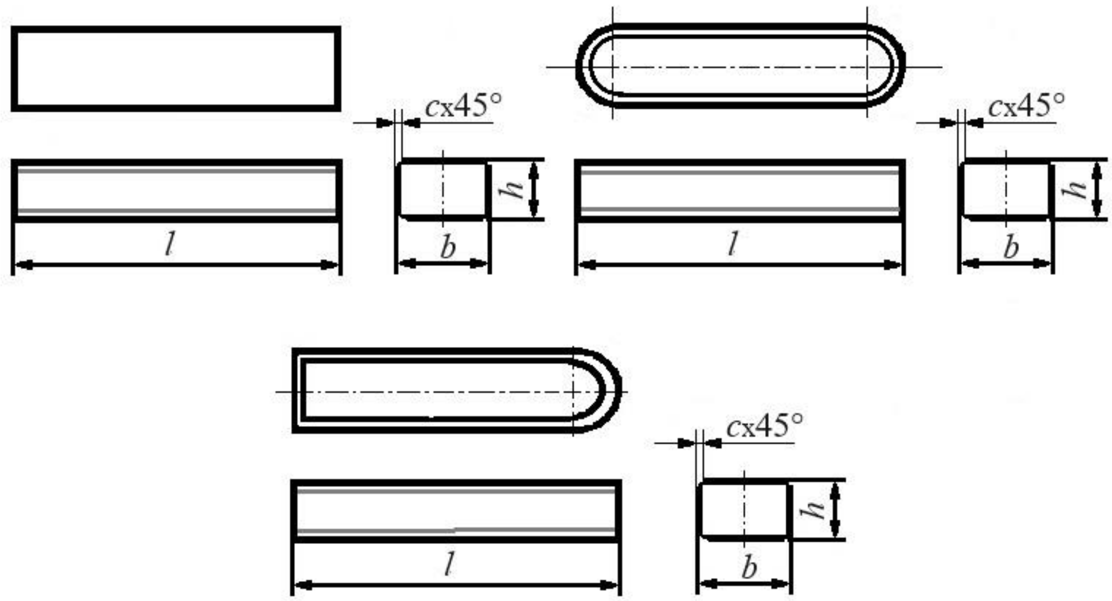


Fig. D.2. High prismatic drains (PN 85001)

Table D.12. High prismatic inlets (PN 85001), mm

Shaft diameter		Dimensions		Groove depth		Length L		Radius r		Chamfering c	
from	to	b	h	shaft t	hub t_1	from	to	from	to	t	t_1
30	38	10	9	5.5	3.8	22	110	0.25	0.4	0.4	0.6
38	44	12	11	7	4.4	28	150				
44	50	14	12	7.5	4.9	36	160				
50	58	16	14	9	5.4	45	180	0.4	0.6	0.6	0.8
58	65	18	16	10	6.4	50	200				
65	75	20	18	11	7.4	56	220				
75	85	22	20	12	8.4	63	250				
85	95	25	22	13	9.4	70	280				
95	110	28	25	15	10.4	80	320	0.7	1.0	1.0	1.2
110	130	32	28	17	11,4	90	360				
130	150	36	32	20	12,4	100	400				
150	170	40	36	22	14,4	100	400				
170	200	45	40	25	15,4	110	450				

Comment 1. Material - drawn steel for drains with an instantaneous strength limit of not less than 500 ÷ 600 MPa.

Comment 2. Length series *l* by PN 85001: 6, 8, 10, 12, 14, 16, 18. 20, 22, 25, 28, 32, 36, 40, 45. 50, 56, 63, 70, 80. 90, 100, 110, 125, 140, 160, 180, 200, 220, 250, 280. 320, 360, 400, 450.

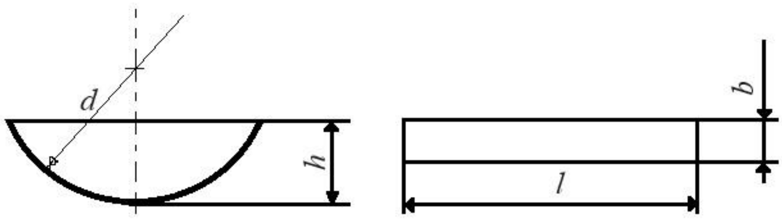


Fig. D.3. Spigot drains (PN 85008)

Table D.13. Spigot drains (PN 85008), mm

Shaft diameter d		Dimensions of drains						Weight, kg	Groove depth	
Communicating the moment	Setting elements	b	h	d_1	l	c or r			shaft	hubs
						max	min		t	t_1
6 to 8	10 to 12	2	2.6	7	6.8	0.16	0.25	0.204	1.8	1.0
		2	3.7	10	9.7			0.414	2.9	
		2.5	3.7	10	9.7			0.510	2.9	
8 to 10	12 to 17	3	3.7	10	9.7			0.612	2.5	1.4
			5	13	12.6			1.05	3.8	
			6.5	16	15.7			1.60	5.3	
10 to 12	17 to 22	4	5	13	12.6			1.40	3.5	1.8
			6.5	16	15.7			2.12	5	
			7.5	19	18.6			3.24	6	
			9	22	21.6			4.10	7.5	
12 to 17	22 to 30	5	6.5	16	15.7	0.25	0.4	2.68	4.5	2.3
			7.5	19	18.6			4.04	5.5	
			9	22	21.6			5.66	7	
			10	25	24.5			6.90	8	
17 to 22	30 to 38	6	9	22	21.6			6.78	6.5	2.8
			10	25	24.5			8.48	7.5	
			11	28	27.3			10.3	8.5	
			13	32	31.4			14.5	10.5	
22 to 30	38 to 44	8	11	28	27.3			13.8	8	3.3
			13	32	31.4			19.3	10	
			15	38	37.1			25.4	12	
30 to 38	44 to 50	10	13	32	31.4	0.4	0.6	24.1	10	3.3
			15	38	37.1			32.3	12	
			16	45	43.1			39.5	13	
			17	55	50.8			45.2	14	
38 to 44	50 to 58	12	19	65	59.1			62.1	16	3.3

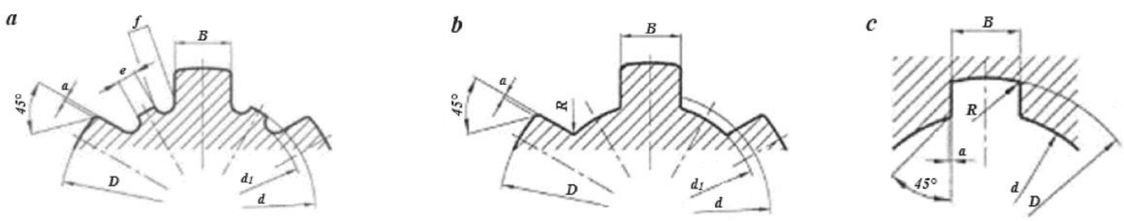


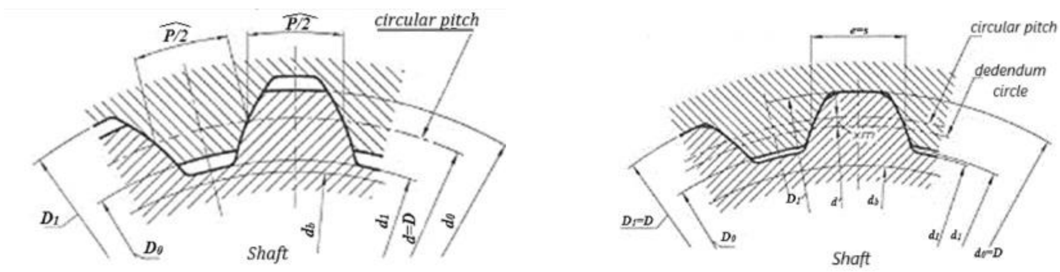
Fig. D.4. Parallel spline connections (PN-M 85017):
a - alignment outline on *d*; *b* - alignment outline on *D* or *B*; *c* - hole outline

Table D.14. Parallel spline connections (PN-M 85017), mm

Nominal size $z \times d \times D$	z	d	D	b	d_1	a	f		r
					No less		Size nom.	tol.	No more than
Lightweight series									
6 × 23 × 26	6	23	26	6	22.1	3.54	0.3	+0.2	0.2
6 × 26 × 30	6	26	30	6	24.6	3.85	0.3	+0.2	0.2
6 × 28 × 32	6	28	32	7	26.7	4.03	0.3	+0.2	0.2
8 × 32 × 36	8	32	36	6	30.4	2.71	0.4	+0.2	0.3
8 × 36 × 40	8	36	40	7	34.5	3.46	0.4	+0.2	0.3
8 × 42 × 46	8	42	46	8	40.4	5.03	0.4	+0.2	0.3
8 × 46 × 50	8	46	50	9	44.6	5.75	0.4	+0.2	0.3
8 × 52 × 58	8	52	58	10	49.7	4.89	0.5	+0.3	0.5
8 × 56 × 62	8	56	62	10	53.6	6.38	0.5	+0.3	0.5
8 × 62 × 68	8	62	68	12	59.8	7.31	0.5	+0.3	0.5
10 × 72 × 78	10	72	78	12	69.6	5.45	0.5	+0.3	0.5
10 × 82 × 88	10	82	88	12	79.3	8.62	0.5	+0.3	0.5
10 × 92 × 98	10	92	98	14	89.4	10.08	0.5	+0.3	0.5
10 × 102 × 108	10	102	108	16	99.9	11.49	0.5	+0.3	0.5
10 × 102 × 120	10	112	120	18	108.8	10.72	0.5	+0.3	0.5
Medium series									
6 × 13 × 16	6	13	16	3.5	12.0	–	0.3	+0.2	0.2
6 × 16 × 20	6	16	20	4	14.54	–	0.3	+0.2	0.2
6 × 18 × 22	6	18	22	5	16.7	–	0.3	+0.2	0.2
6 × 21 × 25	6	21	25	5	19.5	1.95	0.3	+0.2	0.2
6 × 23 × 28	6	23	28	6	21.3	1.34	0.3	+0.2	0.2
6 × 26 × 32	6	26	32	6	23.4	1.65	0.4	+0.2	0.3
6 × 28 × 34	6	28	34	7	25.9	1.70	0.4	+0.2	0.3
8 × 32 × 38	8	32	38	6	29.4	–	0.4	+0.2	0.3
8 × 36 × 42	8	36	42	7	33.5	1.02	0.4	+0.2	0.3
8 × 42 × 48	8	42	48	8	39.5	2.57	0.4	+0.2	0.3
8 × 46 × 54	8	46	54	9	42.7	–	0.5	+0.3	0.5
8 × 52 × 60	8	52	60	10	48.7	2.44	0.5	+0.3	0.5
8 × 56 × 65	8	56	65	10	52.2	2.5	0.5	+0.3	0.5
8 × 62 × 72	8	62	72	12	57.8	2.4	0.5	+0.3	0.5
10 × 72 × 82	10	72	82	12	67.4	–	0.5	+0.3	0.5
10 × 82 × 92	10	82	92	12	77.1	3.0	0.5	+0.3	0.5
10× 92× 102	10	92	102	14	87.3	4.5	0.5	+0.3	0.5
10 × 102 × 112	10	102	112	16	97.7	6.3	0.5	+0.3	0.5

Nominal size $z \times d \times D$	z	d	D	b	d_1	a	f		r
					No less		Size nom.	tol.	No more than
10 × 112 × 125	10	112	125	18	106.3	4.4	0.5	+0.3	0.5
Heavy series									
10 × 18 × 23	10	18	23	3	15.6	–	0.3	+0.2	0.2
10 × 21 × 26	10	21	26	3	18.5	–	0.3	+0.2	0.2
10 × 23 × 29	10	23	29	4	20.3	–	0.3	+0.2	0.2
10 × 26 × 32	10	26	32	4	23.0	–	0.4	+0.2	0.3
10 × 28 × 35	10	28	35	4	24.4	–	0.4	+0.2	0.3
10 × 32 × 40	10	32	40	5	28.0	–	0.4	+0.2	0.3
10 × 36 × 45	10	36	45	5	31.3	–	0.4	+0.2	0.3
10 × 42 × 52	10	42	52	6	36.9	–	0.4	+0.2	0.3
10 × 46 × 56	10	46	56	7	40.9	–	0.5	+0.3	0.5
16 × 52 × 60	16	52	60	5	47.0	–	0.5	+0.3	0.5
16 × 56 × 65	16	56	65	5	50.6	–	0.5	+0.3	0.5
16 × 62 × 72	16	62	72	6	56.1	–	0.5	+0.3	0.5
16 × 72 × 82	16	72	82	7	65.9	–	0.5	+0.3	0.5
20 × 82 × 92	20	82	92	6	75.6	–	0.5	+0.3	0.5
20 × 92 × 102	20	92	102	7	85.5	–	0.5	+0.3	0.5
20 × 102 × 115	20	102	115	8	98.7	–	0.5	+0.3	0.5
20 × 112 × 125	20	112	125	9	104	–	0.5	+0.3	0.5

Comment. For centring on the inside diameter, use designs a and 3, for centring on the outside diameter and spline side surfaces, use design B.



a *b*

Figure D.5. Spline connections of the involute:
a - *S* alignment; *b* - *D* alignment

Table D.15. Convoluted spline connections (PN-ISO 4156:1999), mm

Outer D	m = 1		m = 1.5		m = 2		m = 2.5		m = 3.5		m = 5		m = 10	
	z	x	z	x	z	x	z	x	z	x	z	x	z	x
30	28	0.5	18	0.75	14	0	–	–	–	–	–	–	–	–
32	30	0.5	20	0.25	14	1	–	–	–	–	–	–	–	–
35	34	0	22	0.25	16	0.5	12	1.25	–	–	–	–	–	–
38	36	0.5	24	0.25	18	0	14	0.25	–	–	–	–	–	–
40	38	0.5	26	-0.25	18	1	14	1.25	–	–	–	–	–	–

42	–	–	26	0.75	20	0	16	-0.25	–	–	–	–	–	–
45	–	–	28	0.75	22	-0.5	16	1.25	–	–	–	–	–	–
50	–	–	32	0.25	24	0	18	1.25	–	–	–	–	–	–
55	–	–	36	-0.25	26	0.5	20	1.25	14	1.25	–	–	–	–
60	–	–	38	0.75	28	1	22	1.25	16	0.25	–	–	–	–
65	–	–	–	–	32	-0.5	24	1.25	18	-0.75	–	–	–	–
70	–	–	–	–	34	0	26	1.25	18	1.75	12	2.5	–	–
75	–	–	–	–	36	0.5	28	1.25	20	0.75	14	0	–	–
80	–	–	–	–	38	1	30	1.25	22	-0.25	14	2.5	–	–
85	–	–	–	–	–	–	32	1.25	24	-1.25	16	0	–	–
90	–	–	–	–	–	–	34	1.25	24	1.25	16	2.5	–	–
95	–	–	–	–	–	–	36	1.25	26	0.25	18	0	–	–
100	–	–	–	–	–	–	38	1.25	28	-0.75	18	2.5	–	–
110	–	–	–	–	–	–	42	1.25	30	0.75	20	2.5	–	–
120	–	–	–	–	–	–	46	1.25	34	-1.25	22	2.5	–	–
130	–	–	–	–	–	–	50	1.25	36	0.25	24	2.5	–	–
140	–	–	–	–	–	–	–	–	38	1.75	26	2.5	–	–
150	–	–	–	–	–	–	–	–	42	-0.25	28	2.5	14	0
160	–	–	–	–	–	–	–	–	44	1.25	30	2.5	14	5
170	–	–	–	–	–	–	–	–	48	-0.75	32	2.5	16	0
180	–	–	–	–	–	–	–	–	50	0.75	34	2.5	16	5
190	–	–	–	–	–	–	–	–	–	–	36	2.5	18	0
200	–	–	–	–	–	–	–	–	–	–	38	2.5	18	5

Comment. x - displacement of the initial rail contour, f = 0.1 m – value of chamfer.

Table D.16. Recommended ratio values for different gearboxes

Gearbox	u	
	Closed	Opened
Toothed single-stage:		
cylinder	2 ÷ 6.3	3 ÷ 6.3
conical	2 ÷ 4	–
Toothed two-stage:		
cylinder	6.3 ÷ 40	–
Planetary		
single row	3.15 ÷ 6.3	–
split	6.3 ÷ 18	–
Closed worm	8 ÷ 60	–
Chain	2 ÷ 6	–
Belt	–	–
flat	–	2 ÷ 4
wedge	–	2 ÷ 5
spline	–	2 ÷ 8

Table D.17. Approximate efficiency values

Transmission, assembly	Efficiency	
	Closed	Opened
Toothed single-stage		
cylidry	0.96 ÷ 0.98*	0,93 ÷ 0,95
conical	0.95 ÷ 0.97*	0,92 ÷ 0,94
Planetary		
single row	0.9 ÷ 0.95*	-
split	0.85 ÷ 0.92*	-
Wave	0.8 ÷ 0.92*	-
Snail	0.95(*1-i/200*)*	-
Belt	0.97*	0,92 ÷ 0,95*
Flat, spline	-	0,97*
Wedge-belt	-	0,96*
With roller bearing	0.99 ÷ 0.995	
With plain bearing	0.98 ÷ 0.99	
Coupling	0.98	

Comment. * - values including loss of supports

Table D.18. *4A* closed type asynchronous electric motors (PN-M-88561:1987)

Power, kW	Synchronisation speed, rpm											
	3000			1500			1000			750		
	Size	s, %	$\frac{T_r}{T_z}$	Size	s, %	$\frac{T_r}{T_z}$	Size	s, %	$\frac{T_r}{T_z}$	Size	s, %	$\frac{T_r}{T_z}$
0.55	63B2	8.5	2.0	71A4	7.3	2.0	71B6	10	2.0	80B8	9	1.6
0.75	71A2	5.9		71B4	7.5		80A6	8.4		90LA8	8.4	
1.1	71B2	6.3		80A4	5.4		80B6	8.0		90LB8	7.0	
1.5	80A2	4.2		80B4	5.8		90L6	6.4		100L8	7.0	
2.2	80B2	4.3		90L4	5.1		100L6	5.1		112MA8	6.0	1.8
3.0	90L2	4.3		100S4	4.4		112MA6	4.7		112M8	5.8	
4.0	100S2	3.3		100L4	4.7		112MB6	5.1		132S8	4.1	
5.5	100L2	3.4		112M4	3.7		132S2	3.3		132M8	4.1	
7.5	112M2	2.5		132S4	3.0		132M6	3.2		160S8	2.5	1.4
11.0	132M2	2.3	1.6	132M4	2.8		160S6	2.7	1.2	160M8	2.5	
15	160S2	2.1	1.4	160S4	2.3	1.4	160M6	2.6		180M8	2.5	1.2
18.5	160M2	2.1		160M4	2.2		180M6	2.7		200M8	2.3	
22	180S2	2.0		180S4	2.0		200M6	2.8		200L8	2.7	
30	180M2	1.9		180M4	1.9		200L6	2.1		225M8	1.8	
37	200M2	1.9		200M4	1.7		225M6	1.8		250S8	1.5	

Comment 1. Example of designation of an 11 kW electric motor, synchronous speed 1500 rpm

Electric motor 4A132M4U3

Comment 2. The values of the symbols in the designations: the number 4 indicates the serial number; the letter a - asynchronous motor, the values after the letter - the height of the axis of rotation, mm; the letters *L*, *S* and *M* indicate the setting values after the length; the numbers 2, 4, 6 and 8 indicate the number of poles. The last two symbols *U3* show that the motor is designed for use in a temperate climate.

Comment 3. The *s* column indicates slip in %; the T_r/T_z column indicates the ratio of starting torque to rated torque.

Table D.19. Mechanical properties of selected steels used for gears with hardness < HB 350

Steel grade	Batch diameter, mm	Strength limit, MPa	Yield stress, MPa	Hardness HB	Heat treatment
C35	up to 100 100÷300 300÷500 500÷750	510 590 470 450	270 260 240 230	140 ÷ 187	Normalized
C40	up to 100 100÷300 300÷500 500÷700	550 530 510 490	280 270 260 250	152 ÷ 207	
C45	up to 100 100÷300 300÷500 500÷700	590 570 550 530	300 290 280 270	167 ÷ 217	
C45	40÷60 60÷90 90÷120 180÷250	780÷880 730÷830 680÷780 640÷740	540 440 390 340	223 ÷ 250 207 ÷ 236 194 ÷ 222 180 ÷ 207	Tempered
C50	up to 100 100÷300 300÷500	610 590 570	320 300 290	180÷229	Normalized
	up to 200	790	540	258÷310	Tempered
C55	up to 100 100÷300 300÷500	650 630 610	330 320 310	185 ÷ 229	Normalized
30HGS	up to 60 100÷160 160÷250	980 890 790	840 690 640	215 ÷ 229	Normalized
35H	up to 60 60÷100 100÷200	940 740 690	740 490 440	190 ÷ 241	Normalized
40H	up to 60 100÷200 200÷300 300÷600	980 760 740 690	790 490 490 440	200 ÷ 230	Normalized
40H	up to 120 120÷150 150÷180 180÷250	880÷980 830÷ 930 780÷880 730÷830	690 590 540 490	257 ÷ 285 243 ÷ 271 230 ÷ 257 215 ÷ 243	Tempered
40HN	up to 60 60÷ 100 100÷300 300÷500	980 840 790 740	790 590 570 550	220 ÷ 250	Normalized
40HN	up to 150 150÷180 180÷250	880÷980 830÷930 790÷880	690 590 540	265 ÷ 295 250 ÷ 280 235 ÷ 265	Tempered
Cast alloyed and unalloyed steel					
L35 L40	- -	490 520	270 290	≥ 145	

Steel grade	Batch diamete *r*, mm	Strength limit, MPa	Yield stress, MPa	Hardness HB	Heat treatment
L45	-	540	310	≥ 147	Normalized
L50	-	570	330	≥ 153	
L55	-	590	340	≥ 174	
L40G	-	630	320	155-217 ≥ 174	
L35G	-	590	340	≥ 174	Tempered
L35HGS	-	790	590	≥ 202	
L35HN	-	690	490	219÷269	
L40G2	-	630	320	190÷225	

Table D.20. Mechanical properties of selected steels used for gears with hardness HB≥350.

Steel grade	Heat treatment	HRC tooth surface hardness	Mechanical characteristics of the tooth material	
			R_m	R_e
			MPa	
C20	Carburising	58 ÷ 63	410	240
20G		58 ÷ 63	450	270
12HN2		56 ÷ 58	780	590
15H		58 ÷ 63	690	490
18HGT		58 ÷ 60	980	830
20H		54 ÷ 62	780	640
C40	Surface hardening	38 ÷ 52	550	270
C45		45 ÷ 55	590	330
C50		50 ÷ 57	620	340
40H		50 ÷ 55	740	490
40HN		51 ÷ 57	790	490
38HA	Nitrogenation	50 ÷ 65	880	740
38HMA		50 ÷ 65	980	830
35H	Carbonitriding	48 ÷ 55	830	590
40H		48 ÷ 56	880	640
40HN		50 ÷ 54	900	690

Table D.21. Standardised linear dimensions (ISO 286-1:2010), mm

Rows					Rows					Rows				
Ra5	Ra10	Ra20	Ra40	In.	Ra5	Ra10	Ra20	Ra40	In.	Ra5	Ra10	Ra20	Ra40	In.
10	10	10	10	10.2	40~	40	40	40	41	160	160	160	160	165
–	–	–	10.5	10.8	–	–	–	42	44	–	–	–	170	175
–	–	11	11	11	–	–	45	45	46	–	–	180	180	185
–	–	–	11.5	11.8	–	–	–	48	49	–	–	–	190	195
–	12	12	12	12.5	–	50	50	50	52	–	200	200	200	205
–	–	–	13	13.5	–	–	–	53	55	–	–	–	210	215
–	–	14	14	14.5	–	–	56	56	58	–	–	220	220	230
–	–	–	15	15.5	–	–	–	60	62	–	–	–	240	240
16	16	16	16	16.5	63	63	63	63	65	250	250	250	250	270
–	–	–	17	17.5	–	–	–	67	70	–	–	–	260	290
–	–	18	18	18.5	–	–	71	71	73	–	–	280	280	310
–	–	–	19	19.5	–	–	–	75	78	–	–	–	300	315
–	20	20	20	20.5	–	80	80	80	82	–	320	320	320	330
–	–	–	21	21.5	–	–	–	85	86	–	–	–	340	350
–	–	22	22	23	–	–	90	90	92	–	–	360	360	370
–	–	–	24	24	–	–	–	95	98	–	–	–	380	390
25	25	25	25	27	100	100	100	100	102	400	400	400	400	410
–	–	–	26	26	–	–	–	105	108	–	–	–	420	440
–	–	28	28	29	–	–	110	110	112	–	–	450	450	460
–	–	–	30	31	–	–	–	120	115	–	–	–	480	490
–	32	32	32	33	–	125	125	125	118	–	500	500	500	515
–	–	–	34	35	–	–	–	130	135	–	–	–	530	545
–	–	36	36	37	–	–	140	140	145	–	–	560	560	580
–	–	–	38	39	–	–	–	150	155	–	–	–	600	615

Comment. When selecting sizes, rows with a higher gradation should be preferred (row Ra5), row Ra10 should be preferred, etc.) When selecting sizes larger than 600, the Ra value should be taken from the same rows but an order of magnitude higher. For example: the calculated value is 73.5 mm using the table, we take the value from row Ra 20 ÷ 71 mm.

Table D.22. Recommended combinations of steel grades for pinion and gear at hardness below HB 350

Gearbox	Gear wheel	Gearbox	Gear wheel	Gearbox	Gear wheel
C45	C35 L35 L40 C40 C45	50G	C45 L50 L55 50G (S315)	30HGS	35H 40H L40G
C50	C35 L45 (S275)	–	–	–	–
C55	C45 L55 (S315)	35H or 40H	C50 C55 L55 L35G L40G	40HN	35H 40H L55 L40G

Table D.23. values of the gear width ratio relative to the inter-axial distance Ψ_a (ISO 6336-1:2019).

Ψ_a	0.1	0.125	0.16	0.2	0.25	0.315	0.4	0.5	0.63	0.8	1.0	1.25
Straight tooth gearboxes					$\Psi_a = 0.125 \div 0.25$							
Bevel gearboxes					$\Psi_a = 0.25 \div 0.4$							
Chevron gearboxes					$\Psi_a = 0.5 \div 1.25$							

Table D.24. Values of the involute gear modulus (PN-ISO 54:2001)

Module m_n, mm										
Row 1	0.5	0.6	0.8	1.0	1.25	1.5	2.0	2.5	3.0	4.0
Row 2	0.55	0.7	0.9	1.125	1.375	1.75	2.25	2.75	3.5	4.5
Row 1	5.0	6.0	8.0	10.0	12.0	16.0	20.0	25	32	40
Row 2	5.5	7.0	9.0	11.0	14.0	18.0	22.0	28	36	45

Comment. Second-order values are preferred

Table D.25. values of the coefficient of uneven load distribution along the length of the tooth, $K_{H\beta}$, $K_{F\beta}$

$T_{bd} = \frac{b}{d_1}$	For pinion or wheel hardness < 350HB			With hardness pinion and wheel > 350HB		
	I	II	III	I	II	III
0.2	1.08	1.01	1.00	1.10	1.02	1.00
0.4	1.22	1.05	1.02	1.24	1.06	1.02
0.6	1.40	1.08	1.03	1.46	1.10	1.04
0.8	1.70	1.12	1.05	1.80	1.15	1.07
1.0	2.03	1.17	1.09	2.10	1.23	1.10
1.2	–	1.22	1.11	–	1.36	1.14

Comment. I - cantilever gear arrangement; II - asymmetrical arrangement; III - symmetrical arrangement; T_{bd} - the ratio of the wheel width to its diameter

Table D.26. Inter-axial distance values of cylindrical gears (ISO 6336-1:2019)

Inter-axial distance a_W, mm										
Row 1	40	50	63	80	100	125	160	200	250	315
Row 2	–	–	71	90	112	140	180	225	280	355
Row 1	400	500	630	800	1000	1250	1600	2000	2500	–
Row 2	450	560	710	900	1120	1400	1800	2240	–	–

Comment. Values for Row 2 are preferred

Table D.27. Values of $K_{H\alpha}$ and $K_{F\alpha}$ for bevel gears and chevron gears

Degree of precision	Circular velocity *V*, m/s				
	to 1	5	10	15	20
6	1	1.02	1.03	1.04	1.05
7	1.2	1.05	1.07	1.10	1.12
8	1.06	1.09	1.13	–	–
9	1.1	1.16	–	–	–

Comment. For wheels with straight teeth K_H , $K_{F\alpha} = 1$.

Table D.28. $K_{H\upsilon}$*-values*

Gearbox	Hardness *HB* Tooth surfaces	Circular velocity *V*, m/s			
		to 5	10	15	20
		Degree of accuracy			
		8		7	
About straight teeth	≤350	1.05	–	–	–
	>350	1.10	–	–	–
With oblique teeth and chevron	≤350	1.0	1.01	1.02	1.05
	>350	1.0	1.05	1.07	1.10

Table D.29. Values of tooth form factors Y_F for uncorrected external abutment (ISO 6336:2019)

z or z_e	17	20	25	30	40	50	60	70	80	over 100
Y_F	4.28	4.09	3.90	3.80	3.70	3.66	3.62	3.61	3.61	3.60

Table D.30. $K_{F\upsilon}$*-values*

Degree of accuracy	Hardness *HB* tooth surfaces	Circular velocity *V*. m/s		
		3	3 ÷ 8	8 ÷ 12.5
6	≤ 350	1/1	1.2/1	1.3/1.1
	> 350	1/1	1.15/1	1.25/1
7	≤ 350	1.15/1	1.35/1	1.45/1.2
	> 350	1.15/1	1.25/1	1.35/1.1
8	≤ 350	1.25/1.1	1.45/1.3	-/1.4
	> 350	1.2/1.1	1.35/1.2	-/1.3

Comment. The numerator contains the value of K_F for gears with straight teeth, and the denominator - for gears with oblique teeth.

Table D.31. Basic technical characteristics of single-stage cylindrical gearboxes

Technical characteristics		Gearbox type			
		1CU-100	1CU-160	1CU-200	1CU-250
Transmission		2; 2.5; 3.15; 4; 5; 6.3			
Permissible radial load on the bracket, N	on a high-speed shaft	630	1250	2800	4000
	on a low-speed shaft	2240	4500	6300	9000
Rated torque on low-speed shaft, Nm		315	1250	2500	5000
High-speed shaft speed (no more), min^{-1}		1500			
Efficiency		0.98			
Weight, kg		27	78	135	250

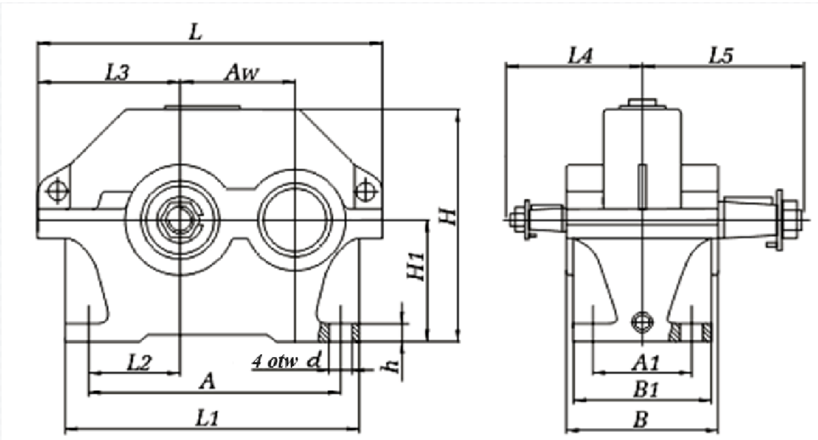


Fig. D.6. Single-stage cylindrical gear 1CU

Table D.32. Overall and connection dimensions of gearbox 1CU

Gearbox	Aw	A	A_1	B	B_1	H	H_1	h	L	L_1	L_2	L_3	L_4	L_5	d
1CU-100	100	224	95	140	132	224	112	22	315	265	85	132	136	155	15
1CU-160	160	355	125	185	175	335	170	28	475	405	136	195	218	218	24
1CU-200	200	437	165	212	200	425	212	36	670	580	165	236	230	265	24
1CU-250	250	545	185	265	250	530	265	40	710	615	212	290	280	315	28

Table D.33. Cylindrical two-stage gearboxes

Technical characteristics		Gearbox type				
		1C2U-100	1C2U-125	1C2U-160	1C2U-200	1C2U-250
Transmission		8; 10; 12.5; 16; 20; 25; 31.5; 40				
Permissible radial load on the bracket, N	on a high-speed shaft	500	750	1000	2240	3150
	on a low-speed shaft	4500	6300	9000	12500	18000
Rated torque on low-speed shaft, Nm		315	630	1250	2500	5000
High-speed shaft speed (no more), min^{-1}		1500				
Efficiency		0.97				
Weight, kg		20	32	95	170	320

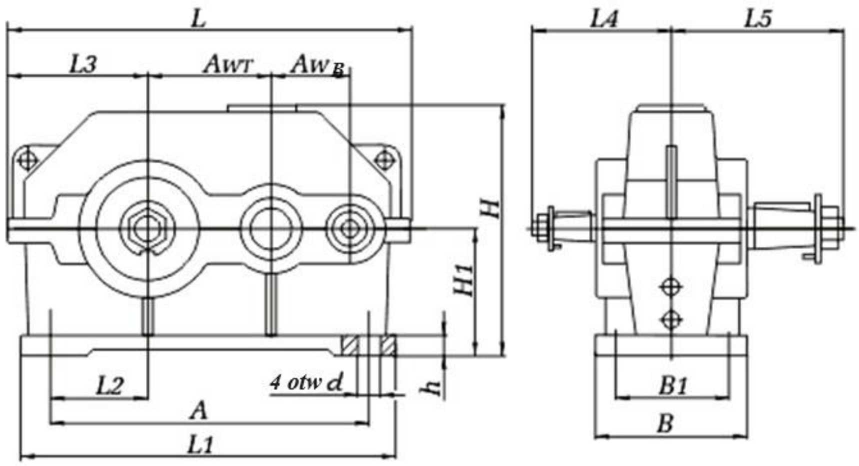


Fig. D.7. Cylindrical two-stage transmission1C2U

Table D.34. Overall and switching dimensions of gearbox 1C2U

Gearbox	Aw_T	Aw_B	A_1	B	B_1	H	H_1	h	L	L_l	L_2	L_3	L_4	L_5	d
1C2U-100	100	80	290	145	109	225	112	20	386	325	85	132	136	165	15
1C2U-125	125	80	335	165	125	270	132	22	440	375	106	155	145	206	19
1C2U-160	160	100	425	195	140	335	170	24	545	475	135	195	170	224	24
1C2U-200	200	125	515	230	165	420	212	30	670	580	165	236	212	280	24
1C2U-250	250	160	670	280	218	515	265	32	825	730	212	290	265	335	28

Table D.35. Cylindrical two-stage gearboxes (with inter-axial distance of the low-speed stage up to 500 mm)

Technical characteristics		Gear types				
		1C2U-315	1C2U-355	1C2U-400	1C2U-450	1C2U-500
Transmission		8; 10; 12.5; 16; 20; 25; 31.5; 40; 50				
Permissible radial load on the bracket, N	on a high-speed shaft	3500	4200	4800	8200	10000
	on a low-speed shaft	30000	40000	45000	71000	10000
Rated torque on the low-speed shaft, Nm		14000	20000	28000	40000	56000
High-speed shaft speed (no more), min^{-1}		1500				
Efficiency		0.98		0.97		
Weight, kg		510	700	930	1530	2090

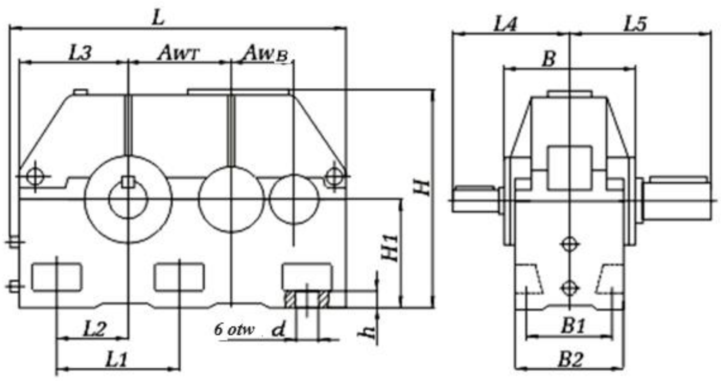


Fig. D8. Cylindrical two-stage gearboxes 1C2У, 1C2N

Table D.36. Overall and connection dimensions of gearbox 1C2U

Gearbox	A_{wT}	A_{wB}	A_1	B	B_1	H	H_1	h	L	L_1	L_2	L_3	L_4	L_5	d
1C2U-315	315	200	395	260	318	685	335	35	1030	370	215	360	300	420	28
1C2U-355	355	225	435	280	360	740	375	35	1160	425	250	400	320	440	28
1C2U-400	400	250	475	330	420	835	425	42	1300	475	280	450	380	500	35
1C2U-450	450	280	630	515	590	955	475	50	1460	530	310	500	500	650	35
1C2U-500	500	315	700	580	650	1055	530	60	1650	615	360	565	530	690	42

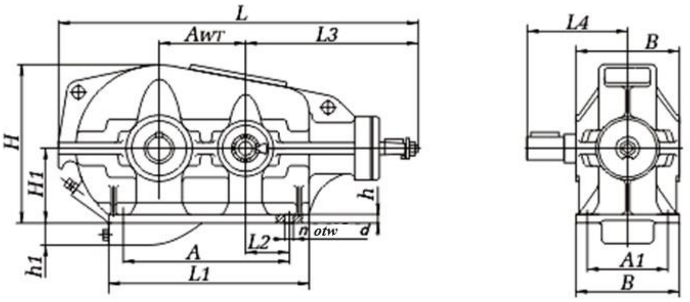


Fig. D.9. Two-stage bevel-cylinder gearboxes of type KC1

Table D.37. Two-stage bevel-cylinder gearboxes

Technical characteristics	Gear types				
	KC1-200	KC1-250	KC1-300	KC1-400	KC1-500
Transmission	6.3; 10; 14; 20; 28				
Permissible radial load on the bracket, N	5100	7000	12000	18000	25000
Rated torque on a low-speed shaft, N ·m	520	1200	2100	5300	9000
Efficiency	0.94				
Weight, kg	186	391	474	980	1740

Table D.38. Overall and connection dimensions of type KC1 gearboxes

Gearbox	A_{WT}	A	A_1	B	H	H_1	h	h_1	L	L_1	L_2	L_3	L_4	d	n
KC1-200	200	375	250	300	435	225	20	-	900	480	85	460	247	17	4
KC1-250	250	480	325	375	515	265	25	-	1170	600	120	625	320	22	4
KC1-300	300	545	350	450	607	315	25	-	1275	680	120	625	385	22	6
KC1-400	400	810	450	526	705	320	35	95	1705	930	212	848	452	26	8
KC1-500	500	990	550	630	877	400	40	100	2085	1160	250	1030	544	33	8

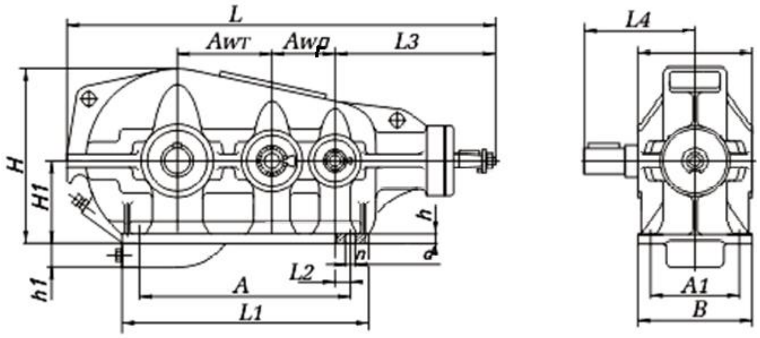


Fig. D.10. Conical-cylindrical three-stage gearboxes type KC2

Table D.39. Tapered-cylinder three-stage gearboxes

Technical characteristics	Gear types			
	KC2-500	KC2-750	KC2-1000	KC2-1300
Transmission	28; 45; 71; 112; 180			
Permissible radial load on the bracket, N	11500	1150	28000	75000
Rated torque on low-speed shaft, N·m	2300	6750	16500	37500
Efficiency	0.91			
Weight, kg	490	1240	2658	5100

Table D.40. Overall and connection dimensions of conical-cylindrical three-stage gearboxes

Gearbox	Aw_T	Aw_P	A	A_1	B	H	H_1	h	h_1	L	L_1	L_2	L_3	L_4	d	n
KC2-500	300	200	705	300	350	600	315	25	-	1300	830	90	460	327	22	6
KC2-750	450	300	1120	470	550	765	335	35	130	1883	1260	120	625	464	33	10
KC2-1000	600	400	1530	600	690	956	400	40	200	2482	1700	165	848	615	33	10
KC2-1300	800	500	2020	740	850	1282	530	50	240	3178	2200	220	1030	790	39	10

Table D.41. Mechanical characteristics of selected steels used for shaft production

Steel	Diameter of semi-finished product mm no less	HB hardness, no less	R_m	R_e	τ_{pl}	R_{-1}	k_s	Heat treatment*
			MPa					
45	Any 120 80	200 240 270	560 800 900	280 550 650	150 300 390	250 350 380	150 210 230	N N N
40H	Any 200 120	200 240 270	730 800 900	500 650 750	280 390 450	320 360 410	200 210 240	N N N
40HN	Any 200	240 270	820 920	650 750	390 450	360 420	210 250	N N
20H	120	240	850	630	240	420	240	Carb, H, ON
12HN3A	120	260	950	700	490	420	210	Carb, H,LT

Comment. * accepted designations: N - normalizing; Carb - carburizing; H - hardening;

LT – stress relieving

Table D.42. Averaged allowable stress values for shafts and axles, MPa

Shaft material	R_m	k_{0g}	k_{-1g}
Carbon steel	400	70	40
	500	75	45
	600	95	55
	700	110	65
Alloy steel	800	130	75
	1000	150	90

Table D.43. Basic dimensions of cylindrical shaft ends (ISO/R 775:1969), mm

Diameter *d* for row		Length *l* for execution		r_1	c
1st	2nd	1	2		
10, 11	-	23	20	0.6	0.4
12, 14	-	30	25	1.0	0.6
16, 18	19	40	28	1.0	0.6
20, 22	24	50	36	1.6	1.0
25, 28	-	60	42	1.6	1.0
32, 36	30, (35), 38	80	58	2.0	1.6
40, 45	42 48	110	82	2.0	1.6
50, 55	(52), (56)	110	82	2.5	2.0
60, 70	63, 65, (71), 75	140	105	2.5	2.0
80, 90	85,95	170	130	3.0	2.5
100, 110, 125	120	210	165	3.0	2.5
140	130, 150	250	200	4.0	3.0
160, 180	170	300	240	4.0	3.0
200, 220	190	350	280	5.0	4.0
250	240, 260	410	330	5.0	4.0
280, 320	300	470	380	5.0	4.0

Comment. Non-recommended values are given in brackets

Table D.44. Values [p] and [$p\upsilon$] for plain bearings

Insert material	υ,m/s	[p], MPa	[$p\upsilon$] MPa· m/s	Application
EN-GJL-HB200	2	0.05	0.1	For working with hardened and normalised shafts
	0.2	9	1.8	
EN-GJL-HB250	3	0.1	0.3	
	0.75	6.0	4.5	
EN-GJL-HB300	3	0.1	0.3	For working with unhardened shafts
	0.75	6	4.5	
EN-GJS-400-15	5	0.5	2.5	

Insert material	υ,m/s	[p], MPa	[$p\upsilon$] MPa· m/s	Application
	1	12	12	For working with hardened and normalised shafts
Ductile cast iron EN-GJS-500-7	5	0.5	2.5	For working with unhardened shafts
	1	12	12	
CuSn10F1	10	15	15	Metal cutting machines, pumps, rolling equipment, gears
CuAl9Fe	8	15	12	
CuSn6Zn6Sn3	8	4 ÷ 6	4 ÷ 6	
CuSn30	10 ÷ 12	20	30 ÷ 90	High variable loads, and imperfect lubrication.
CuSn16Cd3Pb3	2	12	10	Cranes, railways, excavators, crushers, slag carrier liners, cast iron carriers, metal cutting machine spindles
Babbit B83, Babbit B89	60	25	200 ÷ 100	Large loads. Steam turbines, electric machines, turbochargers, roller drives
Babbit B16	6	15	10 ÷ 50	Large loads. Centrifugal pumps, gearboxes, gear racks of rolling mills, metal cutting machines, electric motors – 250 ÷ 750 kW, compressors

Table D.45. Values of the safety factor K_b depending on the nature of the load and the application of the rolling bearings

Charakter of the load	K_b	Application
Light run-out; short-term overloads up to 125% of rated (calculated) load	1.0 ÷ 1.2	Precision gears. Metal cutting machines (except planers, chisels and grinders), hydroscopes. Crane lifting mechanisms. Electric hoists and monorail trolleys. Mechanically driven winches. Small and medium power electric motors. Light fans and blowers.
Moderate run-out; vibration loads; short-term overloads 150 % of rated (calculated) load	1.2 ÷ 1.5	Gears. Gears of all types. Axle boxes for rolling stock. Mechanisms for moving trolleys and cranes. Mechanisms for turning cranes and changing the reach of the jib. Grinder spindles. Electric spindles. Wheels for cars, buses, motorbikes, scooters. Agricultural machinery.
The same, under conditions of increased reliability	1.5 ÷ 1.8	Centrifuges and separators. Axles and traction motors of electric locomotives. Crane motion mechanisms. Wheels of trucks, tractors, tractors, locomotives, cranes and road machinery. High-powered electrical machines. Electrical power equipment.
Loads with significant run-out and vibration; overloads 200 %	1.8 ÷ 2.5	Gear wheels. Crushers and gears. Crank mechanisms. Ball and impact mills. Rolling mills. Powerful fans and extractors.

Charakter of the load	K_b	Application
of rated (calculated) load		
Loads with strong run-out and short-term overload 300 % of rated (calculated) load	2.5 ÷ 3.0	Heavy forging machinery. Sawmill frames. Refrigeration equipment. Working roller conveyors for heavy sectional mills, blooming and slabbing. Hammer mills, crushers.

Table D.46. Temperature coefficient values K_T

Operating bearing temperature °C	≤ 100	125	150	175	200	225	250	350
Temperature coefficient K_T	1.0	1.05	1.10	1.15	1.25	1.35	1.40	1.45

Comment. At t > 120 °C, due to structural changes in the metal, it is necessary to use special bearing materials

Table D.47. Recommended values of basic rolling bearing life L_{10ah} for different types of machinery

Type of machine and nature of work	L_{10ah} , h
Equipment and mechanisms used periodically, agricultural machinery, household appliances	500 ÷ 4000
Mechanisms used for short periods, assembly cranes, construction machinery	4000 ÷ 8000
Precision mechanisms that operate sporadically (auxiliary mechanisms in power plants, conveyors for flow production, lifts, infrequently used metalworking machinery).	8000 ÷ 12000
Part-load single-shift machines (stationary electric motors, gearboxes, crushers)	12000 ÷ 20000
Single-shift, full-load machines (metal-cutting machines, woodworking machines, general technical equipment, cranes, fans, separators, centrifuges, printing equipment)	20000 ÷ 30000
Machinery for round-the-clock use (compressors, pumps, mine hoists, stationary electrical machinery, ship drives, rolling mills, textile machinery)	40000 ÷ 50000
Hydroelectric power plants, rotary kilns, marine engines	60000 ÷ 100000
Machines operating continuously with heavy loads (paper machines, power plants, mine pumps, mushroom shafts of sea-going ships)	100000

Table D.48. X and Y values for bearings

Single- and double-row radial ball bearings					
F_a/C_0	$F_a/VF\ e_r \le$		$F_a/VF\ e_r \ge$		e
	X	Y	X	Y	
0.014				2.30	0.19
0.028				1.99	0.22
0.056				1.71	0.26
0.084				1.55	0.28
0.11	1	0	0.56	1.45	0.30
0.17				1.31	0.34
0.28				1.15	0.38
0.42				1.04	0.42
0.56				1.00	0.44

Angular contact roller bearings. tapered roller bearings and self-aligning roller bearings								
single row				double row				e
$F_a/VF\ e_r \le$		$F_a/VF\ e_r \ge$		$F_a/VF\ e_r \ge$		$F_a/VF\ e_r \le$		
X	Y	X	Y	X	Y	X	Y	
1	0	0.4	$0.4\ \mathrm{ctg}\cdot\alpha$	1	$0.45\ \mathrm{ctg}\cdot\alpha$	0.67	$0.67\ \mathrm{ctg}\cdot\alpha$	$1.5\ \mathrm{tg}\cdot\alpha$

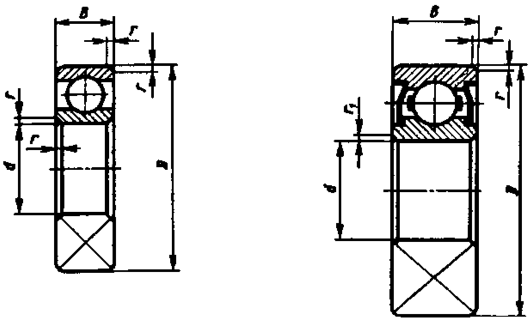
Angular contact ball bearings										
α	*and* F_a/C_0	single row				double row				e
		$F/VF_{ar} \le e$		$F/VF_{ar} \ge e$		$F/VF_{ar} \ge e$		$F/VF_{ar} \le e$		
		X	Y	X	Y	X	Y	X	Y	
	0.014				1.81		2.08		2.94	0.30
	0.029				1.62		1.84		2.63	0.34
	0.057				1.46		1.60		2.37	0.37
	0.086				1.34		1.52		2.18	0.41
12	0.11	1	0	0.45	1.22	1	1.39	0.74	1.98	0.45
	0.17				1.13		1.30		1.84	0.48
	0.29				1.04		1.20		1.69	0.52
	0.43				1.01		1.16		1.64	0.54
	0.57				1.00		1.16		1.62	0.54
	0.015				1.47		1.65		2.39	0.38
	0.029				1.40		1.57		2.28	0.40
	0.058				1.30		1.46		2.11	0.43
	0.087				1.23		1.38		2.00	0.46
15	0.12	1	0	0.44	1.19	1	1.34	0.72	1.93	0.47
	0.17				1.12		1.26		1.82	0.50
	0.29				1.02		1.14		1.66	0.55
	0.44				1.00		1.12		1.63	0.56
	0.58				1.00		1.12		1.63	0.56
18		1	0	0.43	1.00	1	0.02	0.70	1.63	0.57
19							0.92			
α	*and* F_a/C_0	X	Y	X	Y	X	Y	X	Y	e
24										
25				0.41	0.87			0.67	1.44	0.68
26										
30										
35. 36				0.39	0.76		0.78	0.63	1.24	0.80
40				0.37	0.66		0.66	0.60	1.07	0.95
				0.35	0.57		0.55	0.57	0.93	1.14

Comment: i - number of rows of rolling bodies

Table D.49. Selection recommendations for radial ball bearings

ratio F_a/F_{rmax}	Designation and contact angle	The axial component of the radial load S in fractions from F_{rmax}	Attention
0.35 ÷ 0.8	36000; $\alpha = 12°$	0.3 F_{rmax}	Light and super-light series are allowed At very high speeds mostly light series For high speeds, a bearing with the specified contact angle is unsuitable
0.81 ÷ 1.2	46000; $\alpha = 26°$	0.6 F_{rmax}	
> 1.2	66000; $\alpha = 36°$	0.9 F_{rmax}	

Comment. At $F_a/F_{rmax} < 0.35$ single row radial ball bearings are used



a *b*

Fig. D.11. Single row radial ball bearings:
a - 100-200-300-400 (ISO 5753:2009); *b* - 80100-80200 (ISO 5753:2009)

Table D.50. Single row radial ball bearings (ISO 5753:2009), mm

Signs		d	D	B	r	Load capacity, kN		n_{lim}, thousand min^{-1}
						Dynamic, C	Static, C_0	
Very lightweight series								
105		25	47	12	1.0	11.2	5.6	10
106		30	55	13	1.5	13.3	6.8	10
107		35	62	14	1.5	15.9	8.5	8
108	80108	40	68	15	1.5	16.8	9.3	8
109	-	45	75	16	1.5	21.2	12.2	8
110	-	50	80	16	1.5	21.6	13.2	8
111	-	55	90	18	2	28.1	17	8
112	-	60	95	18	2	29.6	18.3	8
113	-	65	100	18	2	30.7	19.6	3.3

Signs		d	D	B	r	Load capacity, kN		n_{lim}, thousand min^{-1}
						Dynamic, C	Static, C_0	
114	-	70	110	20	2	37.7	24.5	6.3
115	-	75	115	20	2	39.7	26.0	5
116	-	80	125	22	2	47.7	31.5	5
117	-	85	130	22	2	49.4	33.5	5
118	-	90	140	24	2.5	57.2	39.0	4
119	-	95	145	24	2.5	60.5	41.5	4
120	-	100	150	24	2.5	60.5	41.5	4
Lightweight series								
205		25	52	15	1.5	14.0	6.95	12.5
206		30	62	16	1.5	19.5	10.0	12.5
207		35	72	17	1.5	25.5	13.7	10
208	80208	40	80	18	2	32.0	17.8	10
209	80209	45	85	19	2	33.2	18.6	8
209A	-	45	85	19	2	36.4	20.1	8
210	-	50	90	20	2	35.1	19.8	8
211	-	55	100	21	2.5	43.6	25.0	6.3
212	80212	60	110	22	2.5	52.0	31.0	6.3
213	80213	65	120	23	2.5	56.0	34.0	5
214	-	70	125	24	2.5	61.8	37.5	5
215	80215	75	130	25	2.5	66.3	41.0	5
216	-	80	140	26	3	70.2	45.0	5
217	-	85	150	28	3	83.2	53.0	5
217A	-	85	150	28	3	89.5	56.5	5
218	80218	90	160	30	3	95.6	62.0	4
219	-	95	170	32	3.5	108.0	69.5	4
219A	-	95	170	32	3.5	115.0	74.0	4
220	80220	100	180	34	3.5	124.0	79.0	4
Medium series								
305		25	62	15	2	22.5	11.4	10
306		30	72	17	2	28.1	14.6	8
307		35	80	19	2	33.2	18.0	8
308		40	90	23	2.5	41.0	22.4	8
309		45	100	25	2.5	52.7	30.0	6.3
310		50	110	27	3	65.8	36.0	6.3
311		55	120	29	3	71.5	41.5	6.3
312		60	130	31	3.5	81.9	48.0	5
313		65	140	33	3.5	92.3	56.0	5
314		70	150	35	3.5	104.0	63.0	5
315		75	160	37	3.5	112.0	72.5	4
316		80	170	39	3.5	124.0	80.0	4
316K5		80	170	39	3.5	130.0	89.0	4
317		85	180	41	4	133.0	90.0	4

Signs		d	D	B	r	Load capacity, kN		n_{lim}, thousand min^{-1}
						Dynamic, C	Static, C_0	
318		90	190	43	4	143.0	99.0	3.2
319		95	200	45	4	153.0	110	3.2
319K5		95	200	45	4	161.0	120.0	3.2
320		100	215	47	4	174.0	132.0	3.2
Heavy series								
405		25	80	21	2.5	36.4	20.4	8
406		30	90	23	2.5	47.0	26.7	6.3
407		35	100	25	2.5	55.3	31.6	6.3
408		40	110	27	3	63.7	36.5	6.3
409		45	120	29	3	76.1	45.5	6.3
410		50	130	31	3.5	87.1	52.0	5
411		55	140	33	3.5	100.0	63.0	5
412		60	150	35	3.5	108.0	70.0	4
413		65	160	37	3.5	119.0	78.1	4
414		70	180	42	4	143.0	105.0	4
415		75	190	45	4	155.0	115.0	4
416		80	200	48	4	163.0	125.0	4
417		85	210	52	5	174.0	135.0	4

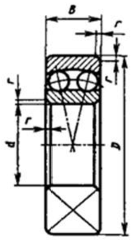


Fig. D.12. Radial spherical double-row ball bearings (ISO 15:1998)

Table D.51. Parameters of double-row spherical radial ball bearings (ISO 15:1998)

Designation	d	D	B	r	Load capacity, kN		e	Y*	Y_0	n_{lim}, thousand. min^{-1}
					C	C_0				
Lightweight narrow series										
1208	40	80	18	2	19.0	8.55	0.22	2.87/4.44	3.01	10
1209	45	85	19	2	21.6	9.65	0.21	2.97/4.6	3.11	8
1210	50	90	20	2	22.9	10.8	0.21	3.13/4.85	3.28	8
1211	55	100	21	2.5	26.5	13.3	0.2	3.23/5.0	3.39	6.3
1212	60	110	22	2.5	30.2	15.5	0.19	3.41/5.27	3.57	6.3

Designation	d	D	B	r	Load capacity, kN		e	Y*	Y_0	n_{lim}, thousand. min^{-1}
					C	C_0				
1213	65	120	23	2.5	31.2	17.2	0.17	3.71/5.73	3.68	5
1214	70	125	24	2.5	34.5	18.7	0.18	3.51/5.43	3.88	5
1215	75	130	25	2.5	39.0	21.5	0.18	3.6/5.57	3.77	5
1216	80	140	26	3	39.7	23.5	0.16	3.94/6.11	4.13	5
1217	85	150	28	3	48.8	28.5	0.17	3.69/5.71	3.87	4
1218	90	160	30	3	57.2	32.0	0.17	3.76/5.82	3.94	4
1220	100	180	34	3.5	63.7	37.0	0.17	3.68/5.69	4.81	3.2
Medium narrow series										
1305	25	62	17	2.0	17.8	6.0	0.28	2.26/3.49	3.36	8
1306	30	72	19	2.0	21.2	7.7	0.26	2.46/3.8	2.58	8
1307	35	80	21	2.5	25.1	9.8	0.25	2.57/3.98	2.69	8
1308	40	90	23	2.5	29.6	12.2	0.24	2.61/4.05	2.74	8
1309	45	100	25	2.5	37.7	15.9	0.24	2.54/3.93	2.66	6.3
1310	50	110	27	3	43.6	17.5	0.24	2.69/4.14	2.8	6.3
1311	55	120	29	3	50.7	23.5	0.23	2.7/4.17	2.82	5
1312	60	130	31	3.5	57.2	26.5	0.23	2.8/4.83	2.93	5
1313	65	140	33	3.5	61.8	29.5	0.23	2.79/4.31	2.92	5
1314	70	150	35	3.5	74.1	35.5	0.22	2.81/4.35	2.95	4
1315	75	160	35	3.5	79.3	38.5	0.22	2.84/4.39	2.97	4
1316	80	170	37	3.5	88.4	42.0	0.22	2.92/4.52	3.06	4
1317	85	180	41	4	97.5	48.5	0.22	2.90/4.49	3.04	4
Medium wide series										
1608	40	90	33	2.5	44.9	15.7	0.43	1.46/2.25	1.52	6.3
1609	45	100	36	2.5	54.0	19.4	0.42	1.51/2.33	1.58	6.3
1610	50	110	40	3	67.7	23.6	0.43	1.48/2.29	1.55	5
1611	55	120	43	3	76.1	28.0	0.41	1.53/2.36	1.6	5
1612	60	130	46	3.5	87.1	33.0	0.4	1.56/2.41	1.63	4
1613	65	140	48	3.5	95.6	38.5	0.38	1.65/2.55	1.73	4
1614	70	150	51	3.5	111.1	44.5	0.38	1.68/2.59	1.76	4
1616	80	170	58	3.5	135.0	58.0	0.37	1.68/2.61	1.76	3.2

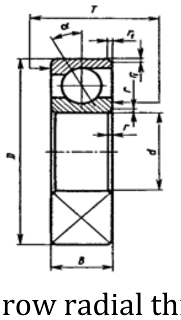


Fig. D.12. Single row radial thrust ball bearings
(ISO 492:2014)

Table D.52. Parameters of single row radial thrust ball bearings (ISO 492:2014)

Designation	d	D	B	T	r	r_1	Load capacity, kN		$n_{\lim}$, thousand $\min^{-1}$
							C	C_0	
Light series narrow α = 12 °									
36208	40	80	18	18	2	1	38.0	23.2	10
36209	45	85	19	19	2	1	31.2	25.1	8
36210	50	90	20	20	2	1	43D	27.0	8
36211	55	100	21	21	2.5	1.2	58.4	34.2	8
36212	60	110	22	22	2.5	1.2	61.5	39.3	6.3
36214	70	125	24	24	2.5	1.2	80.2	54.8	5
36216	80	140	26	26	3	1.5	93.6	65.0	5
36217	85	150	28	28	3	1.5	101.0	70.8	4
36218	90	160	30	30	3	1.5	118.0	83.0	4
36219	95	170	32	32	3.5	2	134.0	95.0	4
Medium narrow series α = 26 °									
46308	40	90	23	-	2.5	1.2	50.8	31.1	6.3
46309	45	100	25	-	2.5	1.2	61.4	37.0	5
46310	50	110	27	-	3	1.5	71.8	44.0	5
46312	60	130	31	-	3.5	2	100.0	65.3	5
46313	65	140	33	-	3.5	2	113.0	75.0	4
46314	70	150	35	-	3.5	2	127.0	85.3	3.2
46316	80	170	39	-	3.5	2	136.0	99.0	3.2
46318	90	190	43	-	4	2	165.0	122.0	2.5
46320	100	215	47	-	4	2	213.0	177.0	2.5
Heavy narrow series α = 36 °									
66408	40	110	27	-	3	1.5	72.2	42.3	5
66409	45	120	29	-	3	1.5	81.6	47.3	5
66410	50	130	31	-	3.5	2	98.9	60.1	3.2
66412	60	150	35	-	3.5	2	125.0	79.5	2.5
66414	70	180	42	-	4	2	152.0	109.0	1.6
66418	90	225	54	-	5	2.5	208.0	162.0	1.25

Table D.53. Corrective reliability coefficient a_1

Reliability S %	90	95	96	97	98	99
Resource designation	L_{10a}	L_{5a}	L_{4a}	L_{3a}	L_{2a}	L_{1a}
a_1	1	0.62	0.53	0.44	0.33	0.21

Table D.54. Correcting material and lubricant coefficient a_{23} (average value)

Bearing type	Working conditions		
	1	2	3
Ball (except spherical)	0.75	1.0	1.3
Tapered roller	0.65	0.9	1.2
Cylindrical roller bearings and spherical roller bearings	0.55	0.8	1.1
Spherical roller	0.35	0.6	0.9

Comment. 1 – normal operating conditions (hydrodynamic lubrication mode not guaranteed, conventional bearing component material, with conventional production technology, slight ring distortion), 2 – hydrodynamic lubrication and slight ring distortion guaranteed, 3 – same lubrication conditions and use of high-quality steel (electro-slagging or vacuum melting).

Table D.55. Tapered single-row roller bearings (ISO 355:2019)

Designation	d	D	T	B	c	r	r_1	Load capacity, kN		e	Y	Y_0	n_{lim}, thous and min^{-1}
								C	C_0				
Light series α = 12 ° ÷ 18 °													
7208	40	80	19.25	19	16	2.0	0.8	46.5	32.5	0.38	1.56	0.86	5
7209	45	85	20.75	20	16	2.0	0.8	50.0	33.0	0.41	1.45	0.8	5
7210	50	90	21.75	21	17	2.0	0.8	56.0	40.0	0.37	1.6	0.88	5
7211	55	100	22.75	21	18	2.5	0.8	65.0	46.0	0.41	1.46	0.8	4
7212	60	110	23.75	23	19	2.5	0.8	78.0	58.0	0.35	1.71	0.94	4
7214	70	125	25.25	26	21	2.5	0.8	96.0	82.0	0.37	1.62	0.89	3.2
7215	75	130	27.25	26	22	2.5	0.8	107.0	84.0	0.39	1.55	0.85	3.2
7216	80	140	28.25	26	22	3.0	0.8	112.0	95.2	0.42	1.43	0.78	3.2
7217	85	150	30.50	28	24	3.0	1.0	130.0	109	0.43	1.38	0.76	2.5
7218	90	160	32.50	31	26	3.0	1.0	158.0	125	0.38	1.56	0.86	2.5
7219	95	170	34.50	32	27	3.5	1.0	168.0	131	0.41	1.48	0.81	2.5
7220	100	180	37.00	34	29	3.5	1.2	185.0	146	0.41	1.49	0.82	2.5
Series average α = 10 ° ÷ 14 °													
7308	40	90	25,25	23	20	2,5	0,8	66	47,5	0,28	2,16	1,18	4
7309	45	100	27.25	26	22	2.5	0.8	83	60	0.28	2.16	1.19	4
7310	50	110	29.25	29	23	3.0	1.0	100	75	0.31	1.94	1.06	4
7311	55	120	31.5	29	25	3.0	1.0	107	81.5	0.33	1.8	0.99	3.2
7312	60	130	33.5	31	27	3.5	1.2	128	96.5	0.3	1.97	1.08	3.2
7313	65	140	36.0	33	28	3.5	1.2	146	112	0.3	1.97	1.08	3.2
7314	70	150	38.0	37	30	3.5	1.2	170	137	0.31	1.94	1.08	3.2
7315	75	160	40.0	37	31	3.5	1.2	180	148	0.33	1.93	1.06	2.5
7317	85	180	44.5	41	35	4.0	1.5	230	196	0.31	1.91	1.05	2
7318	90	190	46.5	43	36	4.0	1.5	250	201	0.32	1.88	1.03	2
Medium series wide α = 11 ° ÷ 16 °													
7608	40	90	35.25	33	29	2.5	0.8	90	67.5	0.3	2.03	1.11	4
7609	45	100	38.25	36	31	2.5	0.8	114	90.5	0.29	2.06	1.13	4

Designation	d	D	T	B	c	r	r_1	Load capacity, kN C	Load capacity, kN C_0	e	Y	Y_0	n_{lim}, thous and min^{-1}
7611	55	120	45.5	44	37	3.0	1.0	160	140	0.32	1.85	1.02	3.2
7612	60	130	48.5	47	39	3.5	1.2	186	157	0.3	1.97	1.08	3.2
7613	65	140	51.0	48	41	3.5	1.2	210	168	0.33	1.83	1.01	3.2
7614	70	150	54.0	51	43	3.5	1.2	240	186	0.35	1.71	0.94	2.5
7615	75	160	58.0	55	47	3.5	1.2	280	235	0.3	1.99	1.20	2.5
7616	85	170	61.5	59	49	3.5	1.2	310	290	0.32	1.89	1.04	2
7618	90	180	67.5	67	54	4.0	1.5	370	365	0.3	1.99	1.2	2
7620	100	215	77.5	73.	61	4.0	1.5	460	460	0.31	1.91	1.65	1.6
Light series wide α = 12 ° ÷ 16 °													
7508	40	80	24.75	24	20	2.0	0.8	56.0	44.0	0.38	1.57	0.87	4
7509	45	85	24.75	24	20	2.0	0.8	60.0	46.0	0.42	1.44	0.79	4
7510	50	90	24.75	24	20	2.0	0.8	62.0	54.0	0.42	1.43	0.78	4
7511	55	100	26.75	25	21	2.5	0.8	80.0	61.0	0.36	1.67	0.92	3.2
7512	60	110	29.75	28	24	2.5	0.8	94.0	75.0	0.39	1.53	0.84	3.2
7513	65	120	32.75	31	27	2.5	0.8	110	98.0	0.37	1.62	0.89	3.2
7514	70	125	33.25	31	27	2.5	0.8	125	101	0.39	1.55	0.85	3.2
7515	75	130	33.25	31	27	2.5	0.8	130	108	0.41	1.48	0.81	2.5
7516	80	140	35.25	33	28	3.0	1.0	143	126	0.40	1.49	0.82	2.5
7517	85	150	38.50	36	30	3.0	1.0	162	141	0.39	1.58	0.85	2
7518	90	160	42.5	40	34	3.0	1.0	190	171	0.39	1.55	0.85	2
7519	95	170	45.5	46	37	3.5	1.2	230	225	0.38	1.56	0.85	2
7520	100	180	49.0	46	39	3.5	1.2	250	236	0.41	1.49	0.82	1.6
Light series wide α = 12 ° ÷ 16 °													
7511A	55	100	26.75	25	21	2.5	0.8	99.0	80.0	0.4	1.5	0.8	3.2
7512A	60	110	29.75	28	24	2.5	0.8	120.0	100	0.4	1.5	0.8	3.2
7513A	65	120	32.75	31	27	2.5	0.8	142.0	120	0.4	1.5	0.8	3.2
75I5A	75	130	33.25	31	27	2.5	0.8	157	130	0.43	1.4	08	3.2
7516A	80	140	35.25	33	28	3.0	1.0	176	155	0.43	1.4	0.8	2.5
7517A	85	150	38.5	36	30	3.0	1.0	201	180	041	1.4	0.8	2
7520A	100	180	49.0	46	39	3.5	1.2	297	280	0.35	1.7	0.9	1.6

Comment: X_0 = 0.5.

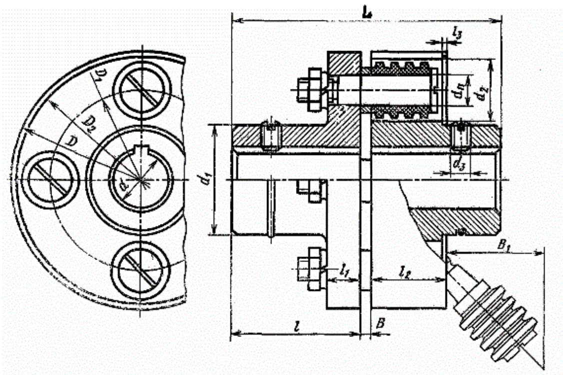


Fig. D.13. Sleeve-finger spring coupling (ISO-R775)

Table D.56. Parameters of bushing-finger spring couplings (ISO-R775)

<table>
<tr><td rowspan="4">T
N·m</td><td rowspan="4">n_{max}
min^{-1}</td><td colspan="11">Dimensions, mm</td></tr>
<tr><td colspan="2" rowspan="2">*d*</td><td rowspan="3">*D* no more</td><td colspan="4">*L* no larger</td><td colspan="4">*l*</td></tr>
<tr><td colspan="8">execution</td></tr>
<tr><td>Row 1</td><td>Row 2</td><td>1</td><td>2</td><td>3</td><td>4</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td rowspan="2">6,3</td><td rowspan="2">8800</td><td>9</td><td rowspan="2"></td><td rowspan="2">71</td><td>43</td><td></td><td>43</td><td>-</td><td>20</td><td>-</td><td>13</td><td>-</td></tr>
<tr><td>10.11</td><td>49</td><td>43</td><td>49</td><td>-</td><td>23</td><td>20</td><td>16</td><td>-</td></tr>
<tr><td rowspan="2">16</td><td rowspan="2">7600</td><td>12.14</td><td rowspan="2"></td><td rowspan="2">75</td><td>63</td><td>53</td><td>63</td><td>-</td><td>30</td><td>25</td><td>20</td><td></td></tr>
<tr><td>16</td><td>83</td><td>59</td><td>83</td><td>59</td><td rowspan="2">40</td><td rowspan="2">28</td><td rowspan="2">30</td><td rowspan="2">18</td></tr>
<tr><td>31,5</td><td>6350</td><td>16.18</td><td></td><td>90</td><td>84</td><td>60</td><td>84</td><td>60</td></tr>
<tr><td>63</td><td>5770</td><td>20.22</td><td></td><td>100</td><td>104</td><td>76</td><td>104</td><td>76</td><td>50</td><td>36</td><td>38</td><td>24</td></tr>
<tr><td rowspan="2">125</td><td rowspan="2">4600</td><td rowspan="2">25.28</td><td rowspan="2">30</td><td rowspan="2">120</td><td>125</td><td>89</td><td>125</td><td>89</td><td>60</td><td>42</td><td>44</td><td>38</td></tr>
<tr><td rowspan="2">165</td><td rowspan="2">121</td><td rowspan="2">165</td><td rowspan="2">121</td><td rowspan="2">80</td><td rowspan="2">58</td><td rowspan="2">60</td><td rowspan="2">38</td></tr>
<tr><td rowspan="2">250</td><td rowspan="2">3800</td><td>32.36</td><td>35,38</td><td rowspan="2">140</td></tr>
<tr><td>40.45</td><td>42</td><td rowspan="2">225</td><td rowspan="2">169</td><td rowspan="2">225</td><td rowspan="2">169</td><td rowspan="5">110</td><td rowspan="5">82</td><td rowspan="5">85</td><td rowspan="5">56</td></tr>
<tr><td>500</td><td>3600</td><td>40.45</td><td>42</td><td>170</td></tr>
<tr><td rowspan="2">710</td><td rowspan="2">3000</td><td>45.50</td><td rowspan="2">48,55</td><td rowspan="2">190</td><td rowspan="3">226</td><td rowspan="3">170</td><td rowspan="3">226</td><td rowspan="3">170</td></tr>
<tr><td>56</td></tr>
<tr><td rowspan="2">1000</td><td rowspan="2">2850</td><td>50.56</td><td>55</td><td rowspan="2">220</td></tr>
<tr><td>63</td><td>60,65,70</td><td>286</td><td>216</td><td>286</td><td>216</td><td rowspan="2">140</td><td rowspan="2">105</td><td rowspan="2">107</td><td rowspan="2">72</td></tr>
<tr><td rowspan="2">2000</td><td rowspan="2">2300</td><td>63.71</td><td>65,70,75</td><td rowspan="2">250</td><td>288</td><td>218</td><td>288</td><td>218</td></tr>
<tr><td>80.90</td><td>85</td><td>348</td><td>268</td><td>348</td><td>268</td><td rowspan="2">170</td><td rowspan="2">130</td><td rowspan="2">135</td><td rowspan="2">95</td></tr>
<tr><td>4000</td><td>1580</td><td>80.90</td><td>85,95</td><td>320</td><td>350</td><td>270</td><td>350</td><td>270</td></tr>
<tr><td rowspan="2">8000</td><td rowspan="2">1450</td><td>100.110</td><td rowspan="2">120</td><td rowspan="2">400</td><td rowspan="2">432</td><td rowspan="2">352</td><td rowspan="2">432</td><td rowspan="2">352</td><td rowspan="3">210</td><td rowspan="3">170</td><td rowspan="3">170</td><td rowspan="3">125</td></tr>
<tr><td>125</td></tr>
<tr><td rowspan="3">16000</td><td rowspan="3">1150</td><td>125</td><td>120</td><td rowspan="3">500</td><td>435</td><td>355</td><td>432</td><td>352</td></tr>
<tr><td>140</td><td>130,150</td><td>515</td><td>415</td><td>515</td><td>415</td><td>250</td><td>200</td><td>205</td><td>155</td></tr>
<tr><td>160</td><td>-</td><td>615</td><td>495</td><td>615</td><td>495</td><td>300</td><td>240</td><td>245</td><td>185</td></tr>
</table>

Comment. Half couplings can be made: 1, 2 - with cylindrical holes for the long and short ends of the shafts; 3, 4 - with tapered holes for the long and short ends of the shafts.

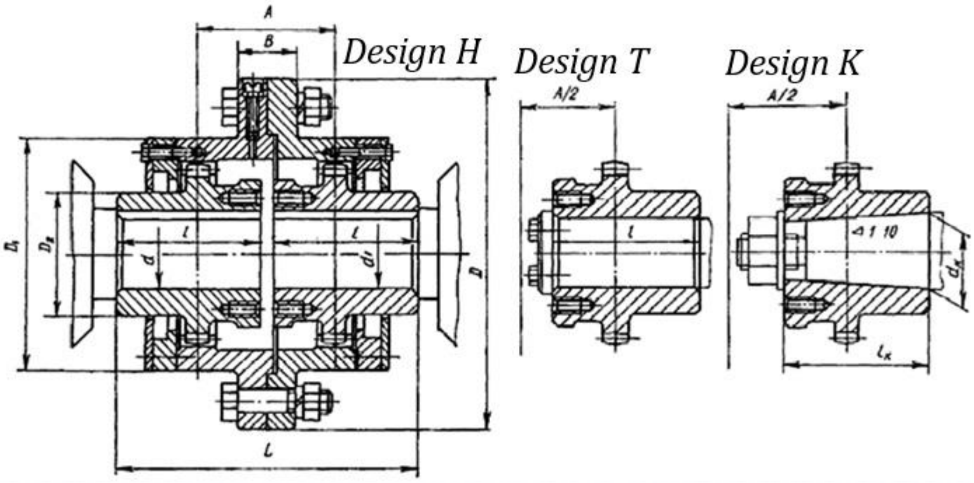


Fig. D.14. Gear couplings (ISO-R773)

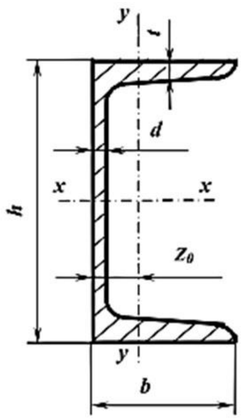
Table D.57. Basic parameters of gear couplings (ISO-R773)

Coupling number	T, Nm	mm										$n_{\max}$, min^{-1}
		d	d_k	d_1	D	D_1	L		l	l_k	A, no less	
							PB	PP				
1	710	40	38	60	170	110	115	115	55	55	49	6300
2	1400	40÷50	55	70	185	125	145	145	70	80	75	5000
3	3150	40÷60	55	90	220	150	170	175	85	80	95	4000
4	5600	45÷75	75	100	250	175	215	215	105	105	125	3350
5	8000	50÷90	95	120	290	200	265	240	115	130	145	2800
6	11800	60÷105	-	130	320	230	255	260	125	-	160	2500
7	19000	65÷120	120	150	350	260	285	290	140	165	185	2120
8	23600	80÷140	150	170	380	290	325	330	160	200	210	1900
9	30000	80÷160	-	190	430	330	335	340	165	-	220	1700
10	50000	80÷180	-	210	490	390	365	370	180	-	245	1400

Comment 1. Coupling type *PB* - for direct connection of shafts; *PP* - for connection of shafts using an intermediate shaft;

Comment 2. Design of half couplings: *H* - without shaft-end fixing; *T* - with shaft-end fixing; *K* - with tapered hole;

Comment 3. The shaft diameter *d of* the intermediate ranges is taken in accordance with D.21 or D.43.



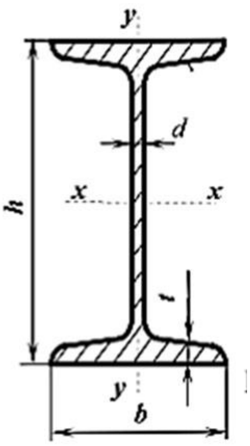
Symbols designation
h - profile height
b - width of shelf
d - wall thickness
t - average thickness of the shelf
F - cross-sectional area
J - moment of inertia
W - strength index
i - radius of inertia
S - static moment of half-section
z_0 - distance from the *y-axis* to the outer wall

Fig. D.15. Hot-rolled steel section (ISO-657-11-1980)

Table D.58. Basic parameters of hot-rolled steel channels (ISO-657-11-1980)

Profile no.	Basic dimensions, mm				F cm^2	J_x cm^4	W_x cm^3	i_x cm	S_x cm^3	J_y cm^4	W_y cm^3	i_y cm	z_0 cm
	h	b	d	t									
5	50	32	4.4	7.0	6.16	22.8	9.1	1.92	5.59	5.61	2.75	0.954	1.16
6.5	65	36	4.4	7.2	7.51	48.6	15.0	2.54	9.0	8.7	3.68	1.08	1.24
8	80	40	4.5	7.4	8.98	89.4	22.4	3.16	13.3	12.8	4.75	1.19	1.31
10	100	46	4.5	7.6	10.9	174	34.8	3.99	20.4	20.4	6.46	1.37	1.44
12	120	52	4.8	7.8	13.3	304	50.6	4.78	29.6	31.2	8.52	1.53	1.54
14	140	58	4.9	8.1	15.6	491	70.2	5.6	40.8	45.4	11.0	1.70	1.67

Profile no.	Basic dimensions, mm				F cm^2	J_x cm^4	W_x cm^3	i_x cm	S_x cm^3	J_y cm^4	W_y cm^3	i_y cm	z_0 cm
	h	b	d	t									
14a	140	62	4.9	8.7	17.0	545	77.8	5.66	45.1	57.5	13.3	1.84	1.87
16	160	64	5.0	8.4	18.1	747	93.4	6.42	54.1	63.6	13.8	1.87	1.80
16a	160	68	5.0	9.0	19.5	823	103	6.49	59.4	78.8	16.4	2.01	2.00
18	180	70	5.1	8.7	20.7	1090	121	7.24	69.8	86	17.0	2.04	1.94
18a	180	74	5.1	9.3	22.2	1190	132	7.32	76.1	105	20.0	2.18	2.13
20	200	76	5.2	9.0	23.4	1520	152	8.07	87.8	113	20.5	2.20	2.07
20a	200	80	5.2	9.7	25.2	1670	167	8.15	95.9	139	24.2	2.35	2.28
22	220	82	5.4	9.5	26.7	2110	192	8.89	110	151	24.1	2.37	2.21
22a	220	87	5.4	10.2	28.8	2330	212	8.99	121	187	30.0	2.55	2.46
24	240	90	5.6	10.0	30.6	2900	242	9.73	139	208	31.6	2.60	2.42
24a	240	95	5.6	10.7	32.9	3180	265	9.84	151	254	37.2	2.78	2.67
27	270	95	6.0	10.5	35.2	4160	308	10.9	178	262	37.3	2.73	2.47
30	300	100	6.5	11	40.5	5810	387	12.0	224	327	43.6	2.84	2.52
33	330	105	7.0	11.7	46.5	7980	484	13.1	281	410	51.8	2.97	2.59
36	360	110	7.5	12.6	53.4	10820	601	14.2	350	513	61.7	3.10	2.68
40	400	115	8.0	13.5	61.5	15220	761	15.7	444	642	73.4	3.23	2.75



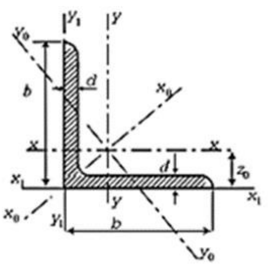
Symbols designation
h - profile height
b - width of shelf
d - wall thickness
t - average thickness of the shelf
F - cross-sectional area
J - moment of inertia
W - strength index
i - radius of inertia
S - static moment of half-section
Z_0 - distance from the *y-axis* to the outer wall

Fig. D.16. Hot-rolled I-section (ISO 657/13)

Table D.59. Hot-rolled I-beam (ISO 657/13)

Profile no.	Basic dimensions, mm				F cm^2	J_x cm^4	W_x cm^3	i_x cm	S_x cm^3	J_y cm^4	W_y cm^3	i_y cm
	h	b	d	t								
10	10	55	4.5	7.2	12.0	198	39.7	4.06	23.0	17.9	6.49	1.22
12	120	64	4.8	7.3	14.7	350	58.4	4.88	33.7	27.9	8.72	1.38
14	140	73	4.9	7.5	17.4	572	81.7	5.73	46.8	41.9	11.5	1.55
16	160	81	5.0	7.8	20.2	873	109	6.57	62.3	58.6	14.5	1.70
18	180	90	5.1	8.1	23.4	1290	143	7.42	81.4	82.6	18.4	1.88
18a	180	100	5.1	8.3	25.4	1430	159	7.51	89.8	114	22.8	2.12
20	200	100	5.2	8.4	26.8	18.40	184	8.28	104	115	23.1	2.07
20a	200	110	5.2	8.6	28.9	2030	203	8.37	114	155	28.2	2.32
22	220	110	5.4	8.7	30.6	2550	232	9.13	131	157	28.6	2.27
22a	220	120	5.4	8.9	32.8	0790	254	9.22	143	206	34.3	2.50

Profile no.	Basic dimensions, mm				F cm²	J_x cm⁴	W_x cm³	i_x cm	S_x cm³	J_y cm⁴	W_y cm³	i_y cm
	h	*b*	*d*	*t*								
24	240	115	5.6	9.5	34.8	3460	289	9.97	163	198	34.5	2.37
24a	240	125	5.6	9.8	37.5	3800	317	10.1	178	260	41.6	2.63
27	270	125	6.0	9.8	40.2	5010	371	11.2	210	250	41.5	2.54
27a	270	135	6.0	10.2	43.2	5500	407	11.3	229	337	50.0	2.80
30	300	135	6.5	10.2	46.5	7080	472	12.3	268	337	49.9	2.69
30a	300	145	6.5	10.7	49.9	7780	518	12.5	292	436	60.1	2.95
33	330	140	7.0	11.2	43.8	9840	597	13.5	339	419	59.9	2.79
36	360	145	7.5	12.3	61.9	13380	743	14.7	423	516	71.1	2.89
40	400	155	8.3	13.0	72.6	19062	953	16.2	545	667	86.1	3.03
45	450	160	9	14.2	84.7	27696	1231	18.1	708	808	101	3.09
50	500	170	10	15.2	100	39727	1589	19.9	919	1043	123	3.23
55	550	180	11	16.5	118	55962	2035	21.8	1181	1356	151	3.39
60	600	190	12	17.8	138	76806	2560	23.6	1491	1725	182	3.54



Symbols designation
b - width of shelf
d - wall thickness
J - moment of inertia
i - radius of inertia
z_0 - distance from the centre of gravity to the outer limits of the shelves

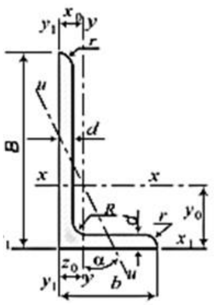
Fig. D.16. Hot-rolled steel angle bar (ISO 657-1:1989)

Table D.60. Basic parameters of hot-rolled steel angles (ISO 657-1:1989)

Profile number	*b*	*d*	Section area cm²	Additional values for the axes								Weight 1m
				x-x		x_0-x_0		y_0-y_0		x_1-x_1	z_0, cm	
	mm			J_x cm⁴	i_x cm	J_{x0max} cm⁴	i_{x0max} cm	J_{y0min} cm⁴	$i_{y0\,min}$ cm	J_{x1} cm⁴		
2	20	3	1.13	0.40	0.59	0.53	0.75	0.17	0.39	0.81	0.60	0.89
		4	1.46	0.50	0.58	0.78	0.73	0.22	0.38	1.09	0.64	1.15
2.5	25	3	1.43	0.81	0.75	1.29	0.95	0.34	0.49	1.57	0.73	1.12
		4	1.86	1.03	0.74	1.62	0.93	0.44	0.48	2.11	0.76	1.46
2.8	28	3	1.62	1.16	0.85	1.84	1.07	0.48	0.55	2.2	0.80	1.27
3	30	3	1.74	1.45	0.91	2.30	1.15	0.60	0.59	–	0.85	1.36
		4	2.27	1.84	0.90	2.92	1.13	0.77	0.58		0.89	1.78
3.2	32	3	1.86	1.77	0.97	2.80	1.23	0.74	0.63	3.26	0.89	1.46
		4	2.4.	2.26	0.96	3.58	1.21	0.94	0.62	4.39	0.94	1.91
3.5	35	3	2.04	2.35	1.07	3.72	1.35	0.97	0.69		0.97	1.60
		4	2.67	3.01	1.06	4.76	1.33	1.25	0.68	–	1.01	2.10
		5	3.28	3.61	1.05	5.71	1.32	1.52	0.68		1.05	2.58
4	40	3	2.35	3.55	1.23	5.63	1.55	1.47	0.79	6.35	1.09	1.85
		4	3.08	4.58	1.22	7.26	1.53	1.90	0.78	8.53	1.13	2.42
		5	3.79	5.53	1.20	8.75	1.54	2.30	0.79	10.73	1.17	2.97
4.5	45	3	2.65	5.13	1.39	8.13	1.75	2.12	0.89	9.04	1.21	2.08
		4	3.48	6.63	1.38	10.50	1.74	2.74	0.89	12.10	1.26	2.73
		5	4.29	8.03	1.37	12.70	1.72	3.33	0.88	15.30	1.30	3.37
5	50	3	2.96	7.11	1.55	11.30	1.95	2.95	1.00	12.40	1.33	2.32

Profile number	b	d	Section area cm^2	Additional values for the axes								Weight 1m
				x-x		x_0-x_0		y_0-y_0		x_1-x_1	z_0, cm	
	mm			J_x cm^4	i_x cm	J_{x0max} cm^4	i_{x0max} cm	J_{y0min} cm^4	$i_{y0\,min}$ cm	J_{x1} cm^4		
		4	3.89	9.21	1.54	14.60	1.94	3.80	0.99	16.60	1.38	3.05
		5	4.80	11.20	1.53	17.80	1.92	4.63	0.98	20.90	1.42	3.77
		6	5.69	13.07	1.52	20.72	1.91	5.43	0.98	–	1.46	4.47
5.6	56	4	4.38	13.10	1.73	20.80	2.18	5.41	1.11	23.30	1.52	3.44
		5	5.41	16.00	1.72	25.40	2.16	6.59	1.10	29.20	1.57	4.25
6.3	63	4	4.96	18.90	1.95	29.90	2.45	7.81	1.25	33.10	1.69	3.90
		5	6.13	23.10	1.94	36.60	2.44	9.52	1.25	41.50	1.74	4.81
		6	7.28	27.10	1.93	42.90	2.43	11.20	1.24	50.00	1.78	5.72
7	70	4.5	6.20	29.0	2.16	46.0	2.72	12.0	1.39	51.0	1.88	4.87
		5	6.86	31.9	2.16	50.7	2.72	13.2	1.39	56.7	1.90	5.38
		6	8.15	37.6	2.15	59.6	2.71	15.5	1.38	68.4	1.94	6.39
		7	9.42	43.0	2.14	68.2	2.69	17.8	1.37	80.1	1.99	7.39
		8	10.70	48.2	2.13	76.4	2.68	20.0	1.37	91.9	2.02	8.37
7.5	75	5	7.39	39.5	2.31	62.6	2.91	16.4	1.49	69.6	2.02	5.80
		6	8.78	46.6	2.30	73.9	2.90	19.3	1.48	83.9	2.06	6.89
		7	10.10	53.3	2.29	84.6	2.89	22.1	1.48	98.3	2.10	7.96
		8	11.50	59.8	2.28	94.9	2.87	24.8	1.47	113.0	2.15	9.02
		9	12.80	66.1	2.27	105.0	2.86	27.5	1.46	127.0	2.18	10.10
8	80	5	8.63	52.7	2.47	83.6	3.11	21.8	1.59	93.2	2.17	6.78
		6	9.38	57.0	2.47	94.0	3.11	23.5	1.58	102.0	2.19	7.36
		7	10.80	65.3	2.45	104.0	3.09	27.0	1.58	119.0	2.23	8.51
		8	12.30	73.4	2.44	116.0	3.08	30.3	1.57	137.0	2.27	9.65
9	90	6	10.60	82.1	2.78	130.0	3.50	34.0	1.79	145.0	2.43	8.33
		7	12.30	94.3	2.77	150.0	3.49	38.9	1.78	169.0	2.47	9.64
		8	13.90	106.0	2.76	168.0	3.48	43.8	1.77	194.0	2.51	10.90
		9	15.60	118.0	2.75	186.0	3.46	48.6	1.77	219.0	2.55	12.20
10	100	5	12.80	122.0	3.09	193.0	3.88	50.7	1.99	214.0	2.68	10.10
		7	13.80	131.0	3.08	207.0	3.88	54.2	1.98	231.0	2.71	10.80
		8	15.60	147.0	3.07	233.0	3.87	60.9	1.98	265.0	2.75	12.20
		10	19.20	179.0	3.05	284.0	3.84	74.1	1.96	333.0	2.83	15.10
		12	22.80	209.0	3.03	331.0	3.81	86.9	1.95	402.0	2.91	17.90
		14	26.30	237.0	3.00	375.0	3.78	99.3	1.94	472.0	2.99	20.60
		16	29.70	264.0	2.98	416.0	3.74	112.0	1.94	542.0	3.06	23.30
11	110	7	15.20	176.0	3.40	279.0	4.29	72.7	2.19	308.0	2.96	11.90
		8	17.20	198.0	3.39	315.0	4.28	81.8	2.18	353.0	3.00	13.50
12.5	125	8	19.7	294	3.87	469	4.87	122	2.49	516	3.36	15.5
		9	22.0	327	3.86	520	4.86	135	2.48	582	3.40	17.3
		10	24.3	360	3.85	571	4.84	149	2.47	649	3.45	19.1
		12	28.9	422	3.82	670	4.82	174	2.46	782	3.53	22.7
		14	33.4	482	3.80	764	4.78	200	2.45	916	3.61	26.2
		16	37.8	539	3.78	853	4.75	224	2.44	1051	3.68	29.6
14	140	9	24.7	466	4.34	739	5.47	192	2.79	818	3.78	19.4
		10	27.3	512	4.33	814	5.46	211	2.78	911	3.82	21.5
		12	32.5	602	4.31	957	5.43	248	2.76	1097	3.90	25.5
16	160	10	31.3	774	4.96	1229	6.25	319	3.19	1356	4.30	24.7
		11	34.4	844	4.95	1341	6.24	348	3.18	1494	4.35	27.0
		12	37.4	93	4.94	1450	6.23	376	3.17	1633	4.39	29.4
		14	43.3	1046	4.92	1662	6.20	431	3.16	1911	4.47	34.0
		16	49.1	1175	4.89	1866	6.17	485	3.14	2191	4.55	38.5
		18	54.8	1299	4.87	2061	6.13	537	3.13	2472	4.63	43.0
		20	60.4	1419	4.85	2248	6.10	589	3.12	2756	4.70	47.4
18	180	11	38.8	1216	5.60	1933	7.06	500	3.59	2128	4.85	30.5
		12	42.2	1317	5.59	2093	7.04	540	3.58	2324	4.89	33.1
20	200	12	47.1	1823	6.22	2896	7.84	749	3.99	3182	5.37	37.0
		13	50.9	1961	6.21	3116	7.83	805	3.98	3452	5.42	39.9
		14	54.6	2097	6.20	3333	7.81	961	3.97	3722	5.46	42.8

Profile number	b	d	Section area cm²	Additional values for the axes x-x: J_x cm⁴	x-x: i_x cm	x_0-x_0: J_{x0max} cm⁴	x_0-x_0: i_{x0max} cm	y_0-y_0: J_{y0min} cm⁴	y_0-y_0: $i_{y0\,min}$ cm	x_1-x_1: J_{x1} cm⁴	z_0, cm	Weight 1m
	mm											
		16	62.0	2363	6.17	3755	7.78	970	3.96	4264	5.54	48.7
		20	76.5	2871	6.12	4560	7.72	1182	3.93	5355	5.70	60.1
		25	94.3	3466	6.06	5494	7.63	1438	3.91	6733	5.89	74.0
		30	111.5	4020	6.00	6351	7.55	1688	3.89	8130	6.07	87.6
22	220	14	60.4	2814	6.83	4470	8.60	1159	4.38	4941	5.93	47.4
		16	68.6	3175	6.81	5045	8.58	1306	4.36	5661	6.02	53.8
25	250	16	78.4	4717	7.76	7492	9.78	1942	4.98	8286	6.75	61.5
		18	87.7	5247	7.73	8337	9.75	2158	4.96	9342	6.83	68.9
		20	97.0	5765	7.71	9160	9.72	2370	4.94	10401	6.91	76.1
		22	106.1	6270	7.69	9961	9.69	2579	4.93	11464	7.00	83.3
		25	119.7	7006	7.65	11125	9.64	2887	4.91	13064	7.11	94.0
		28	133.1	7717	7.61	12244	9.59	3190	4.89	14674	7.23	104.5
		30	142.0	8177	7.59	12965	9.56	3389	4.89	15753	7.31	111.4



Symbols designation
B - width of the larger arm
b - width of the smaller arm
d - wall thickness
J - moment of inertia
i - radius of inertia
x_0, y_0 - distance from the centre of gravity to the outer limits of the shelves

Fig. D.17. Hot-rolled unequal-area steel angle bar

Table D.61. Basic parameters of hot-rolled unequal steel angles (ISO/R 657-2:1968)

Profile no.	Dimensions mm: B	b	d	Cross-section cm²	J_x cm⁴	i_x cm	J_y cm⁴	i_y cm	J_{umin} cm⁴	$i_{u\,min}$ cm⁴	Tilt angle u tgα	J_{x1} cm⁴	J_{y1} cm⁴	x_0 cm	y_0 cm	Weight 1m kg
2.5/1.6	25	16	3	1.16	0.7	0.78	0.22	0.44	0.13	0.3	0.392	–	–	0.42	0.86	0.91
3.2/2	32	20	3	1.49	1.52	1.01	0.46	0.55	0.28	0.43	0.382	–	–	0.49	1.08	1.17
			4	1.94	1.93	1.00	0.57	0.54	0.35	0.43	0.374			0.53	1.12	1.52
4/2.5	40	25	3	1.89	3.06	1.27	0.93	0.7	0.56	0.54	0.385	–	–	0.59	1.32	1.48
			4	2.47	3.93	1.26	1.18	0.69	0.71	0.54	0.381			0.63	1.37	1.94
			5	3.03	4.73	1.25	1.41	0.68	0.86	0.53	0.374			0.66	1.41	2.37
5/3.2	50	32	3	2.42	6.18	1.6	1.99	0.91	1.18	0.7	0.403	–	–	0.72	1.6	1.9
5.6/3.6	56	36	4	3.58	11.4	1.78	3.7	1.02	2.19	0.78	0.406	23.2	6.25	0.84	1.82	2.81
			5	4.41	13.8	1.77	4.48	1.01	2.66	0.78	0.404	29.2	7.91	0.88	1.86	3.46
6.3/4	63	40	4	4.04	16.3	2.01	5.16	1.13	3.07	0.87	0.397	33.0	8.51	0.91	2.03	3.17
			5	4.98	19.9	2.00	6.26	1.12	3.72	0.86	0.396	41.4	10.8	0.95	2.08	3.91
			6	5.90	23.3	1.99	7.28	1.11	4.36	0.86	0.393	49.9	13.1	0.99	2.12	4.63
			8	7.68	29.6	1.96	9.15	1.09	5.58	0.85	0.386	66.9	17.9	1.07	2.20	6.03
7/4.5	70	45	5	5.59	27.8	2.23	9.05	1.27	5.34	0.98	0.406	56.7	15.2	1.05	2.28	4.39
7.5/5	75	50	6	7.25	40.9	2.38	14.6	1.42	8.48	1.08	0.435	83.9	25.2	1.21	2.44	5.69
			8	9.47	52.4	2.35	18.5	1.40	10.9	1.07	0.430	112	34.2	1.29	2.52	7.43
8/5	80	50	5	6.36	41.6	2.56	12.7	1.41	7.58	1.09	0.387	84.6	20.8	1.13	2.60	4.99
			6	7.55	49.0	2.55	14.8	1.40	8.88	1.08	0.386	102	25.2	1.17	2.65	5.92
9/5.6	90	56	5.5	7.86	65.3	2.88	19.7	1.58	11.8	1.22	0.384	132	32.2	1.26	2.92	6.17
			6	8.54	70.6	2.88	21.2	1.58	12.7	1.22	0.384	145	35.2	1.28	2.95	6.70
			8	11.18	90.9	2.85	27.1	1.56	16.3	1.21	0.380	194	48.7	1.36	3.04	8.77
10/6.3	100	63	6	9.59	98.3	3.20	30.6	1.79	18.2	1.38	0.393	198	49.9	1.42	3.23	7.53
			7	11.1	113	3.19	35.0	1.78	20.8	1.37	0.392	232	58.7	1.46	3.28	8.70
			8	12.6	127	3.18	39.2	1.77	23.4	1.36	0.391	266	67.6	1.50	3.32	9.87
			10	15.5	154	3.15	47.1	1.75	28.3	1.35	0.387	333	87.5	1.58	3.40	12.1
11/7	110	70	6.5	11.4	142	3.53	45.6	2.00	26.9	1.53	0.402	286	74.3	1.58	3.55	8.98

			8	13.9	172	3.51	54.6	1.98	32.3	1.52	0.400	353	92.3	1.64	3.61	10.9
12.5/8	125	80	7	14.1	227	4.01	73.7	2.29	43.4	1.76	0.407	452	119	1.80	4.01	11.0
			8	16.0	256	4.00	83	2.28	48.8	1.75	0.406	518	137	1.84	4.05	12.5
			10	19.7	312	3.98	100	2.26	59.3	1.74	0.404	649	173	1.92	4.14	15.5
			12	23.4	365	3.95	117	2.24	69.5	1.72	0.400	781	210	2.00	4.22	18.3
14/9	140	90	8	18.0	364	4.49	120	2.58	70.3	1.98	0.411	727	104	2.03	4.49	14.4
			10	22.2	444	4.47	146	2.56	85.5	1.96	0.409	911	245	2.12	4.58	17.5
16/10	160	100	9	22.9	606	5.15	186	2.85	110	2.20	0.391	1221	300	2.23	5.19	18.0
			10	25.3	667	5.13	204	2.84	121	2.19	0.390	1359	335	2.28	5.23	19.8
			12	30.0	784	5.11	239	2.82	142	2.18	0.388	1634	405	2.36	5.32	23.6
			14	34.7	897	5.08	272	2.80	162	2.16	0.385	1910	477	2.43	5.40	27.3
18/11	180	100	10	28.3	952	5.80	276	3.12	165	2.42	0.375	1933	444	2.44	5.88	22.2
			12	33.7	1123	5.77	324	3.10	194	2.40	0.374	2324	537	2.52	5.97	26.4
20/12.5	200	125	11	34.9	1449	6.45	446	3.58	264	2.75	0.392	2920	718	2.79	6.50	27.4
			12	37.9	1568	6.43	482	3.57	285	2.74	0.392	3189	786	2.83	6.54	29.7
			14	43.9	1801	6.41	551	3.54	327	2.73	0.90	3726	922	2.91	6.62	34.4
			16	49.8	2026	6.38	617	3.52	367	2.72	0.388	4264	1061	2.99	6.71	39.1

Table D.62. Hot-rolled round steel sections (EN 10060:2003)

Diameter d, mm	Cross-sectional area, cm^2	Weight of 1 m length of wire rod, kg	Diameter d, mm	Cross-sectional area, cm^2	Weight of 1 m length of wire rod, kg
5.0	0.196	0.154	29.0	6.605	5.185
5.5	0.238	0.187	30.0	7.069	5.549
6.0	0.283	0.222	31.0	7.548	5.925
6.3	0.312	0.245	32.0	8.043	6.313
6.5	0.332	0.261	33.0	8.553	6.714
7.0	0.385	0.302	34.0	9.079	7.127
8.0	0.503	0.395	35.0	9.621	7.553
9.0	0.636	0.499	36.0	10.179	7.990
10.0	0.785	0.617	37.0	10.752	8.440
11.0	0.950	0.746	38.0	11.341	8.903
12.0	1.131	0.888	39.0	11.946	9.378
13.0	1.327	1.042	40.0	12.566	9.865
14.0	1.539	1.208	41.0	13.203	10.364
15.0	1.767	1.387	42.0	13.854	10.876
16.0	2.011	1.578	43.0	14.522	11.400
17.0	2.270	1.782	44.0	15.205	11.936
18.0	2.545	1.998	45.0	15.904	12.485
19.0	2.835	2.226	46.0	16.619	13.046
20.0	3.142	2.466	47.0	17.349	13.619
21.0	3.464	2.719	48.0	18.096	14.205
22.0	3.801	2.984	50.0	19.635	15.413
23.0	4.155	3.262	52.0	21.237	16.671

Diameter d, mm	Cross-sectional area, cm^2	Weight of 1 m length of wire rod, kg	Diameter d, mm	Cross-sectional area, cm^2	Weight of 1 m length of wire rod, kg
24.0	4.524	3.551	53.0	22.062	17.319
25.0	4.909	3.853	54.0	22.902	17.978
26.0	5.309	4.168	55.0	23.758	18.650
27.0	5.726	4.495	56.0	24.630	19.335
28.0	6.158	4.834	58.0	26.421	20.740
60.0	28.274	22.195	140.0	153.938	120.841
62.0	30.191	23.700	145.0	165.130	129.627
63.0	31.173	24.470	150.0	176.715	138.721
65.0	33.183	26.049	155.0	188.692	148.123
67.0	35.257	27.676	160.0	201.062	157.834
68.0	36.317	28.509	165.0	213.825	167.852
70.0	38.485	30.210	170.0	226.980	178.179
72.0	40.715	31.961	175.0	240.528	188.815
73.0	41.854	32.855	180.0	254.469	199.758
75.0	44.179	34.680	185.0	268.803	211.010
78.0	47.784	37.510	190.0	283.529	222.570
80.0	50.266	39.458	195.0	298.648	234.438
82.0	52.810	41.456	200.0	314.159	246.615
85.0	56.745	44.545	210.0	346.361	271.893
87.0	59.447	46.666	220.0	380.133	298.404
90.0	63.617	49.940	230.0	415.476	326.148
92.0	66.476	52.184	240.0	452.389	355.126
95.0	70.882	55.643	250.0	490.874	385.336
97.0	73.898	58.010	260.0	530.929	416.779
100.0	78.540	61.654	270.0	572.555	449.456
105.0	86.590	67.973	–	–	–
110.0	95.033	74.601	–	–	–
115.0	103.869	81.537	–	–	–
120.0	113.097	88.781	–	–	–
125.0	122.719	96.334	–	–	–
130.0	132.732	104.195	–	–	–
135.0	143.139	112.364	–	–	–

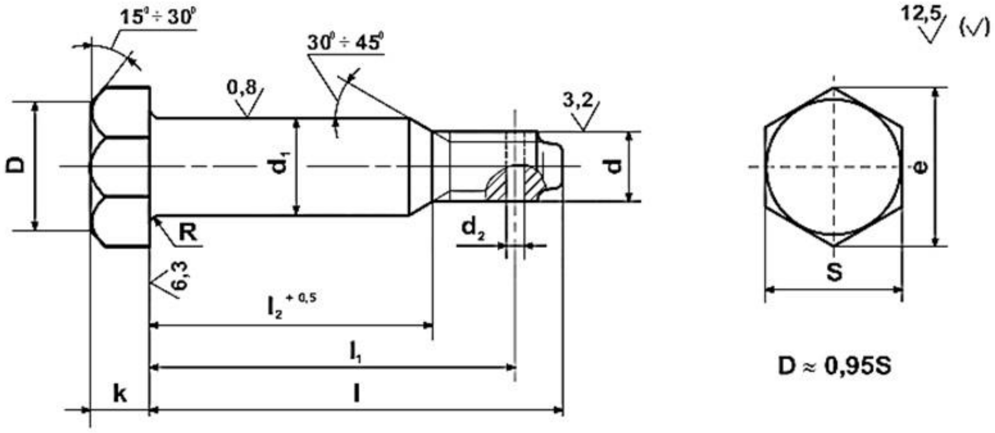


Fig. D.18. Hexagon head screw

Table D.63. Hexagon head bolts, accuracy class A, for reaming holes (ISO 898-1)

Nominal thread diameter *d*		6	8	10	12	(14)	16	(18)	20	(22)	24	(27)	30	36	42	48
Thread pitch	ordinary	1	1.25	1.5	1.75	2		2.5			3		3.5	4	4.5	5
	finely wound	–	1	1.25		1.5					2			3		
Bar diameter d_1		7	9	11	13	15	17	19	21	23	25	28	32	38	44	50
Keyway dimension *S*		10	12	14	17	19	22	24	27	30	32	36	41	50	60	70

Comment. Screw dimensions in brackets are not recommended for use.

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