Jan Długosz University of Częstochowa
Scientific Issues, Mathematics XII, Częstochowa 2007

GEOMETRICAL ACTIVITIES AS A TOOL FOR STIMULATING MATHEMATICAL THINKING OF 4-7 YEARS OLD CHILDREN

Ewa Swoboda

Institute of Mathematics, Rzeszow University
Rejtana Street 16A, 35-959 Rzeszow, Poland
e-mail: eswoboda@univ.rzeszow.pl

Rationale

In the constructivist approach to teaching mathematics, great emphasis is put on the way how children use the language. Talking during lesson is perceived from two different perspectives: as the tool for communication (social function) and - as the tool for shaping and determining the thinking process. Talk is not a result of a fully developed thought - although is created through a course of word statement.

When leading my own research on geometrical intuition of 4 - 7 years old children I put a hypothesis that geometrical activities can be used as the tool for stimulating mathematical thinking in that group of children. Theoretical analyses and survey of literature supporting this hypothesis are as following:

[1] Following quotation from the main Vygotskian book Thinking and Speaking (1989, p. 333) summarizing one of important aspects of his theory about relation between thoughts and words: The child's thought, precisely because it is born as a dim, amorphous whole, must find expression in a single word. The child chooses a word as an eligible dress for his thought. As his thought becomes more differentiated, the child is less apt to express it in single words but constructs a composite whole. Conversely, progress in speech to the differentiated whole of a sentence helps the child's thoughts to progress from a homogeneous whole to well-defined parts.

[2] Teaching the mathematical thinking means - among others - teaching how to analyse some phenomena, how to perceive relations and properties. Those relations should construct the whole in the domain of

analysed situation, and if go beyond of its scope, they should build a structure, the net of general properties. In general, mathematical thinking consists of finding rules, perceiving a general in the particular.

[3] Theories about teaching geometry underlines the idea, that at the first level of geometrical knowledge a child perceives geometrical object as a whole. That idea is a base for van Hiele theory, in a similar way M. Hejny describes the "pre-concept level". So, the first geometrical thoughts are global, amorphous whole. On the next levels, attendants and relations start to exist as independent phenomena: the amorphous whole becomes more differentiated, more and more separated elements constitute whole.

[4] Small children' learning mainly consists of acquiring experiences by own activities. The special importances have for them all information, gathered by perception. This is a main reason why geometrical concepts, built by perception, are closer to child' abilities than arithmetical ones.

Methodology

The aim of my research was as following: to study children's language while they are solving geometrical tasks packed with relations and properties.

The research was lead in 2004 - 2007, among pre-school children at different ages. Teachers were told not to interrupt children's work. From our previous experience we know that very often a teacher wants to give children some "hints", but in fact - his/her remark only disturbs children's way of thinking.

Example 1.
Ania, (6 years old) draws the geometrical pattern. Before the work she prepares all felt-tip pens she needs and while drawing she chooses colors very carefully.
A teacher starts to talk with the girl during the work, bur she only answers on questions without any symptoms of willingness to say something.
Girl starts to work without any word. She is very concentrated on drawing.

[1] Teacher : what is this, this figure?
[2] Ania: (silence)
[3] Teacher: so, how many sides does it have? (she shows sides by her finger)
[4] Ania: (counts loudly): 1,2,3, - three - angles?

Examples of children' work

Analyzing children' work we can put hypothesis that their competence grows with age, both in drawing and in argumentation. I will illustrate this statement by examples.

4 years old children

Example 2. (Krystian, Michal)
T: I have prepared a pattern for you. Do you know, how to continue the drawing?
Krystian: A little person.
Michal: House.
T: How do you know it?
K: Because I know everything.

In this situation children take a space on a paper as a place for their own free creativity. They did not treat this task as a continuity pattern with regularities and relations between figures. They ignored information given by a teacher that they have "to continue"what is already began. Those children are not ready to perceive (and look for) regularities yet.
But the next dialog shows that the patterns can be used as a good tool for discovering regularities.

Example 3. (Krystian, Pawel, Grzes, 4 years old)
T: I prepared this pattern for you. Look carefully on it - do you know how to continue this drawing?
Krystian: In my book for painting I had an airplane.
Pawel: I know!! I can do it too!! I can do it too!
Grzes: How do you know it?
P: Because here it is shown (he points his finger at dotted figures).

The first boy - Krystian - is still thinking about "free drawing". But the other two boys are already in the world of regularities. For Pawel, the doors to this world have just opened: he discovered the relations, he understood the task, but he was not able to play in this game. The only what he could do was to repeat the motif (he coated the dotted line in the second motif). But his reaction (a big enthusiasm: I know!, I can do it too!!) shows that he was delighted by understanding the task. The member of the same group - Grzes, created the pattern where - in spite of many manual obstacles - a lot of relations between figures are retained.

5 years old children

In this group of children, the ability of talking about perceiving relation is on the higher level than in the group of 4 year old children. This is the group that more often utter some statements about patterns and the way how to continue it. They support their own utterances by gestures - this means that they used gestures as symbols for coding some relations between figures.

Example 4. Ola, 5 years old, Natalka 4 years old
T: I have prepared you a pattern. Do you know how to continue this drawing?
Ola: Big - small - big - small.

T: And how will you draw it?
O: [draws in the air by her finger the way how she will draw it]
T: And what does Natalka think about Ola's statement? Do you agree with Ola?
Natalka: [she nods that she agrees and she shows by the finger how she will draw]
T: So, how will you continue the drawing?
Na: Big - small - big - small

We can observe very interesting phenomena during this dialog. Natalka, younger than Ola, accepts arguments, vocabulary and gestures given by Ola and repeats them as her own. But in her work she is independent. She tries to depict the general structure of the pattern. She is critical towards her own work - she makes some corrections during drawing. Repeated triangles put in an "upside down"position show that this way of putting them was especially difficult for her. This phenomenon can be treated as an argument that although geometrical properties are within her reach, the verbal utterances about them are difficult.
For children, building their own, independent argumentation is difficult at this stage. It is shown in the net dialog

Examples 5. (Kuba, Wojtek, 5 yeasr old)
T: Do you know, how to continue?
Kuba: yes [Wojtek - silence]
T: So, how?
K: Triangles.
T: Could you tell me, how will you draw those triangles, Kuba?
K: Hey! I have an idea!! Triangle - triangle - circle - circle, can I do it like this?
T: And what do you think, Wojtek, about Kuba's idea?
Wojtek [disorientated] Do I have to continue by drawing circles?
K: No, the next is triangle and circle.
T: Do you agree with Kuba's opinion?
W: I agree.
T: But look carefully and say what do you think. What will you draw next?
W: Squares.

Kuba is a very creative boy. He discovered, that in this task some regularities exist, but he did not want to continue patter. He decided to make his own pattern and suggests to his colleague to work accordingly to his idea. This situation throws Wojtek off balance - he did not see any connection between this what he sees and this what Kuba says.
But this is not true that at this age children are not able to think independently and critically. These abilities are presented by the boy in the next dialogue.

Example 6. (Patryk, Bartek 5 year old)
T:I prepared a pattern for you. Do you know how to continue this pattern?
Bartek: I will draw like this [he creates a shape of triangle by the finger in the air]
T: And next?
B: [he "drawsą small triangle in the opposite position]
T: And next?
B: [he shows a big one]
T: And next?
B: [he shows the small one, in the opposite position]
T: And what is Patryk's idea?
Patryk: I will draw one like this - one like that- one like this - one like that [he shows by fingers elements from the motif]
T: And how do you know Patryk how to draw?
P: Because it is shown.
... .
B: [he looks at the Patryk's work] You have to make the same angles, not so big ones [he shows a big triangle, because Partyk draws big triangles first]
B: I will join those triangles [he draws big triangles first, after that he puts the small ones]

6 years old children

Utterances in this group of children are more accurate. They express more details and relationships. The argumentation is also more exact and independent. This is an example:

Example 7. (Iza i Ola - 6 years old)
T: Do you know how to continue the drawing?
Iza: yes
Ola: yes
T: Could you say Iza, what will you draw and how?
I: I will draw it like this [she shows the pattern prepared by the teacher]
N: And Olga?
O: Here one square and upwards, and here the next square and downwards ...
I: Following the pattern ...
O: Because here are these dots so follow the pattern.....

Summary

The process of creation of the argumentation by 4-7 years old children in the geometrical environment consists of a few levels:

- Perceiving regularities

- Realizing regularities
- Verbalizing regularities.

Words are representations of concepts and ideas. Using words requires an intellectual effort as well as while using symbols. Geometrical symbol is "in the middle"between an abstractive symbol and a real thing. Geometrical figures, at the early stage of its understanding, can be treated as a not-finished picture of a real object. A square, even when drawn very correctly, can be interpreted by child as a ńot-finished window"(Hejny, 1993) but as a symbol it can be used very correctly in communication. Parallelity, as a relation between two objects, is perceived both by a child and by a mathematician, although a mathematician would connect the eligible sign with the abstract idea and a child would say: "two sticks ale lying equally". Gruszczyk-Kolczyńska writes: "the process of coding and en-coding in teaching mathematics starts from a very high level of abstraction and requires skills in operational reasoning on the real level (p. 19). In my opinion, this statement relates to arithmetical formulas. Geometry as rhythms and patterns gives a chance to code and encode rules and formulas at a very low level. Of course, it is not an easy way. The passage from perception, through geometrical symbolic representation to verbal mathematical description is very long, and often strange to children. Sometimes, the moment a child sees the pattern, it comments on it in a spontaneous way. The child says: it is easy, I can do it without any problem. After correct drawing, during a talk with a teacher he or she reacts: why do you ask me: how do you know how to draw it? - it is shown, it is obvious. But their effort they put in forming the utterances that describe the rule of drawing, has a very import didactical meaning. Children start to transform these relations and connections into words. Previously, they realized them without words, often in non-conscious way. By the talk, these relations gain a status of existence. They emerge gradually from experience and start to be the facts related to the mathematical world.

References

[1] P. van Hiele. Structure and Insight. A Theory of Mathematics Education, Academic Press Inc. London, 1986.

[2] M. Hejný. The Understanding of Geometrical Concepts, Proceedings of the 3rd Bratislava International Symposium on Mathematical Education, BISME3. Comenius University, Bratislavia, 1993.

[3] L.S. Wygotski. Myślenie i Mowa, Biblioteka Klasyków Psychologii, PWN, Warszawa, 1989.