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PROBLEMS TEACHER'S PRACTICE FORMING MATHEMATICAL ACTIVITY AND CREATIVITY OF THE GIFTED PUPILS

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1. Introduction

The words of P. J. Taylor (2003), who states that *mathematics and mathematics education, and teaching to a high standard, are the keys to solving the world's existing problems and planning for the future* have inspired this work. The subject of the work, **Problems teacher's practice forming mathematical activity and creativity of the gifted pupils**, is submerged in the issue of one of the contemporary trends in researching the methodology of teaching maths, which is called: **activity and creativity in teaching mathematics - theory, diagnosis and methodology, prospects**. This work is the extension and some kind of modification of the previous work of mine, see: A. Pardała (2006). In this work I refer to certain synthesis of the knowledge to that point, see: A. Pardała (2003, 2004, 2006), and I also articulate one of its' aspects - the crucial importance of teacher's intervention on activity and creativity of a student, who is solving a mathematical problem. And then I synthetically present findings of research, assessing teacher's impact on stimulating a gifted student, in particularly which was done for doctoral thesis by E. Śmietana (2005). In summery of my work, I put forward some remarks and final reflections related to mathematical activity and creativity in gifted students' education.

2. The aim and tasks of the work, understanding its basic concepts

The aim of the work is: 1) outline the current state of research and experience in the scope of stimulating a pupil's activity and creativity who is just solving a mathematics problem, 2) point out the extent and directions of how to use the teacher's intervention in the process of solving a problem by a gifted pupil.

The task of the work is to try to find answers to the following questions: 1) How advanced are current experiments and research into mathematical education of gifted students? 2) What are the didactic recommendations to improve the practice of forming a pupil's mathematical creativity.

In my work I make references to my synthesis of the knowledge of the present state of research into creativity in teaching mathematics and school and teacher experience in this respect. Here, I refer to the results found in the literature and hitherto completed Polish research works. I suggest the following list of the concepts found in this work: mathematical creativity, mathematics teacher's intervention, see: A. Pardała (1984, 2004, 2006), and its types, see: E. Śmietana (2005). **The very term "intervention"** explains what kind of situation is meant here. It is as follows: a pupil has just got a problem and is solving it possibly all by himself (cooperation with the whole class or a group of pupils is also possible). However, at same moments, the teacher joins in by asking questions, making suggestions, comments, gestures, reflections, etc. Each activity of that kind is called the teacher's intervention in the problem solving process.

3. Formation of creativity in mathematics teaching and the prospects of mathematics education

The document, *50 Years of CIEAEM: Where we are and where we go, Manifesto 2000 for the Year of Mathematics*, contains, among other things, some diagnoses and views on the evolution of notions, experience and intentions to improve mathematics teaching. Part II of the document, Where we go?, focuses on some problems, ideas and directions of the future work of those concerned with: 1) development of mathematics for all and its popularization, 2) revitalization of the consciousness of democratic society and winning its support for a preferential treatment of mathematics education, 3) modernization of the existing base for mathematics education taking into account its position in the modern mathematized world. And the core of the diagnosis of *Where we go?* is the following:

1) A re-evaluation of the aims of general education has taken place; from a universal education for the elite to an education for all. This immediately implies a change in the aims of mathematics education and its mission which must: a) ensure an understanding of "mathematization processes"in society, b) create a clear and critical assessment of the role mathematics plays currently and its application in social environment. Further on, the authors of the document ask questions defining this scope of research, some of which are:

- How can both mathematics teaching and learning be represented not only as an introduction into some great ideas of our culture but also into the criticism of their content and application?
- What kind of research on mathematics didactics could contribute to the creation of a new outlook on mathematics teaching practice?
- How to make society aware of the fact that mathematics teaching develops responsibility and gives free rains to a democratic vision of introducing new forms of social contact, communication and dialogue?

2) Views on mathematics education are of dichotomic, bipolar nature. On one hand, mathematics is still one of those school subjects which causes strong anxiety, aversion and feeling of incompetence and which is difficult and senseless for most pupils. Hence they consider themselves "mentally handicapped"in this field and doomed to failure. O the other hand, for some parents, pupils and politicians mathematics requires particular aptitude and is considered to be only for the chosen.

Hence "mathematical abilities", ą talent for mathematics", a natural gift for mathematical thinkingór ą natural interest in mathematicsąre seen as something very rare among pupils or in society. And this makes mathematics a natural factor of social selection, which only aggravates a feeling of anxiety and aversion to it. Further on, in the above mentioned document , there are quite definite views on the didactic aspect of the problem: **mathematics as a means of social selection**. In my opinion, one can find a strong thesis formulated here: as long as the social focus is on "the talented", the majority will not be properly educated. In particular, the authors pose some crucial investigative questions, some of which are:

- *Should we keep the highly selective framework and methods of education, but give up the privileged position of the subject as part of the core of general education? Or do we seek to keep mathematics at the core of the curriculum but find ways of teaching the subject to all students? How to overcome the limitations of this dichotomy?*
- *The notions "mathematical ability", "individual differencesąnd the "gifted pupiląre ideologically collective constructions based on convictions or prejudices, as a possible vehicle of purpose and interest. Moreover, the prejudice of "mathematical giftednessżeadily associates itself with other hereditary features such as gender and ethnicity - how can we act against that?*

The above outlined ideas and trends of updating mathematics teaching and stimulating mathematical creativity correspond to the experience gained by a French association, Math Pour Tous, described in the work by L. Beddou and C. Mauduit (2001). In this concept the teaching of mathematics is based on investigative activity and dedication of as many pupils and students as possible. Here, one tries to follow or imitate some behaviours, patterns and principles characteristic for scientific inquiry, such as: discovering by a sking questions; learning by research; stimulating creativity and imagination; appreciating the importance of error in learning; learning how to listen to, get across and exchange ideas, etc.. An academic teacher is a supervisor, puts forward a number of tasks and problems whose solution would not make use of the already ącquired knowledge". The teacher running such a workshop is obliged to get the work of particular pupils or groups of pupils (twin groups) to progress. The results must be presented on-line or at a conference. The pioneers of the concept are G. Polya, I. Lakatos, I. David, E. Marchisotto, Z. Krygowska and others. The unique nature of the concept and the educative action "Math en Jeans"(Math Pour Tous) consists in its being addressed to all the pupils and students concerned and not only to their elite - i.e. would-be research workers or professional mathematicians. Besides, the teacher and the pupil (pupils) begin from the same level. The pupil, getting an open problem, has an impression of doing new things and demonstrates his emotional attitude by saying: "I have solved", "I have found". Here is an example of the problem - **Conway's Sofa**, solved during this educational activity:

Let us consider a corridor consisting of two parts, either 1 metre long, at an angle of 90° to each other. Along the corridor we want to carry a sofa represented by an undeformable flat figure S. An example of the figure may be a square with each side 1 m long. What is the largest possible area of a sofa with area exceeding 1 m , which can be carried along the corridor? What can be said about the problem when the corridor consists of several parts? What happens when the angles at which *the corridor turns are not right angles?*; see: L. Beddou and C. Mauduit (2001), p. 24. At the beginning young people find the problem rather difficult: how to mathematically describe the situation and the movement of the figure?; how to construct examples of the figures or sets of figures meeting the conditions set out?; is there any relation between the shape and the surface plane of the figures and vice versa? Concretization or extension of the problem seems only too natural here. A successful attempt at solving it takes some mathematical skills and activities as well as further studies. One can notice the participants change their attitude to the problem and its solution as: 1) it is necessary to substantiate such things as: "it is obvious that ... ", or "it can be seen that ... "; 2) one has to be open to the reasoning of the others; they are or may be right; 3) one must be aware of the extent of the obtained solution (partial solution, solution for a set of figures and description of their properties, knowledge of only some theorems useful for finding the solution, etc.). The below mentioned research papers on mathematics didactics are also related to the problem of how to form pupils' mathematical creativity. R.A. Utiejewa (2001) reminds us that "mental energy"forms the basis of man's abilities although the concept of ability is not unequivocally described even today. The authoress calls up L. Terman's (1959) research work which confirmed the idea that the main distinguishing mark of ability is intellect characterized by its seven components including a logical-mathematical one. Besides, contemporary psychologists are unanimous that one should distinguish creative mathematical abilities from exceptional mathematical abilities (mathematical genius) which occur at most in 1% of the pupils. R.A. Utiejewa tested 2000 Russian pupils aged 10 - 15 and found out only 8% of this population has mathematical talent. And her model of differentiated mathematics teaching in the case of creativity endowed pupils assumes spotting them early (in the fifth form at the latest) in order to offer them the right conditions ensuring the optimum development of their mathematical potential in their school environment (class, school). The main purpose of such concept is to individually form scientific and mathematical activities of creative pupils taking into account: motivated activity of both the pupil and the teacher, teaching through setting problems and individual attention to each talented pupil. The basic organizational features are: "group workór "individually differentiated"work. **The authoress of the concept also gives four indispensable conditions of its successful realization: 1)** special mathematics programmes focused on the interests and creative aptitudes of the pupils which should be constantly updated considering individual capabilities (also work pace) of the pupils; **2)** important methodical changes in the teacher's work and teaching and a change in his relation with mathematically talented pupils both during the lesson and while organizing their individual study; **3)** special programmes to prepare a would-be mathematics teacher during his university education for how to professionally work with talented pupils; **4)** providing the teachers working with gifted pupils with all the necessary educational aids (literature, guide books, course books, collections of problems, periodicals, etc.). In the summary of the article she emphasizes an important aspect of creativity pedagogy: working out programmes, together with their relevant methodology, for mathematically talented pupils.

The work of E. Jagoda, D. Panek and A. Pardała (2001) corresponds to the above. The authors point to the growing impact of the media technology and the

Internet on the state of education in the world and the forms of pupils' development and education. For this reason tele-education can be a successful means of work with a talented pupil. And further, the paper also signals some shortcomings affecting creativity pedagogy and gifted pupils because the didactics for gifted pupils has not been consolidated and the knowledge of school and teaching practice, i.e. practically functioning programmes, their realization, methodics of identifying, educating and developing mathematically gifted pupils is not sufficient. That may be an encouragement to carry on research on mathematics didactics, and enrich it. To prove it the authors żevealęxamples of the methodics of work with gifted primary and secondary school pupils. The evidence of the efficiency of the methods are: **1)** an early detection of mathematical interests of the pupils, their willingness to demonstrate that they are capable of noticing something in mathematics or in a maths problem that others can not see; **2)** correct interaction between teacher and student, i.e. the most brilliant students can be a mathematics teacher's helpers (his subject assistants); **3)** a relevant selection of the teaching material and problem material, forms of educating and developing talented pupils (mathematics societies are preferred) and making them more active (usually through mathematics competitions); **4)** work timetable and a programme of the mathematics society suitable for talented pupils.

In the Rzeszów region, where I live, for example a man of pedagogical success with respect to work with the mathematically talented pupil is A. Bysiewicz, a mathematics teacher in M. Kopernik Secondary School, Krosno. He is the teacher of Jarosław Wrona, the winner of the silver medal in 2002 in the XLIII International Mathematics Olympiad in Glasgow. Another successful educationist is W. Rożek, a teacher of mathematics in KEN Comprehensive School Complex, Stalowa Wola. He is known as the teacher and tutor of a large group of finalists of Mathematics Olympiads. One of them is Tomek Czajka, the prize-winner of the domestic and international final of a Mathematics Olympiad. What is more, Tomek Czajka was the mainstay of the three-person representation of Warsaw University that won, in March 2003, the world championships in computer programming in Beverly Hills.

Here is W. Rożek's work timetable for a secondary school mathematics society intended for gifted pupils. Mathematics activities are carried on in two groups. The first one includes first form pupils and those from upper forms who have just decided to enlarge their mathematics knowledge. Here I have introduced basic notions related to the content of Olympic problems. In particular, they are as follows: 1) Number theory: number congruence, small Fermat theorem, 2) Problems related to chess-board colouring , 3) Dirichlet's drawer theorem , 4) Inequalities and means, 5) Geometry: lines, circles, polygons, Tales arcs, points and lines in a triangle, geometric transformations. In the other group including pupils from upper forms the knowledge is enlarged with such notions as: 1) Power of a point in relation to the circle, 2) Inversion, 3) Brioncham theorem, Cevy theorem, Menelaos theorem, 4) Monotone sequences and inequality proving, 5) Viete formulas, 6) Functional equation, 7) Recurential equations.

Another form of support and formation of pupils' creativity in the Rzeszów region, generally, formation of creativity in mathematics teaching is the affiliation of mathematics school societies and mathematics competitions in the district (e.g. H. Steinhaus Mathematics Competition in Jasło) by the Rzeszów Section of the Polish Mathematics Society. Selected staff of mathematicians - members of the Section and academic teachers of Rzeszów colleges-offer essential and educational help to the members and tutors of the societies. They also organize seminars, workshops and mathematical competitions for them.

Another supporter of the development of pupils' mathematical creativity is the School Superintendent Office, which organizes seminars for the executive staff and teachers, e.g. **Education and care of a gifted pupil**, in order to demonstrate and popularize the local experience and achievement in this respect. It is that kind of activity resorting both to past experience and looking ahead, carried on by headmasters and outstanding teachers of not only mathematics but also other subjects that forecasts good progress in spotting, educating and developing mathematically gifted pupils and students.

The work of S. Grozdev (2003) synthetically presents reflections and Bulgarian experiences in the field of stimulation of students' mathematical activity and creativity. The author reveals that the background of the methodology of work with students at the stage of the preparation to domestic and international Mathematics Olympiads is based on the implementation of advices of H. Freudenthal, J. Piaget, H. Poincare and Bulgarian educator I. Ganchev. The first of the mentioned above states that *in mathematical teaching the taught one should pass through the following stages: first stage - instinctive rediscovery, second - conscious application, third - formal definition*, see: H. Freudenthal(1973). With reference to the first stage, J. Piaget says that *a complete acquire of knowledge occurs in the rediscovery process only, which needs creation of problem situations.* Following H. Poincare, however, to create means to distinguish and choose. And further S. Grozdev admits that successes of the Bulgarian students in the International Mathematical Olympics result mainly from the following actions:

1) *Our special attention to the żevisionąctivity is connected with the so called a hierarchic approach for investigating and systematizing of students' cognitive activities in the preparation for Olympiads. The żevisionąctivity, which is discussed in the present note, is concretized in individual reading through personal notebooks and is in a direct relation with "keeping a notebookąctivity. The latter activity is a result of the above mentioned search, collection, investigation and systematization of topics, methods and problems. In its turn the żevisionąctivity influences 9up along the vertical) successful problem solving, creativity and scientific research.*

2) The main task in the preparation of gifted students for a successful participation in Mathematics Olympiads is to stimulate their cognitive and will for individual work and research. Some people say that is very simple to become a scientist.

4. Results of research on teacher's intervention

Now I would like to outline the results of empirical research on teacher's practice of forming pupils' activity and creativity based on E. Śmietana's (2005) doctor thesis.

E. Śmietana (2005) - a secondary school mathematics teacher - in his doctoral thesis deals with the effect of the teacher's intervention on the mathematical activity of a talented pupil solving a problem. He carries out an individual teaching experiment in a group of secondary pupils and then proves the thesis: *the teacher's distracting interventions increase the mathematical activity of the pupil solving the* *problem and sometimes an indispensable help in solving an untypical mathematics problem.* The author formulated four aims for the experiment: "1) finding effective teacher interventions in the process of solving a mathematical problem by a mathematically talented pupil, 2) identifying the activity blocks that occur in the process, pinpointing their causes and suggesting the ways of eliminating them, 3) analysis of the pupil's behaviour while solving the problem after the teacher's distracting intervention and a description of his mathematical activity, 4) evaluation of the effect of the distracting intervention on the solution of the problem", ibid, p. 378. He achieved these goals applying, among other things, a method of stimulating the process of solving a mathematics problem (an algebra or geometry one) through properly prepared scenarios of the intervention. The impact and effectiveness of the intervention is precisely analysed and described in view of the revealed activity aspects of the pupil or his activity blocks. The behaviour dynamics of the tested pupils is clearly presented in specific blocks diagrams. The interesting conclusions which E. Śmietana has drawn from the qualitative didactic analysis should be further verified in more numerous group, at different levels in the working conditions typical of a mathematics teacher. The core of his conclusions is as follows:

1) Opening interventions appeared to be useful and effective in the methodics of solving mathematics problems. They resulted in the pupil's being receptive to the areas of the applied knowledge and triggered the association that helped to solve the problem, for example, the pupil associated the basic problem with an equivalent one, which caused a change or extension of the pupil's knowledge.
2) Opening interventions have the following characteristics: a) they do not always properly stimulate the pupil's mathematical activity (they may arise a feeling of being distracted, which hampers the activity necessary to solve the problem or confuses the pupil how to use the knowledge that is still new or "unfriendly"), b) they are not necessarily effective in the case of all types of mathematics problems and all mathematically gifted pupils, c) they can occur at any stage of a problem being solved and frequent occasions.

3) In the process of developing the skill of solving mathematics problems little attention has yet been paid to divergent (distracting) interventions, causing divergent thinking that, in the case of a talented pupil, positively increases his mathematical activity. The research subject of E. Śmietana's is in the same line as A. Pardała's doctoral dissertation (1984) which I describe as: **Didactical problems of the teacher's intervention in the process of teaching mathematics to pupils**. E. Śmietana successfully introduces some terms from the analysed literature into his research scope defined by the subject and aim of the research. For example, he takes such terms as convergent thinking and divergent thinking from psychological literature, see e.g.: J.P. Gilford (1978), whereas the notion: the teacher's intervention, its forms and kinds, understood as convergent intervention which consolidates the pupil's knowledge necessary to solve a mathematics problem, comes from mathematics didactics and becomes the object of his research, see: A. Pardała (1984). The result of his investigation is enriching the practice teaching a gifted pupil and introducing new terms: *opening intervention* and *divergent intervention* into mathematics didactics, which helps describe the process of solving a mathematics problem by a gifted pupil.

5. Summary, remarks and final conclusions

Taking the above into consideration and historically analyzing the evolution of the education and development of pupils' abilities and strategies of educating exceptionally talented pupils, one can distinguish **four types of didactic and organizational activities** that can be recommended: **1)** making the development of gifted pupils faster, which means increasing the pace of their teaching and learning, **2)** providing them with a greater amount of knowledge, i.e. expanding the scope of the subject taught, **3)** offering them a more advanced knowledge but only slightly more advanced than the level of their current knowledge and personal expectations, **4)** forming pupils' creativeness in the process of teaching a given subject.

All the above suggestion being accepted, the very process of educating gifted pupils resembles a realization of a sequence of consecutive organizational and educational tasks and the formation of pupils' creativeness (also mathematical creativeness) is its target. Such a system of education of gifted pupils can be also adapted to talented students, however it takes: 1) multilevel teaching and varied teaching content, 2) fully qualified staff supervising their development.

The concept of forming the pupil's mathematical creativeness in mathematics teaching is integrally connected with the idea of developing creative mathematical activities of the pupils which should be realized, according to M. Klakla (2002, pp. 48-49), through activities regarding intellectual, didactic and evaluation aspects, with the teacher responsible for planning and guidance. M. Klakla's concept (addressed to secondary school pupils) consists of two steps i.e. teaching as well as developing elements of creative mathematical activity and creative mathematical activities of particular pupils and appropriately selected teaching programme and unique methodology of attaining a particular kind of creative mathematical activity. This is also the result of the selection of the problems and initial examples, multistage problems and paradigmatic examples.

The effectiveness of the formation of the pupil's creativeness in mathematics teaching and work with a talented pupil is also conditioned by a useful cooperation with the school, mathematics teacher and his experts (methodics advisors, mathematicians and mathematics educators, educationists and creativity psychologists). At the school level, a solidly prepared and well organized programme of work with the gifted pupil and in particular with a mathematically gifted pupil should be indispensable. Such a programme should include the accepted concept of education and development of gifted pupils and make use of long-standing experience of a given school as well as the break-through solutions of the Society of Creative Schools and the Association of Active Schools based on the pedagogy of individual differences. While creating the programme, the following targets can be defined: **1)** attracting and spotting talented or exceptionally gifted pupils, **2)** abandoning the class-lesson system in favour of other forms of encouragement and work with pupils who are exceptionally mathematically gifted, **3)** special forms of work with the pupil or pupils who have a particular talent for mathematics.

Special forms of work ensuring the effectiveness of the formation of the pupil's mathematical creativeness include: 1) extra activities, i.e. those performed outside lesson hours adapted to the individual needs and interests of mathematically gifted pupils, **2)** mathematics societies whose members come from forms of different levels and where the motivation is the intention to broaden mathematical knowledge and exchange of ideas, **3)** an individual teaching programme for a particularly talented pupil who was a success in mathematical competitions or in the Olympiad, **4)** September meetings and cooperation (at school or by correspondence) between former Olympiad participants from a particular school or region and their younger schoolmates interested in taking part in a Mathematics Olympiad or individual contacts between the latter and research workers, mathematicians who once attended the same school, **5)** participation of exceptionally talented pupils in the activities (meetings and workshops, lectures and seminars) organized by the mathematics departments (Institutes, Faculties) at school for higher deduction or confiding mathematically gifted pupils in the care of the mathematicians from those colleges, 6) participation in educational activities organized by the National Found for Children and contacts between the most talented young people in this country and the leading representatives of Polish mathematics and science.

The above examples of international and domestic experiences from building the mathematical activity and creativity of students confirm the existence of many forms and possibilities. Additionally, they reveal the related methodics and problems of teachers' practice. Those examples are some kind of proofs, which enrich the practice of stimulating mathematical activity and creativity of students. Moreover, they confirm that it is not just about looking for only one optimal and effective way of their creation. It is rather about **healthy competition** that gives school, as well as students wider vision for the reached level of mathematical activity and creativity. Following P.J. Taylor (2003) *activities related to competitions can also give a student a rare opportunity to meet students from other schools, broadening their perspectives and making new friends. Activities such as camps and mathematical ćirclesćan very much enrich the student's interest and motivation in mathematics, potentially propelling them to a level far beyond the classroom, and since mathematics is such a big subject, not prejudicing the material they will learn in the school in the following year. Giving a student access to the right books and journals can also achieve the same result.*

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