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# FROM MIRROR REFLECTION TO THE CONCEPT OF LINE SYMMETRY ON THE PLANE

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## Rationale

In this paper I would like to discuss the growth of one of the geometrical concepts: mirror reflection and the way the child pass during the process of discovering certain properties leading to the concept of line symmetry.

Mathematical definition of this transformation concerns the entire plane and refers to the point. For children more natural is local symmetry being a transformation of a limited figure into the limited figure. At the beginning it is reduced to the situation in which the figure has not common points with the line of symmetry. Therefore it is considered as a mirror reflection. Perception is of a great importance in the process of learning and teaching on this stage.

The problem of the research work is: how the passage between visual perception of shapes on a plain or relation of the shape towards another on a plain and in contrast to noticing the dynamism of the relation look. In Poland this problem has significant didactic consequence e.g. in teaching line symmetry. Children start making ink - stains and paper cut - outs (10-year-old students) and they observe mirror reflections (they use a mirror). During the activities the terms are introduced: mirror reflection, symmetric figures and a line of symmetry.

The concept of mirror reflection and all names were introduced in the examples: *'During making ink stains there is the same shape on each half of the paper. Shapes are placed in the way that they are the mirror reflections of each other. A bend line of the paper specifies the placement of a mirror.'*

Figures having a line of symmetry are considered to be specific examples of symmetric figures (two halves have the same shape and they are mirror reflection). In the next stage (11 years old) students deal with figures having a line of symmetry, they learn about concept of line symmetry of a figure and

congruent figure (as figures that overlap). As you can see, the intention of conception is internalisation of the activities done and leading to acceptation of definition of line symmetry.

The authors of this conception assumed that a figure and its reflection are of the same shape. Motivation of an ink stain leads to the fact that the copied figure is of the same size. The assumption that the figure and its reflection are of the same shape may cause some kind of cognitive conflict. In the sense of child's perception and by creating the environment meaningful it may be not the same shape. Potentially, the cognitive conflict which can be discovered by every child, but can not be identified or named, causes problem with the specialization of the concept and with leading to subtle understanding of reality.

I will analyse some specific characteristic situations observed in following stages of my scientific work, which are supposed to present the evolution of understanding the mirror reflection by children through acquiring experiences. I will also show how children discovered properties which preserve in mirror reflection (shape and size) and the ones which change themselves (orientation of the figure). Research lasted 4 years. Detailed description of whole research and research tools one can find in [1–3].

## Methodology

My research has embraced four kinds of situations which could be named as:

[1] the diversity of objects,

[2] identical objects placed differently;

[3] two congruent figures placed differently and the process of overlapping one figure with another one;

[4] the process of overlapping one figure with another one.

Properly constructed game for 2 students was the instrument of research on the each stage. The first stage of research was devoted to verify in what way children will talk about the mirror reflection on the plane, to what extend they will differentiate and how they will describe this relation. A board game, which was based on choosing objects and pointing common features, was the diagnostic instrument. Diversity of objects on the board enabled to indicate different features which match the objects. One of them could be the specific placement on the plane such as the relation of the mirror reflection. On the second stage of research I limited the diversity of objects. I placed 2 families of congruent figures. They were placed differently but in a characteristic way

(line symmetry, glide symmetry, rotary symmetry, translation). The placement would have been the common feature. I introduced dynamism in the third and fourth stage of research. Children overlapped one figure with the another. The movement was in the 2 dimension space. Transformations made by children can be examples of geometric transformations on the plane [5].

**Situation: the diversity of objects**

The pupils' goal was to find figures the same, in a certain respect and explain his/her choice by pointing on the feature combining one object with the other one. The students are required to look and describe common characteristics of the objects in the pictures.
It turned out that **in the case of specific geometric figures** (triangles, rectangles) there was no need to refer to the placement of objects and connecting it with the relation of the mirror reflection. Children at this age had enough geometrical knowledge and knew the properties of figures so that they did not need to join identical triangles in pairs because of the specific placement. Figures had a lot of properties - the same number of sides, angles, having an acute angle etc. It was enough to join them in pairs and point out the common feature. Whereas taking other objects into consideration (not figures) which a child can not give a specific geometrical name to, I noticed different argumentation.

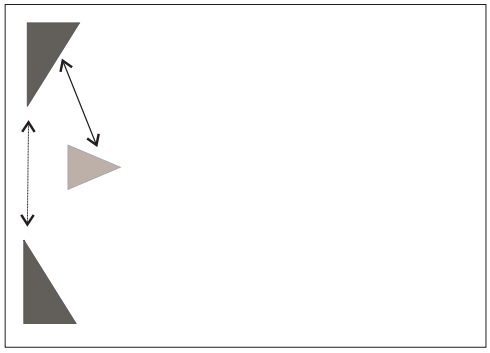


Fig. 1. Example of the choise of two figures which are elements of the set of triangles.

**Example 1.**

*Sabrina looks at objects on fields 55 and 62 - musical note having "tummiesón different sides. She goes to the other side of the table, but finally before making a move she says:*

S11: ... Not these because it is on a different side....
*She wonders about a selection. She changes her mind and selects two loops (57, 30) saying:*
S12: They are different but they both have this (shows a loop).

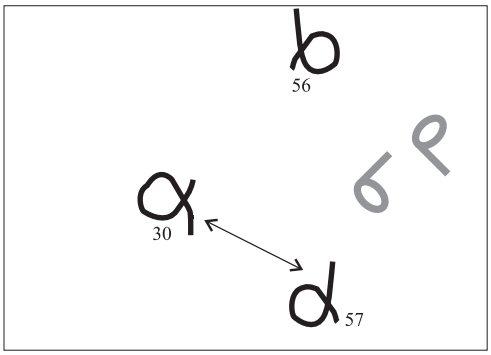



Fig. 2. The objects on a board.

Sabrina noticed a difference in the orientation of the objects-that makes them different physical objects, despite the same shape (Fig. 1). She decided to choose objects which we can name colloquially as loops, according to their shape identity. She considered other objects of the same shape but different orientation. She rejected these figures discovering that they differ too much. (S9: 'because it is on the other side'). Her choice indicates clearly that the difference in the orientation was the factor diversifying the objects strongly. The identical shape proved to be the weaker feature combining the objects.

**Example 2.**

*K3: I'm choosing 19 and 49, because these are the same notes, but this (19) is in different side.*
*KK4: 62 and 55, because they are in different position, but they are empty inside.*

Kuba referred to two properties: shape and position. However, he decided on the basis of one more fact which he could not precise. There were three more areas presenting the same objects as chosen, but he did not point them (Fig. 2). He also did not choose the figure in the privileged position the vertical - the horizontal level which is differentiated by the majority of pupils. He did not indicate the third figure which had an orientation consistent with the two chosen. The explanation of this fact is not clear. He could have decided to choose consciously the two objects which were relatively the closest to each

other and demanded only a little rotation in the mind to discover that they are *'the same notes'* (K3). He differentiated them from among the rest which were not exactly *'the same'*. Consistent orientation of objects decided about his choice. The difference in the position was noticed and emphasised but it did not disqualify the objects identity.

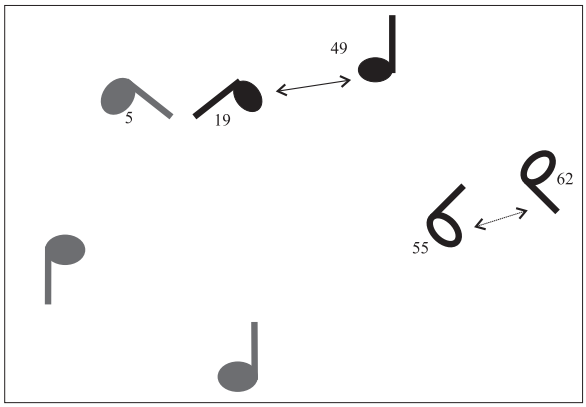



Fig. 3. Objects19 and 49 and objects 55, 62 on a board. The other objects with the same shape are marked gray color.

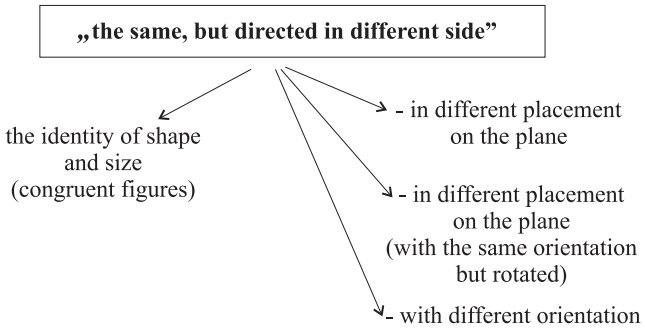
Kacper - the second student - clearly followed Kuba making his choices. He found the objects of the same shape on the board. He chose empty figures in order not to follow his opponent's choice. 'Empty inside' was the feature combining them. Kacper did not name it specifically. He emphasised different position in his argumentation. The same shape was a deciding factor. He pointed to differently situated figures but which were identical.

**Example 3.**

*A66: I'm choosing 55, 62 because they are like notes empty inside. (Compare with Fig. 3)*
*B67: These are not notes. There is a stick placed incorrectly in one of them.*
*A68: Yes, they are. If you drew this (62) and then reversed the board, it would be that! This is similar to a note. These figures are similar to notes and similar to one another. They are* **even the same**, *but their sticks are placed differently.*
*B69: But you did not say it like that at the beginning.[ ... ]*
*A71: You only have to look from the other side at this board* **and you can see the same** - *are you convinced now?*
*B72: That is all right.*

Adela explained how we should look at the one of the figures to notice that they are identical. The crux of the choice was the relation a figure to a figure. This relation was of a dynamic character. The figures were the same, because they were their own images in some transformation (here it was mental transformation), which did not change neither the shape nor the size, but only 'reverse on the other side'. Adela's justification enabled her to find the feature combining the figures of identical shape and size, but different orientations. Both girls recognised the identity of the figures, despite different orientations (stage of 'being (directed) in different side').

Relatively often students used the description 'the same but directed in different side'. Its meaning was different and embraced three kinds of situations:




**Situation: congruent objects placed differently**

**Example 4.**

*Filip does not understand how it is possible to refer to the position. He asks if it is correct to say that they have the same angles. Krzysiek makes a choice:*
*K1: 5, 15 -they are in* **the same identical position**.
*F2: 19, 15 - because the position is the same (he points the direction of the position of each figure, he places his hand in a way that his fingers indicate the pointed parts of the figure) [ ... ]*
*K5: 22, 26 - they are placed in the same way, horizontally, in this direction (he shows the direction of the position of each figure separately, but it is the same direction). There is (a higher pointed part) on the right.*

Chosen figures were their own images in translation (Fig. 4). For the student they ware in ***the same position***. When there was no choice of figures in translation, explanations appeared:

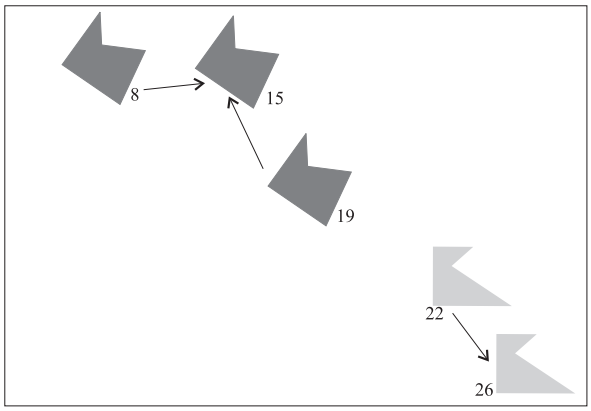



Fig. 4. The location of the objects from areas: 5, 15, 19, 22, 26 on the board.

*K12: 29, 24 - here is the centre (he is pointing the line between figures) and they ... as if ... are diverging, they look like they have bigger obtuse.*
*F13: What? I do not agree!*
*K14: They are placed in a similar way, but on the other sides. This one is placed in the same way, but to the left (he is pointing to the object from area24), and this to the right (on the area 29).*

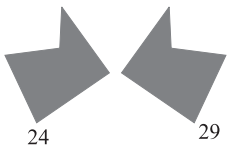



Fig.5.

Student's utterances were much elaborated. It indicates his difficulties in describing the specific position of the figure (Fig. 5). The distinction of the axis of symmetry was not difficult - Krzysiu used the term 'centre'. The position of two figures was referred to this imagined 'centre' - the axis of symmetry. We can combine diverging of the figures with their movement understood as the parallel shift of each figure. In the case of this movement one figure does not overlap the other one. They diverge in the same way but one to the left and the second to the right.

**Situation: two congruent figures placed differently and the process of overlapping one figure with another one**

The figure remained the same. The situation was changing. By using a computer I introduced some specific kind of movement on the plane and combined

a static situation with overlapping of a figure with another one. Moves were performed by selecting an operation: *shift* (the figure is moved to the right, to the left, up or down), *rotate* (clock or counterclockwise), *reflect* (vertically or horizontally). I observed some characteristic students' behaviour.

**Example 5.**

In the problem on Fig. 6 it would be enough to reflect the figure. After thinking a while the student recognized different orientation of figures. He/she knew that only by *reflecting* it is possible to change the orientation of a figure and that it is enough to use the operation *reflect* only once. The student reflected the figure horizontally and then by *rotating* obtained a translation placement.

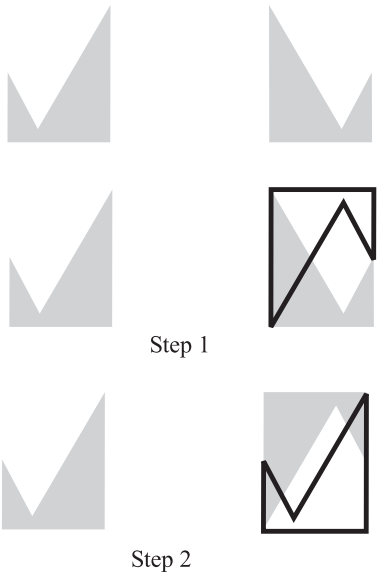



Fig. 6. An example of student's solution.

In the situations showed on Fig. 7 overlapping a figure with another one requires both: reflecting and rotating one of the figures. This arrangement of figures required from the student a while of thinking. It indicated moment of a deeper analysis of the figures' configuration, imaginative transforming of the figure, predicting the effects of the transformations, comparing certain characteristic of figures. After thinking a while the student reflected a figure vertically and rotated it to the translation position.

In those both situations the student had to change his/her perception of sameness of figures with operations of overlapping one figure with the other one. It required a certain mathematical strategy. Very soon, strategy of trans-

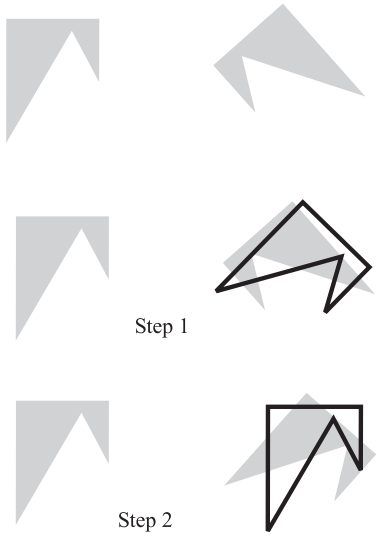



Fig. 7. An example of student's solution.

forming arose: through reflections, rotations attain the parallel arrangement of the figures, and late on it sufficed to translate them. Arranging the figures in a position of mirror reflection seemed too difficult. The operation *reflect* fulfil only role: it enabled to change the orientation of a figure. The placement of a figure on a plane after reflecting did no matter.

**Situation: the process of overlapping one figure with another one**

I had to perform the process of overlapping one figure with another one a little. It would not be available to use the operation *reflect* as a thirst transformation (it was blocked). In order to putt students' attention on the movement, the figures had different shape in each task.

It was a cognitive conflict between a visual analysis of a placement of figures and a possibility of movement. The students recognized a need of reflection but it was not possible to perform this kind of operation firstly (Fig. 8). They had to begin with rotating a figure. Finding a specific arrangement of a figure was the focus of their attention. They arranged the figure in parallel manner and then reflected it vertically or horizontally (they made a vertical reflection manly). After that transformation they were surprised because they would not be able to reach a good placement (Fig. 8, steps 1-2). They went on to rotating but it was a mistake rotating a figure twice. It turned out that they did not reach a good position for reflection. A lot of experiments enabled the students to find a new strategy of overlapping a figure with another one by rotating fist and then reflect the figure (Fig. 9).

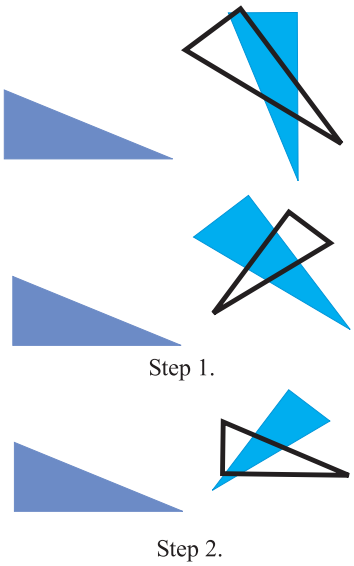



Fig. 8. An example of student's solution.

It was a long way of investigation of arranging the figure in a position of mirror reflection in respect to horizontal or vertical line. It appeared that the students could recognise or intuitively guess some relation between figures; however these relations were not appropriate to the beforehand given line of symmetry.

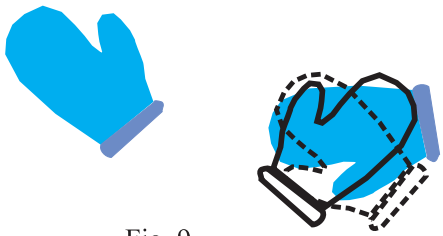


Fig. 9.

## Conclusions

Analysis of my research results enabled me to determine the existence of some cycles in the development of the concept of symmetry on the plane by 10-12 years old children. The thirst cycle bases on perception. Students require them to build the basic concepts [4]. It is necessary to supply the specific

situations concentrating their attention on the placement of figures and the relationships included in these placement to the students. The goal is not for a student to simply memorise the definition of line symmetry but to understand how to describe what he/she can see and understand the relation of a figure to another; to be able to replace movement with a relation between two objects as well as relation to movement. It is close to the actions become a process so the individual can describe or reflect upon all the steps in the transformation without necessarily performing them.

The *perceived object* [6] is the object based on perceptual information - seeing figures in a specific relations like mirror reflection, physically cutting figures and putting them appropriately, observing reflection in the mirror, in the water etc. The *conceived object* occurs when there is a reflection on perceptions and actions, so the focus is no longer on the specific physical manifestations but on the actions and processes performed upon them. In the process of learning and teaching from mirror reflection to line symmetry there are some specific efforts needed. The aim of them is to teach to discover definition conditions to use mathematics as a language to describe the real world.

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