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Modeling of Crystal Defects in Nonlocal Elasticity: A Review*

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Abstract
A review of studies concerning models of crystal defects in solids is presented. The emphasis is on describing imperfections in nonlocal elastic continuum. Nonlocal theory reduces to the classical theory of elasticity in the long wave-length limit and to the atomic lattice theory in the short wave-length limit.

1 Introduction

At the present time, it is generally recognized that the deformation process occurs at various structural levels with their own scale lengths. Because of the peculiarities of deformation it is necessary to use specific physical concepts and mathematical tools at every structural level, while events progressing at different scale levels are interdependent.

The perfect crystal is a completely symmetric infinite structure with atoms placed precisely on the lattice points. Every error in atom placement results in an imperfect crystal. Real crystals contain a large amount of defects. The defects of crystal lattice can be classified according to their dimension.

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– **Zero-dimensional (point) defects.** The lattice site which contains no atom is called a vacancy. Atom in a nonlattice position is called an interstitial. A foreign atom present on a lattice site is called a substitutional impurity. A foreign atom present in a nonlattice position is called an interstitial impurity [1, 2].

– **One-dimensional (line) defects.** A chain of point defects is the sipmlest example of line defects. An edge dislocation is the result of the lattice distortion caused by the insertion of an extra half-plane of atoms part way into the crystal. The distortion of the lattice is primary localized in the vicinity of the edge of the plane. A screw dislocation transforms the flat atom planes of the crystal into a spiral ramp [3–5]. Along with dislocations (translational defects) disclinations (rotational defects) also represent line imperfections [6, 7].

– **Two-dimensional (surface) defects.** Since any real crystal is finite, the ideal lattice structure must be terminated by a surface. The atoms in the vicinity of the surface do not see a completely symmetric situation, thus creating two-dimensional imperfection. In addition to normal external surfaces, a crystal may also have internal surfaces, such as stacking faults or grain boundaries [8, 9]. A grain boundary is that region of disorder which separates a lattice of one orientation from a lattice of a different orientation. It is possible to have a grain boundary of a special low-energy sort, called a twin, in which the atoms across the surface bear special relation to one another similar to that in the perfect crystal. A stacking fault is related to the difference between a face-centered cubic crystal and a hexagonal closely packed crystal. Some of the atoms in the vicinity of the stacking fault bear a relation to one another similar to the hcp crystal instead of the fcc crystal. Various type internal cracks, slits, slots and cuts are also among surface defects [10].

– **Three-dimensional (volume) defects.** Voids, foreign inclusions and inhomogeneities fall into the category of volume defects [11, 12].

2 Modeling of imperfections

In recent years the problem of modeling stress and strains fields produced by imperfections in a crystal has attracted much attention of a number of researchers. An interaction between various kind defects and an interaction of a defect with an applied loading are mainly due to the elastic intercation. In an elastic solid the abovementioned imperfections, as a rule, are modeled by certain distribution of concentrated forces or by certain distribution of the tensor of plastic strain incompatibility.

The simplest model of point defect in an elastic body is the center of dilatation [1, 11]. More complicated multipole model was proposed in [13] (see also [14]).

Straight dislocations were first considered in [15, 16]. The Italian mathematician Vito Volterra introduced [15] the theory of the elastostatic stress and displacement fields created by dislocating solids. This involves making a cut in a solid, displacing its surfaces relative to one another by some fixed amount, and joining the sides of the cut back together, filling in with material as necessary. In other words, defect of the Volterra type can be formed by cutting a surface S in a body and the subsequent rigid relative translation with the Burgers vector $\mathbf{b}$ and the rotation with the Frank vector Ω of two sides of the cut. The corresponding plastic distortion $\boldsymbol{\beta}^{\mathbf{P}}$ has the form obtained by Mura [17]

$$\boldsymbol{\beta}^{\mathbf{P}} = -\delta(\mathbf{S})(\mathbf{b} + \Omega \times \mathbf{r}), \tag{1}$$

where $\delta(\mathbf{S})$ is the vector delta-function lumped on the surface S.

In 1934 Polanyi [18], Taylor [19] and Orowan [20] realized that the plastic deformation of solids could be explained in terms of the theory of Volterra dislocations. The insight was critical in developing the modern science of solid mechanics. Considerable study is being given to dislocations in the frame-work of crystal imperfections and solid mechanics [3–5, 21–25].

Dislocation loops of the prismatic type with the Burgers vector normal to a loop plane may be produced after quenching or irradiation and can play an important part in the formation of dislocation networks and in the production of nuclei of a brittle fracture [26, 27]. Glide dislocation loops with the Burgers vector in a loop plane are formed by Frank-Read source mechanism or by stress concentration and are of importance in investigation of plastic deformation [28].

Stress field due to the circular prismatic dislocation loop in an infinite isotropic medium has been obtained by Kroupa [26]. Corresponding problem for the circular glide dislocation loop has originally been studied by Kröner [13] and Keller (their equations have been examined and corrected by Marcinkowski and Sree Harsha [28]). In succeeding years interest in dislocation loops in elastic media has been maintained [29]; for a comprehensive review, see [30].

At the present time, in parallel with dislocations, disclinations have attracted a large amount of interest in a variety of fields due to their numerous applications. They have been receiving the attention of many researchers, in particular in the context of their applications to polymers [31], liquid crystals [32], biological structures [33], grain boundaries [34], amorphic solids [35], rotation plastic deformation [7] (see also a review article of Romanov and Vladimirov [6] and a book of Likhachev and Khairov [36]). Straight disclinations (Volterra distorsions of the 4th, 5th and 6th kind [15] with the Frank vector Ω) have being studied intensively by de Witt [37–40] and Likhachev and Khairov [36].

Elastic fields and energies of circular twist and wedge disclinations have been investigated in the frame-work of classical elasticity in [31,41–44] For comprehensive review and additional references, see also a study of Kolesnikova and Romanov [30].

Classical elasticity solutions for cracks were obtained by many authors; we refer to the works [45–49]. The elastic stress fields around loaded cracks, slip bands, kink bands, twins all have qualitatively similar features when viewed on a siutable scale. This is because they all represent the same type of incompatibility in a solid that caused by the Somigliana dislocation. Under appropriate condition, it is expected that any type of these defects to be a source of any of the others [50]. The similarity of the piled up group of slip dislocations to the freely slipping crack was recognized in the development of the theory of crystal dislocations. A crack with a normal displacement discontinuity can be descripted by a continuous distribution of freely climbing dislocations. Arrays of dislocations and disclinations are also used to describe stacking faults and grain boundaries. For details and additional references, see [7, 50].

A limitation of the abovementioned solutions is that the stress fields have nonphysical singularities and the elastic energy diverges if one does not cut the defect core. The classical solutions cannot

describe the situation in the strongly distored region in the vicinity of the defect. Moreover, the classical elasticity is not valid in the immediate vicinity of the imperfection and fails to explain the phenomena at the atomic scale.

The pure discrete lattice method of calculation of lattice distortion produced by a single vacancy was originally developed by Kanzaki [51] and has been further applied and extended by Hardy [52] and Flocken and Hardy [53, 54]. This method requires the extensive computer calculations, especially for change in the material properties. The current state of research on computer simulation of point defects in solids is reviewed in [55-57].

In the semi-discrete approach [58, 59] a portion of a crystal centered on the point defect is treated as a discrete lattice, and for remainder of the crystal an elastic continuum model is used. Johnson and Brown [60] have proposed more precisely version of the semi-discrete method, in which crystal is devided into three regions: a discrete region, an elastic continuum and an intermediate region. In the discrete region the atoms near the defect are treated as classical particles, one deals with with the individual displacements of the atoms, and the potential energy is calculated using interatomic potentials. In the intermediate region the displacements of atoms are determined by the elastic displacement field, but energy is subject o the interatomic potential. Finally, in the elastic region both displacements and energy are calculated from the theory of elasticity. Various refinements of the semi-discrete method have been described by Lidiard [61] (see also [62]).

However, one does not have any *a priory* knowledge of how large to make the discrete region. At the same time, the equations governing the displacements of the atoms of the discrete region increase rapidly in number and complexity as the volume of this region increases. In addition, there are an infinite number of solutions to the Lamé equation, and boundary conditions are needed to determine which are applicable to a given problem, but boundary conditions at the interface between regions cannot be simply specified.

The discrete lattice methods have been used with success by Celli [63], Boyer and Hardy [64] for modeling an isolated screw dislocation in crystals with various lattice structure. For review of the researches on computer simulation of dislocation core, see also [62, 65, 66] and the references therein. Such an approach has shed light on

the physical aspect of the problem, however, atomistic models, as a rule, are concerned with the simplist geometry, depend on a choice of interatomic potentials, require extensive computer calculations and involve difficulties in passing from one scale level to another.

Resonably far from the dislocation line the distortion of the crystal is adequately described by the elasticity theory, but in the vicinity of the dislocation line the continuum elasticity description fails. The dislocation core is defined as the region of the material where the crystal lattice has significant distirtions and is practically of the order of lattice parameter.

Many efforts have been made to improve classical elastic solutions for dislocations, for instant combining the elastic and discrete approaches for better description of high distored region near defect. The simplest model is the Frenkel-Kontorova model [67] in which the dislocation is considered as a set of particles coupled by nearest-neighbour elastic interaction and moving in a periodic potential. Another model which takes into account the discrete structure of the crystal and describes the core of the dislocation is the Pierls-Nabarro model [68, 69]. In this model the dislocation structure is described by the misfit function. The assumption of the model is that the dislocation is characterized by the elastic energy due to a finite density of dislocations and the misfit energy which results from the nonlinear atomic interaction in the glide plane. A review of the Frenkel-Kontorova and Pierls-Nabarro models for dislocations and their generalizations was made by Hirth and Lothe [5] (see also [70-73]).

The semi-dicsrete approach according to which the crystal with dislocation is devided into two parts (a discrete lattice and an elastic continuum) has been discussed extensively in [14, 74, 75]. But as a rule all improved solutions correspond to the straight dislocations.

Though the literature concerning various models of the dislocation core is very extensive, that for the disclination core is not numerous. We can only mention studies of Doyama and Cotteril [76] and Mikhailin and Romanov [77] on computer simulation of straight disclinations (see also [78]). As far as we can judge, no other attempts have been made to improve the situation in the vicinity of the disclination line, especially for circular disclinations.

The solutions for the crack problems obtained in the frame-work of classical elasticity have non-physical singularities at the crack tip. The regular attempts to improve elastic solutions have attracted much

attention. We mention models of Leonov and Panasyuk [79, 80], Dugdale [81] and Barenblatt [82] in which compressional cohesive stresses are introduced in a small region in the vicinity of the crack tip.

In recent years there has appeared a number of studies of cracks in solids based on atomistic models. We may distinguish between the following broad categories of such models. The lattice-statics approximation [83–85] deals with an infinite number of atoms interacting with highly idealized interatomic potentials. From the viewpoint of idealized geometry and interatomic force laws this model surves to provide insight into those features which are introduced by the discrete atomic character of crystalline material without entering into the characteristics pecular to a concrete material. In the molecular–dynamics approach [86–89] differential equations of motion are solved numerically for a sufficiently large number of atoms interacting with a given potential. In connection with a choise of such a number the following problem arises: how large enough should be this number for numerical simulation of the corresponding infinite system. It is only with the advent of modern computers that atomistic investigation became possible. Both the lattice-statics and molecular-dynamics methods only concern one-dimensional and two-dimensional cases because, even using modern computers, it is impossible to treat a three-dimensional lattice of any reasonable size.

The hybrid approach [90–93] involves division of non–bounded solid into two regions. The region I in the vicinity of the crack is considered as a lattice crystal, and the remainder, the region II, is treated as an elastic continuum. There is a question concerning the size of the region I requiered for this to be a valid assumption. Another problem of the hybrid method consists in jointing descrete and continuum solutions. Both the rigid interface method and the flexible interface method, extensively discussed in the literature [14], have been used. The finite element–atomistic coupling scheme [94–96] also belongs to the hybrid methods. This scheme creats an atomistic model embedded in the finite element continuum with a two–layer transition zone in which the lattice and the finite element continuum overlap. In the inner layer the lattice is made to dictate the boundary conditions for the continuum. In the outer layer the atoms are made to coincide and move in a coordance to the finite element solution. Merits and demerits as well as plausibility of various atomistic models have also been discussed in [85, 89, 97–100].

3 Nonlocal elasticity

Recently, great advances have been made in crystal defect research by application of nonlocal elasticity. Several versions of non–local continuum mechanics based on various suggestions have been proposed by Kröner [101, 102], Podstrigach [103], Eringen [104, 105], Edelen [106], Kunin [107], Rogula [108], and others.

Starting from interrelated equations describing elasticity and diffusion (or heat transfer) Podstryhach [103] excluded the chemical potential (or the temperature) from the constitutive equation for the stress tensor and obtained the stress-strain relation containing spatial and time derivatives. In the infinite medium this relation can be integrated using the Fourier–Laplace transform, and the final result has the nonlocal form. Kröner [101], Kröner and Datta [102] and Kunin [107] started from discrete lattice and interpolated functions of discrete argument by special continuous functions. The stress-strain relation in such a quasicontinuum is non-local. Comprehensive review and additional references can be found in [109–110].

As a matter of fact, non–locality (in the broad sense) has various origin. A medium with the couple-stress tensor and non-symmetric stress tensor can also be considered as non-local. In this case the corresponding "weak non-local theory" (using the terminology of [107]) with its own scale length parameter is built. But stress and couple-stress fields caused by imperfections in such a medium also have singularities.

The nonlocal theory of elasticity takes into account interatomic long-range forces. The stress at a reference point in the body depends not only on the strain at this point but also on the strains at all other points of the body. Thus, the relation between the stress tensor and the strain tensor has the integral form.

The governing equations for the static case and for a linear isotropic non-local elastic solid are the following [104, 110]:
the equilibrium equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{\sigma}_{\mathrm{nl}}(\mathbf{x}) = -\mathbf{f}_{\mathrm{nl}}(\mathbf{x}), \tag{2}$$

the integral relation between the nonlocal $\boldsymbol{\sigma}_{\mathrm{nl}}(\mathbf{x})$ and local $\boldsymbol{\sigma}(\mathbf{x})$ stress tensors

$$\boldsymbol{\sigma}_{\mathrm{nl}}(\mathbf{x}) = \int_{\mathbf{V}} \alpha(|\mathbf{x}' - \mathbf{x}|, c)\, \boldsymbol{\sigma}(\mathbf{x}')\, \mathrm{d}\mathbf{V}(\mathbf{x}') \tag{3}$$

the Hooke law for the local stress tensors

$$\boldsymbol{\sigma}(\mathbf{x}') = \lambda \ \mathrm{tr}\, \mathbf{e}(\mathbf{x}')\, \mathbf{I} + 2\mu \mathbf{e}(\mathbf{x}'), \tag{4}$$

the geometrical relation

$$\mathbf{e}(\mathbf{x}') = \frac{1}{2}\big[\boldsymbol{\nabla}' \mathbf{u}(\mathbf{x}') + \mathbf{u}(\mathbf{x}') \boldsymbol{\nabla}'\big]. \tag{5}$$

the compatibility condition

$$\boldsymbol{\nabla}' \times \mathbf{e}^{\mathrm{p}}(\mathbf{x}') \times \boldsymbol{\nabla} = -\boldsymbol{\eta}. \tag{6}$$

Here $\mathbf{x}$ and $\mathbf{x}'$ are the reference and running points, $\mathbf{u}$ is the displacement vector, $\mathbf{e}$ is the linear strain tensor, $\mathbf{f}_{\mathrm{nl}}$ is the body force, $\mathbf{e}^{\mathrm{p}}$ is the plastic strain tensor, $\boldsymbol{\eta}$ is the incompatibility tensor, λ and μ are Lamé constants, $\boldsymbol{\nabla}$ and $\boldsymbol{\nabla}'$ are the gradient operators with respect to $\mathbf{x}$ and $\mathbf{x}'$, correspondingly, $\mathbf{I}$ denotes the unit tensor. The volume integral in (3) is over the region occupied by the solid.

The weight function (the nonlocal modulus) $\alpha(|\mathbf{x}' - \mathbf{x}|, c)$ depends on a distance $|\mathbf{x}' - \mathbf{x}|$ between the reference $\mathbf{x}$ and running $\mathbf{x}'$ points, includes the parameter c connected with a characteristic length ratio a/l , where a is an internal characteristic length and l is an external characteristic length and has the following properties

(i) $\alpha(|\mathbf{x} - \mathbf{x}'|, c)$ has a maximum at $\mathbf{x} = \mathbf{x}'$.

(ii) $\alpha(|\mathbf{x} - \mathbf{x}'|, c)$ attenuates rapidly with $|\mathbf{x} - \mathbf{x}'|$ to zero.

(iii) $\alpha(|\mathbf{x} - \mathbf{x}'|, c)$ is a continuous function of $|\mathbf{x} - \mathbf{x}'|$ with a bounded support V.

(iv) $\alpha(|\mathbf{x} - \mathbf{x}'|, c)$ is a delta sequence and in the classical limit $c \to 0$ becomes the Dirac delta function

$$\lim_{c \to 0} \alpha(|\mathbf{x} - \mathbf{x}'|, c) = \delta(|\mathbf{x} - \mathbf{x}'|).$$

(v) For $c \to 1$ non-local theory agrees with atomic lattice dynamics.

(vi) $\int_V \beta(|\mathbf{x} - \mathbf{x}'|, \zeta) \ \mathrm{d}v(\mathbf{x}') = 1.$

Eringen [111] found several forms of nonlocal modulus giving a perfect match with the Born-Kármán model of the atomic lattice dynamics and the atomic dispersion curves. In the present paper we employ the following two-dimensional

$$\alpha(|\mathbf{x}' - \mathbf{x}|, c) = \frac{1}{2\pi c^2} K_0 \left(\frac{|\mathbf{x}' - \mathbf{x}|}{c} \right) \tag{7}$$

and three-dimensional

$$\alpha(|\mathbf{x} - \mathbf{x}'|, c) = \frac{1}{8(\pi t)^{3/2}} \exp \left(-\frac{|\mathbf{x} - \mathbf{x}'|^2}{4t} \right) \tag{8}$$

nonlocal moduli. In Eqn (7) $K_0(x)$ is the modified Bessel function. The constant t in Eqn (8) is connected with the nonlocality parameter c.

The nonlocal kernel (6) is the fundamental solution of the Helmholtz equation

$$\frac{1}{2\pi} \left(\Delta - \frac{1}{c^2} \right) K_0 \left(\frac{r}{c} \right) = \delta(\mathbf{x}) \tag{9}$$

From Eqns (3), (7) and (9) we obtain

$$\Delta \boldsymbol{\sigma}_{\rm nl} - \frac{1}{c^2} \boldsymbol{\sigma}_{\rm nl} = -\frac{1}{c^2} \boldsymbol{\sigma}. \tag{10}$$

In such a manner solving the nonlocal elasticity problem reduces to solving the nonhomogeneous Helmholtz equation.

The nonlocal kernel (8) is the Green function of the diffusion equation in which the parameter t plays a role of "time". This suggests that the stress tensor satisfies the diffusion equation

$$\frac{\partial \boldsymbol{\sigma}_{\rm nl}}{\partial t} - \Delta \boldsymbol{\sigma}_{\rm nl} = 0 \tag{11}$$

under the "initial" condition

$$t = 0 : \quad \boldsymbol{\sigma}_{\rm nl} = \boldsymbol{\sigma}. \tag{12}$$

It should be noted that in fact the parameter t is not the time, but the nonlocality constant. Indicating the Laplace transform with respect to the parameter t by an asterisk we obtain

$$\Delta \boldsymbol{\sigma}^*_{\rm nl} - s \boldsymbol{\sigma}^*_{\rm nl} = -\boldsymbol{\sigma}, \tag{13}$$

where s is the transform variable.

In the case of vanishing body force the Kröner stress function tensor $\boldsymbol{\kappa}$ can be introduced

$$\frac{1}{2\mu}\boldsymbol{\sigma}^*_{\rm nl} = \Delta\boldsymbol{\kappa} + \frac{1}{1-\nu}\left[\boldsymbol{\nabla\nabla}(\mathrm{tr}\,\boldsymbol{\kappa} - (\Delta\mathrm{tr}\,\boldsymbol{\kappa})\mathbf{I})\right] \tag{14}$$

where ν is the Poisson ratio.

The tensor $\boldsymbol{\kappa}$ satisfies the following equation

$$(\Delta - s)\Delta^2\boldsymbol{\kappa}^* = -\boldsymbol{\eta} \tag{15}$$

under the supplementary assumption

$$\boldsymbol{\nabla}\cdot\boldsymbol{\kappa} = 0. \tag{16}$$

In the case of vanishing incompatibility tensor the Boussinesq-Galerkin vector $\mathbf{w}$ can be used

$$2\mu\mathbf{u} = 2(1-\nu)\Delta\mathbf{w} - \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{w}). \tag{17}$$

In this case

$$\boldsymbol{\sigma}^*_{\rm nl} = (\nu\mathbf{I}\Delta - \boldsymbol{\nabla\nabla})\boldsymbol{\nabla}\cdot\mathbf{w} + (1-\nu)\Delta(\boldsymbol{\nabla}\mathbf{w} + \mathbf{w}\boldsymbol{\nabla}) \tag{18}$$

and

$$(\Delta - s)\Delta^2\mathbf{w}^* = -\frac{1}{1-\nu}\mathbf{f}. \tag{19}$$

Another method of solving nonlocal elasticity problems consists in direct integration of the corresponding local elasticity solution. It should be noted that equation (3) is written in invariant tensor form. In Cartesian coordinates integration can be carried out immediately. Otherwise, the nonlocal stress field is determined by transforming coordinates from curvilinear at the running point $\mathbf{x}'$ to the rectangular at this point, shifting tensors to the reference point $\mathbf{x}$, transforming from rectangular coordinates at the point $\mathbf{x}$ to the curvilinear ones at this point, and the subsequent integration over the region occupied by the solid using an appropriate nonlocal modulus. It should be noted that even in the case of simple kernels this methods needs calculation of complicated integrals.

For example, in the case of cylindrical coordinates the following integrals [112] are of fundamental importance:

$$\int_{-\infty}^{\infty} \exp\left[-\frac{(z'-z)^2}{4t} - \xi|z'|\right] \mathrm{sign}z'\,\mathrm{d}z' =$$

$$= -\sqrt{\pi t}\exp\left(t\xi^2\right)\ S(\xi,|z|,t)\,\mathrm{sign}z,$$

$$\int_{-\infty}^{\infty} \exp\left[-\frac{(z'-z)^2}{4t} - \xi|z'|\right] |z'|\,\mathrm{sign}z'\,\mathrm{d}z' =$$

$$= \frac{2}{\xi}\sqrt{\pi t}\exp\left(t\xi^2\right)\ U(\xi,|z|,t)\,\mathrm{sign}z,$$

$$\int_{-\infty}^{\infty} \exp\left[-\frac{(z'-z)^2}{4t} - \xi|z'|\right]\,\mathrm{d}z' = \sqrt{\pi t}\exp\left(t\xi^2\right)\ T(\xi,|z|,t),$$

$$\int_{-\infty}^{\infty} \exp\left[-\frac{(z'-z)^2}{4t} - \xi|z'|\right] |z'|\,\mathrm{d}z' =$$

$$= \frac{1}{\xi}\sqrt{\pi t}\exp\left(t\xi^2\right)\left[2Q(\xi,|z|,t) - T(\xi,|z|,t)\right], \tag{20}$$

The functions $Q(\xi,|z|,t)$, $S(\xi,|z|,t)$, $T(\xi,|z|,t)$, $U(\xi,|z|,t)$ were introduced in [113]. For the sake of convenience we present them here:

$$Q(\xi,|z|,t) = \frac{1}{2}\left(1-2t\xi^2\right)T(\xi,|z|,t) -$$

$$-\frac{1}{2}\xi|z|S(\xi,|z|,t) + \frac{2\xi\sqrt{t}}{\sqrt{\pi}}P(\xi,|z|,t),$$

$$U(\xi,|z|,t) = \frac{1}{2}\xi|z|T(\xi,|z|,t) + t\xi^2 S(\xi,|z|,t),$$

$$S(\xi,|z|,t) = \exp(\xi|z|)\,\mathrm{erfc}\left(\xi\sqrt{t} + \frac{|z|}{2\sqrt{t}}\right) -$$

$$-\exp(-\xi|z|)\,\mathrm{erfc}\left(\xi\sqrt{t} - \frac{|z|}{2\sqrt{t}}\right),$$

$$T(\xi,|z|,t)=\exp(\xi|z|)\operatorname{erfc}\left(\xi\sqrt{t}+\frac{|z|}{2\sqrt{t}}\right)+$$
$$+\exp(-\xi|z|)\operatorname{erfc}\left(\xi\sqrt{t}-\frac{|z|}{2\sqrt{t}}\right),$$
$$P(\xi,|z|,t)=\exp\left(-\xi^2t-\frac{z^2}{4t}\right). \tag{21}$$

It should be noted that these function also appear as inversions of the Laplace transform with respect to the parameter t [114]:

$$\mathcal{L}^{-1}\left\{\exp\left(-\sqrt{\xi^2+s}|z|\right)\right\}=\frac{|z|}{2\sqrt{\pi}t^{3/2}}P(\xi,|z|,t),$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\exp\left(-\sqrt{\xi^2+s}|z|\right)\right\}=\frac{1}{2}\left[S(\xi,|z|,t)+2\mathrm{e}^{-\xi|z|}\right],$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\exp\left(-\sqrt{\xi^2+s}|z|\right)\right\}=\frac{1}{2\xi^2}\left[U(\xi,|z|,t)+(2t\xi^2-\xi|z|)\mathrm{e}^{-\xi|z|}\right],$$

$$\mathcal{L}^{-1}\left\{\frac{1}{\sqrt{\xi^2+s}}\exp\left(-\sqrt{\xi^2+s}|z|\right)\right\}=\frac{1}{\sqrt{\pi t}}P(\xi,|z|,t),$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s\sqrt{\xi^2+s}}\exp\left(-\sqrt{\xi^2+s}|z|\right)\right\}=\frac{1}{2\xi}\left[-T(\xi,|z|,t)+2\mathrm{e}^{-\xi|z|}\right],$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2\sqrt{\xi^2+s}}\exp\left(-\sqrt{\xi^2+s}|z|\right)\right\}=$$
$$=\frac{1}{2\xi^3}\left[Q(\xi,|z|,t)-(1+\xi|z|-2t\xi^2)\mathrm{e}^{-\xi|z|}\right]. \tag{22}$$

The functions $Q(\xi,|z|,t)$, $S(\xi,|z|,t)$, $T(\xi,|z|,t)$, $U(\xi,|z|,t)$ were studied in [113], where their integral representations were derived allowing us to obtain solutions more amenable to numerical quadrature.

Theory of nonlocal elasticity indicates its power in the study of point defects [115-119], straight edge [120] and screw [121, 122] dislocations, straight wedge and twist disclinations [123], circular prismatic and glide dislocation loops [113, 124], circular twist, rotation and wedge disclination loops [125, 126], line cracks [127–130] and volume defects [131]. Additional discussion can be found in the papers [132–137].

4 Conclusions

1. Nonlocal elasticity takes into account interatomic long-range forces. Nonlocal theory reduces to the classical theory of elasticity in the long wave-length limit and to the atomic lattice theory in the short wave-length limit.
2. The nonlocal theory of elasticity makes it possible to apply a unified approach to the study of phenomena at the micro and macro levels.
3. In contrast to the many generalized models that improve the description of only one specific type of defects, in the context of nonlocal theory of elasticity one can consider various types of defects from a single point of view.
4. Nonlocal stresses, in contrast to the local ones, are not singular at the point of defect location.
5. To compute the elastic energy of the defect in a nonlocal medium there is no need to cut the region of its kernel.
6. The kernel of a defect is commensurable with the parameter that characterizes the nonlocality.
7. As a rule, the stresses have a maximum at a certain distance from the defect. The value of this maximum is completely realistic from the physical point of view.

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