# MAGNETIC INTERLAYER COUPLINGIN THE SUPERLATTICES WITH ROUGH INTERFACES

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## INTRODUCTION

Recent progress in low dimensional condensed matter physics revealed that dimensionality has a great influence on the physical characteristics. Many laminar systems like Ag/Cu(001) overlayer or $GaAs/Al_xGa_{1-x}As$ quantum wells and/or superlattices, as the layer thickness decreases, show dimensional crossover from 3D to almost 2D behaviour [1-3]. Generally, the dimension of these systems changes with the monolayer coverage, wire thickness or temperature. In the case of rough interfaces, a nonintegral dimension of the stratified system can be interpreted in terms of fractal geometry (Haussdorff dimension [4]).
The aim of this paper is to study the indirect magnetic interactions in multilayers which show nonintegral dimensionality. We will calculate the RKKY exchange integral between magnetic ions in a metallic system of nonintegral dimension. Having that, we will find expression for the interlayer coupling between two ferromagnetic layers, with rough interfaces, across metallic nonmagnetic spacer.

## RKKY INTERACTION IN A SYSTEM OF FRACTIONAL DIMENSIONALITY

The starting point for any description of metallic-like magnetic systems is the case of dilute alloys, when a few TM or RE ions are immersed in the sea of host conduction electrons. The effective interaction between RE or TM localized moments is mediated via the free electrons. Within perturbative approach, the RKKY interaction between magnetic moments of the magnetic ions ($\vec{\mu}_i$ and $\vec{\mu}_j$) can be written as [5]

$$H(R_{ij}) = \frac{1}{2}A^2\chi(R_{i,j})\vec{\mu}_i \cdot \vec{\mu}_j, \qquad (1)$$

where $\chi(R_{ij})$ is the nonuniform static susceptibility. The explicit form of the $\chi(R_{ij})$ is given by [6]

$$\chi(R_{ij}) = -T\sum_l G(i\omega_l, R)^2 , \qquad (2)$$

where $\omega_l = \pi T(2l+1)$ are the Matsubara frequencies and the electronic Green's function is

$$G(i\omega,\vec{R}) = \int \frac{d^\alpha \vec{k}}{(2\pi)^2} \frac{e^{i\vec{k}\cdot\vec{r}}}{i\omega - \varepsilon_k}, \qquad (3)$$

with $\varepsilon_k$ being the free electron spectrum. We find that the RKKY exchange integral $J(r)$ in a $\alpha D$ system is given by

$$J(r) = J_o r^{2-\alpha}(J_{\alpha/2-1}(k_F r)Y_{\alpha/2-1}(k_F r) + J_{\alpha/2}(k_F r)Y_{\alpha/2}(k_F r)), \quad (4)$$

with $J_\nu(x)$ and $Y_\nu(x)$ being the Bessel and Neumann functions [7].

## ROUGHNESS OF THE INTERFACES

The interfaces can exhibit roughness, which can modify the interlayer coupling. Till now, only the special case of correlated steps on a surface has been considered [8]. In the following, we will present an alternative approach based on the concept of fractals. The surface/interface roughness often exhibits a self-affine structure [9,10] and different scaling behaviour can be found as a function of thickness and lateral length scale *L*. In this case, the interface is characterized by the mean-square average roughness (height-correlation function) $\xi(L)$ :

$$\xi(L) = \left[\frac{1}{L}\sum(z_j - \rho)^2\right]^{1/2}, \qquad (5)$$

and the scaling of the roughness parameter is given by $\xi(L) \approx L^\beta$, with $\beta$ being a fraction.

In our approach, we will consider a trilayer in which the outer ferromagnetic layers are separated by a nonmagnetic of average thickness $\rho$. We assume that at least one of the interfaces ($F_1^\beta$ or $F_2^\beta$) is self-similar and its dimension equals $2+\beta$ (with $0<\beta<1$). As usually, we assume that the interlayer coupling between layers is mediated by the free charge carriers of the central layer. The restraint conditions imposed (by the interface potentials) on the motion of free particles cause that the *k*-space of their eigenstates shows fractional (spectral) dimensionality. This means that the magnetic interaction between two ionic moments, that belong to different magnetic layers, is described by the formula (4).

The magnetic interlayer coupling can be obtained by summing contributions from all pairs of moments $\vec{\mu}_i$ and $\vec{\mu}_j$, with *i* and *j* running over $F_1^\beta$ and $F_2^\beta$. The interlayer coupling energy $E_{1,2}$ per unit measure of the interface can be expressed by the formula

$$E_{1,2} = I_{1,2}^{\alpha,\beta}\cos\Theta_{1,2}. \qquad (6)$$

The interlayer exchange coupling integral $I_{1,2}^{\alpha,\beta}$ is given by

$$I_{1,2}^{\alpha,\beta} = J_0 \sum_{j\in F_2^\beta} \chi_\alpha(R_{0j}). \qquad (7)$$

In view of Eq.(5), we find that volume element of $F_2^\beta$ behaves as $dV_2^\beta \approx |\vec{r}-\vec{\rho}|^{1+\beta} d|\vec{r}-\vec{\rho}|$. Thus, Eq. (7) can be rewritten as

$$\sum_{j\in F_2^\beta} \chi_\alpha(R_{0j}) \to \int_{F_2^\beta} |\vec{r}-\vec{\rho}|^{1+\beta} \chi(k_F r) d|\vec{r}-\vec{\rho}|. \qquad (8)$$

The term $|\vec{r}-\vec{\rho}|^{1+\beta}$ can be expanded in power series of $|\vec{r}|$ and $|\vec{\rho}|$. Since $\beta$ is a fraction, we should make use of the fractional version of the Taylor formula [11]:

$$f(x) = \sum_{j=0}^{n-1} \frac{(D^{\alpha+j}f)(a)}{\Gamma(\alpha+j+1)}(x-a)^{\alpha+j} + R_n(x). \qquad (9)$$

The symbol $(D^{\alpha+j}f)(x)$ denotes the derivative of fractional order $j+\alpha$ of the real function $f(x)$. The fractional calculus is a powerful tool in theoretical studies of systems which show fractional dimensionality [1-3,12,13]. The Riemann-Louville diffintegral $DI^\alpha$ is defined as follows [11,14,15]:

$$(I_0^\alpha f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^\infty \frac{1}{(x-t)^{\alpha-1}} f(t)dt, \qquad (10)$$

and is a fractional counterpart of derivative of fractional order $D^\alpha = d^\alpha/dx^\alpha$ (for $\alpha > 0$) or fractional integral $I^\alpha$ (for $\alpha < 0$).

For the case of large separations, the leading term of the interplane coupling can be calculated if we limit ourselves to the first term of expansion, i.e., $|\vec{r}-\vec{\rho}|^{1+\beta} \approx |\vec{r}|^{1+\beta}$. Thus, if we make use of the identities [7 p. 111 Eq. 65 and p. 20 Eq. 56], the interlayer exchange integral (7) and (8) reads

$$I_{1,2}^{\alpha,\beta} = I_0 \int_0^\infty dt \int_\rho^\infty r^{2-\alpha+\beta} J_{\beta-1}(2k_F r \,\mathrm{ch}t) dr. \qquad (11)$$

Integral (11) can be calculated strictly only for some values of $\alpha$ and $\beta$. In the case of arbitrary $\alpha$ and $\beta$, only approximate formulae for the interlayer coupling parameter $I_{1,2}^{\alpha\beta}$ can be obtained. To calculate $I_{1,2}^{\alpha\beta}$, let us recall the identity fulfilled by fractional diffintegrals of the Bessel functions [11 p. 48]:

$$(DI_{0^+}^\lambda)[x^{\mu/2} J_\mu(\sqrt{x})] = 2^\lambda x^{(\mu+\lambda)/2} J_{\mu+\lambda}(\sqrt{x}). \qquad (12)$$

Having identity (12), we can integrate over $\rho$ in Eq. (11) using the formula for fractional diffintegration by parts [11 p. 42]:

$$\int_a^b \phi(x) I_{a^+}^\alpha \psi(x) dx = \int_a^b \psi(x) I_{b^-}^\alpha \phi(x) dx. \qquad (13)$$

If we account that $I_0^\lambda x^\mu \approx x^{\mu+\lambda}$ [11 p.140 ], Eq. (11) can be reduced to

$$I_{1,2}^{\alpha\beta} = I_0(\rho)^{\mu+1} \int_0^\infty (\mathrm{ch}t)^{-2-\beta/2+\alpha} J_\mu(2k_F \rho \,\mathrm{ch}t) dt. \qquad (14)$$

In the case of arbitrary $\alpha$ and $\beta$, the integration over variable $t$ in Eq. (14) cannot be performed in a direct way. However, if we have $\alpha = 2+\beta/2$, Eq. (14) takes the following form:

$$I_{1,2}^{\alpha\beta} = I_0(\rho)^{\nu+1} J_{\nu/2}(k_F\rho) Y_{\nu/2}(k_F\rho), \qquad (15)$$

where $\nu = 1+\beta/2$. The condition $\alpha = 2+\beta/2$ seems to be very restrictive. However, some layered systems exhibit continuous dimensional crossover when external conditions (e.g., temperature or magnetic field) are changed. This means that in such systems, this peculiar condition can always be fulfilled. In the case of arbitrary $\alpha$ and $\beta$, the integration over *t* in Eq. (14) cannot be performed directly. Fortunately, with the help of fractional analysis, we can transform the integral (14) to a more simple form, which allows us to draw some conclusions concerning the interlayer coupling. Setting $x = (2k_F\rho \,\mathrm{ch}t)^2$ and using the identity

$$\frac{d^\lambda}{dx^\lambda} = (2\rho \,\mathrm{ch}t)^{-2\lambda} \frac{d^\lambda}{d(k_F^2)^\lambda}, \tag{16}$$

which results from the definition of fractional derivatives, we can rewrite Eq. (14) in the following form:

$$I_{1,2}^{\alpha\beta} = I_0(\rho)^{\alpha-1}(k_F)^{-\nu}\int_0^\infty dt \frac{d^\lambda}{d(k_F^2)^\lambda}((k_F)^\nu J_\nu(2k_F\rho \,\mathrm{ch}t)) \,. \tag{17}$$

If we change order of differentiation and integration, the integration over *t* leads to

$$I_{1,2}^{\alpha\beta} = J_0(\rho)^{\alpha-1}\frac{d^\lambda}{d(k_F^2)^\lambda}[(k_F)^\nu J_{\nu/2}(k_F\rho)Y_{\nu/2}(k_F\rho)] \,, \tag{18}$$

with $\lambda = \alpha - 2 - \beta/2$ and $\nu = 1+\beta/2$.

Result (18) represents the leading term of the interlayer coupling constant (i.e., term which dominates at large $\rho$). The other term of expansion (8)–(9) can be calculated in a similar way as the result (18). However, since we have assumed that the interface $F_2^\beta$ is self-affine, it is evident that our calculations are valid for the superlattices with relatively thick spacer layers. In this case, it suffices to study the properties of the leading term (18). Both expressions (15) and (18) show oscillatory behaviour determined by the oscillations of the Bessel functions $J_{\nu/2}(k_F\rho)$ and $Y_{\nu/2}(k_F\rho)$. Similarly, as in the case of systems with integral dimension, the oscillation period is directly related to the $2k_F$ wave vector.

The fact that expressions (15) and (18) are analytical functions of $\alpha$ and $\beta$ allows us to discuss the effect of dimensionality on the interlayer coupling. Detailed analysis indicates that interlayer coupling constant $I_{1,2}^{\alpha,\beta}$ is strongly influenced by changes of the spectral dimension of the spacer layer. In the case of $\alpha = 3$, $\beta = 0$, the envelope function falls off with the spacer thickness *d* as $I_{1,2} \approx d^{-2}$, while for $\alpha = 2$ it decays as $I_{1,2} \approx d^{-1}$. Thus the strength of the interlayer coupling varies during dimensional crossover. This indicates a new way, in which properties of magnetic multilayers can be manipulated. In many layered system, the spectral dimension changes (dimensional crossover) when some external parameters like, e.g., temperature or magnetic field are varied. Thus, by proper choice of the external fields, we are able to influence the strength of interlayer coupling, an effect important in the construction of the electronic devices. The interface roughness $\beta > 0$ acts in a similar way as decreasing $\alpha$.

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