# SPACE-CHARGE-PERTURBED AND SPACE-CHARGE-LIMITED CURRENT TRANSIENTS IN DISORDERED SOLIDS

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## ABSTRACT

The formulae determining space-charge-perturbed currents (SCPCs) and transient space-charge-limited currents (TSCLCs) in disordered solids have been rederived. The calculations are based on the simplified multiple-trapping (MT) model, under the assumption of strongly non-equilibrium trapped carrier distribution. The obtained formulae concern mainly the initial current decay and the effective transit time of the carriers. The analytical results given here, as well as those obtained previously, are compared with the results of Monte Carlo simulation of SCPCs and TSCLCs for the exponential trap distribution.

## INTRODUCTION

The dispersive character of excess carrier transport (see, e.g. [1]) is a specific feature of many disordered solids. According to the multiple-trapping (MT) model, this phenomenon is due to equilibration of charge carriers over energetically distributed trapping states. The dispersive transport is most commonly studied using the time-of-flight method. The sample is sandwiched between two electrodes with a constant voltage applied, and the carriers are generated by a light pulse. The carrier motion in the sample induces a current transient in the measuring circuit. The majority of experiments concerns the case of negligible field distortion in the sample (so called small signal regime). Otherwise, two limiting cases can be distinguished. The carriers are either a) generated in the sample by a short light pulse or b) continuously generated or injected into the sample, which results in the vanishing electrical field at the nearer electrode. The corresponding current transients are respectively called: a) space-charge-perturbed currents (SCPCs) and b) transient space-charge-limited currents (TSCLCs) [2].

Up to now, only some approximate analytical [3-5] and numerical [6] solutions of MT transport equations, including space-charge effects, were obtained. In [3-4] and [5] the expressions for dispersive SCPCs and TSCLCs were derived under the assumption of strongly non-equilibrium and quasi-equilibrium trapped carrier distribution, respectively. The validity of some results seems, however, to be questionable, since they are based on the concept of effective time-dependent carrier mobility, and thus ignore the carrier packet dispersion. In this communication, the formulae for dispersive SCPCs and TSCLCs are rederived in a somewhat different way, using strongly non-equilibrium approach. The corresponding formulae obtained in the framework of the quasi-equilibrium approach are considered briefly in the Appendix. The results given here, and those obtained in [3-5], are compared with the results of Monte Carlo simulation of SCPCs and TSCLCs for the exponential trap distribution.

## THEORY

### Transport equations

The MT carrier transport in the presence of space-charge effects is described by the continuity equation:

$$j(t) = e\mu_0 n(x,t)E(x,t) + \kappa\kappa_0 \frac{\partial E(x,t)}{\partial t}, \qquad (1)$$

the Poisson's equation:

$$\frac{\partial E(x,t)}{\partial x}=\frac{e}{\kappa\kappa_0}\left[n(x,t)+n_t(x,t)\right], \tag{2}$$

and an equation governing the trapping/detrapping processes. The meaning of the notations being used is: $x$ and $t$: the space and time variables; $n(x,t)$ and $n_t(x,t)$: the free and trapped carrier densities; $E(x,t)$: the electric field strength; $j(t)$: the total current density; $e$: the elementary charge; $\mu_0$: the free carrier mobility; $\kappa$: the dielectric constant; $\kappa_0$: the permittivity of free space. In the following calculations, we shall assume that $n(x,t) << n_t(x,t)$.

The simplified MT equation in the strongly non-equilibrium approximation reads [7,8]:

$$\frac{\partial}{\partial t}\left[\frac{n_t(x,t)}{\Phi(t)}\right]\approx n(x,t), \tag{3}$$

where the function $\Phi(t)$ is defined as:

$$\Phi(t)\approx C_t\int_{\varepsilon_0(t)}^{\infty}N_t(\varepsilon)\mathrm{d}\varepsilon. \tag{4}$$

Here, $\varepsilon$ is the energy variable, $N_t(\varepsilon)$ is the trap density per energy unit, $C_t$ is the carrier capture coefficient, and $\varepsilon_0(t) = kT\ln(1.8\nu_0 t)$ is the demarcation level (with $k$ the Boltzmann constant, $T$ – the sample temperature, and $\nu_0$ – the frequency factor). The energy $\varepsilon_0(t)$ is the minimum trap depth for which the carrier release is negligible up to the time $t$. One can show that Eq. (3) describes two processes: carrier capture in the energy region $\varepsilon > \varepsilon_0(t)$ and carrier release from the traps of depth $\varepsilon_0(t)$. The considered approximation is adequate, if the trap density decreases with energy sufficiently slowly.

As a model trap distribution, leading to the dispersive carrier transport, we chose the exponential distribution

$$N_t(\varepsilon)=\frac{N_{tot}}{kT_c}\exp\left(-\frac{\varepsilon}{kT_c}\right), \tag{5}$$

where $N_{tot}$ is the total trap density and the characteristic temperature $T_c$ determines the decay rate of the trap density.

Current transients in initial time region

Let us consider at first the SCPC and TSCLC transients in the initial time region, for times shorter than the effective carrier transit time $\tau_e$. Integrating Eq. (1) over the sample thickness $L$ and using Eq. (3) we get:

$$j(t) = \frac{e\mu_0}{L}\int_0^L \frac{\partial}{\partial t}\left[\frac{n_t(x,t)}{\Phi(t)}\right]E(x,t)\mathrm{d}x. \qquad (6)$$

For the initial stage of the carrier transport Eq. (6) can by approximated by:

$$j(t) = \frac{e\mu_0}{L}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{\Phi(t)}\right]\int_0^L n_t(x,t)E(x,t)\mathrm{d}x. \qquad (7)$$

This follows from the fact, that omitted term is proportional to the integral:

$$\int_0^L \frac{\partial n_t(x,t)}{\partial t}E(x,t)\mathrm{d}x,$$

which vanishes for $t \ll \tau_e$ in the small signal case, and is generally expected to be negligible. Making use of Eq. (2), Eq. (7) can be rewritten as:

$$j(t) = \frac{\kappa\kappa_0\mu_0}{2L}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{\Phi(t)}\right]\left[E^2(L,t) - E^2(0,t)\right]. \qquad (8)$$

On the other hand, for $t \ll \tau_e$ from Eq. (1) we also get:

$$j(t) = \kappa\kappa_0\frac{\mathrm{d}E(L,t)}{\mathrm{d}t}. \qquad (9)$$

In the case of SCPC, the relationship $E(L,t) - E(0,t) = \sigma_0/\kappa\kappa_0$ holds for $t \ll \tau_e$ ($\sigma_0$ – the surface density of injected charge), whereas in the case of TSCLC the boundary condition has the form of $E(0,t) = 0$. Comparing Eqs. (8) and (9) we obtain the differential equations for $E(L,t)$, which are easy to solve. Then, the current transients may be calculated from (8) or (9).

The resulting formulae for dispersive SCPC and TSCLC are:

$$j(t) = j_0\tau_0\left(1-\frac{\eta}{2}\right)\frac{\mathrm{d}}{\mathrm{d}t}\left\{\exp\left[\frac{\eta}{\tau_0\Phi(t)}\right]\right\}, \quad (10)$$

$$j(t) = j_0\tau_0\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{1-1/2\tau_0\Phi(t)}\right], \quad (11)$$

respectively, where the current density $j_0 = \kappa\kappa_0 V^2/L^3$, $\tau_0 = L^2/\mu_0 V$ is the free carrier time-of-flight, and $\eta = \sigma_0 L/\kappa\kappa_0 V$ is the relative efficiency of carrier generation ($0 < \eta \leq 1$).

Current transients in final time region

The formula determining SCPC transient for $t >> \tau_e$ was derived in [4,5]. In the final time region the carrier retrapping is negligible. Therefore, the measured current is proportional to the emission current from traps of depth $\varepsilon_0(t)$. The corresponding formula reads:

$$j(t) = \frac{j_0\tau_0^2}{2}\left[-\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t}\right]. \quad (12)$$

The formula for final TSCLC decay in the strongly non-equilibrium case was obtained in [3]. The total charge in the sample is then constant and the decrease in free carrier density is due to progressive carrier thermalization. The result is of the form:

$$j(t) = \frac{9j_0}{8}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{\Phi(t)}\right]. \quad (13)$$

In fact, the values of numerical coefficients in the above equations depend on the spatial distribution of trapped carriers for $t >> \tau_e$. The coefficients 1/2 and 9/8 in Eqs. (12) and (13) were obtained under the assumptions $n_t(x,t) \approx$ const, and $n_t(x,t) \propto x^{-1/2}$, respectively.

Effective carrier transit times

In the case of dispersive transport there is no sharp transition between the initial and final courses of current transients (cf. next Section). For this reason, the effective transit time $\tau_e$ of the carriers cannot be defined uniquely. In [4-5] the transit time $\tau_e$ was calculated in similar manner as for non-dispersive transport [2], using the concept of effective, time-dependent carrier mobility. It was concluded that for SCPC the time $\tau_e$ becomes somewhat shorter with increasing injected charge.

We think that a more proper way is to calculate $\tau_e$ from the intersection points of the extrapolated curves, representing the current transients for $t \ll \tau_e$ and $t \gg \tau_e$ . In the case of SCPCs, comparing Eqs. (10) and (12), one gets the transcendental equation:

$$(1-\eta/2)\exp[\eta/\tau_0\Phi(\tau_e)] = \tau_0^2\Phi^2(\tau_e)/2, \tag{14}$$

the solution of which is:

$$\tau_0\Phi(\tau_e) = c(\eta). \tag{15}$$

The function $c(\eta)$ depends very weakly on the injection efficiency $\eta$: $c(0) = 2^{1/2} \approx 1.414$, $c(0.545) \approx 1.455$ (the maximum value) and $c(1) \approx 1.422$. Therefore, the time $\tau_e$ is almost independent of the injected charge.

In the case of TSCLCs, using Eqs. (11) and (13), one obtains a similar formula:

$$\tau_0\Phi(\tau_e) = 1.5. \tag{16}$$

Since the function $\Phi(t)$ monotonically decreases with time, the transit time is slightly shorter than for SCPCs.

Exponential trap distribution

For the exponential distribution of traps (5) the formula (4) yields:

$$\Phi(t) \approx \frac{1}{\tau_t}(1.8\nu_0 t)^{-\alpha}, \tag{17}$$

where $\tau_t = 1/C_t N_{tot}$ is the mean carrier trapping time, and $\alpha = T/T_c$ is the dispersion parameter. Then, according to the previous formulae, the current transients behave as follows:

$$j(t) \propto t^{-(1-\alpha)} \exp\left(\eta A t^{\alpha}\right), \quad t << \tau_e, \tag{18}$$

$$j(t) \propto t^{-(1+\alpha)}, \quad t >> \tau_e, \qquad \text{(SCPC)} \tag{19}$$

and:

$$j(t) \propto t^{-(1-\alpha)}\left(1 - Bt^{\alpha}\right)^{-2}, \quad t << \tau_e, \tag{20}$$

$$j(t) \propto t^{-(1-\alpha)}, \quad t >> \tau_e. \qquad \text{(TSCLC)} \tag{21}$$

Here, $A = \tau_t(1.8\nu_0)^{\alpha}/\tau_0$, and $B = A/2$. It is seen that the initial decay rate of SCPC diminishes with increasing injection efficiency $\eta$, whereas the final decay rate remains unchanged. Also, the TSCLC decays in the initial time region somewhat slower than the function $t^{-(1-\alpha)}$. The effective carrier transit time fulfils the relationship:

$$\tau_e \propto \left(L^2 / V\right)^{\alpha}, \tag{22}$$

both in the case of SCPC and TSCLC. The analogous formulae, with slightly different numerical coefficients, can also be obtained in the quasi-equilibrium approximation (see Appendix).

## NUMERICAL RESULTS

In order to verify the accuracy of given formulae, we carried out numerical calculations of SCPCs and TSCLCs for the exponential trap distribution. The method of Monte Carlo simulation of the MT carrier transport in the presence of space-charge effects has been described in [9]. The calculations were performed for several values of dispersion parameter $\alpha$ and, in the case of SCPCs, for two values of injection level, $\eta = 0.1, 1.0$. The value $\eta = 0.1$ corresponds roughly to the small-signal mode whereas $\eta = 1.0$ is the highest possible injection level. The carrier number in the simulations ranged from 500 to 50000. In the figures, the analytical and numerical results are showed by solid lines and points, respectively, and the carrier transit times are marked by the arrows. The current transients are presented in terms of the relative current intensity $I(t)/I_0$; $I(t) = j(t)S$, $I_0 = j_0S$, where $S$ is the sample cross-section area.

Figure 1 shows the current transients calculated for of $\alpha$ = 0.3. The analytical results are obtained using the non-equilibrium formulae given in previous Section. In the case of SCPCs, the influence of space charge on the form of current transient is very unsignificant, due to large dispersion of the carrier packet. This can be seen in Figure 2, which presents the time evolution of the trapped carrier density, as well as of the

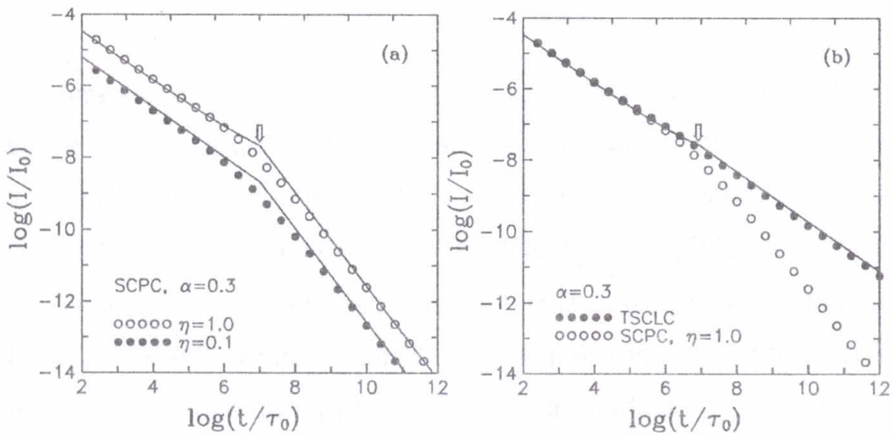


Figure 1. SCPC and TSCLC transients calculated for exponential trap distribution with $\alpha = 0.3$. $\tau_t/\tau_0 = 5\cdot10^{-3}$, $\nu_0\tau_0 = 1.0$.

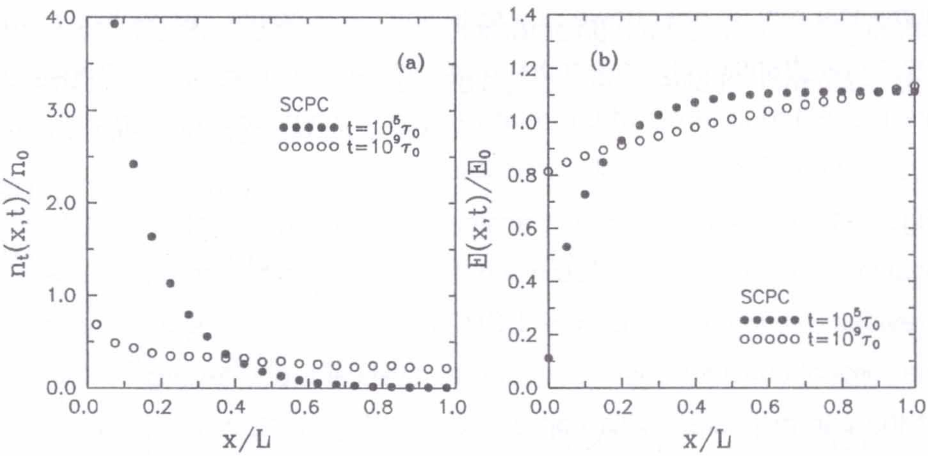


Figure 2. Time evolution of the trapped carrier density and the electric field in the case of SCPC. $\eta$ = 1.0, other parameters are the same as in Fig. 1. $n_0 = \sigma_0/eL$, $E_0 = V/L$.

resulting field in the sample. Figure 3 presents the time dependence of the total charge $Q(t)$ in the sample. The formulae for $Q(t)$ may be easily derived from the equations given in Section 2. It should be noted that the normalized plots of $Q(t)$ vs $t$ for SCPCs, corresponding to $\eta$ = 0.1 and $\eta$ = 1.0, almost superimpose. This proves that the transit time $\tau_e$ is independent of the carrier injection level. Figure 4 shows the SCPCs and TSCLCs obtained for higher value of dispersion parameter, $\alpha$ = 0.7. The analytical results correspond to quasi-equilibrium approximation (see Appendix). Here, the influence of injected charge on the form of SCPC curves is more distinct than for $\alpha$ = 0.3 (Fig. 1). Finally, Figure 5 shows the current transients for intermediate value of $\alpha$,

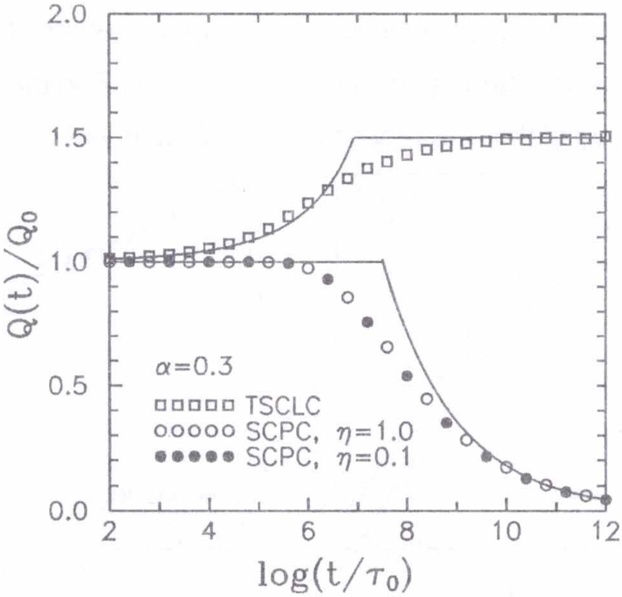


Figure 3. Time dependence of the total charge $Q(t)$ in the sample; $Q_0$ = $Q(0)$. The calculation parameters are the same as in Fig. 1.

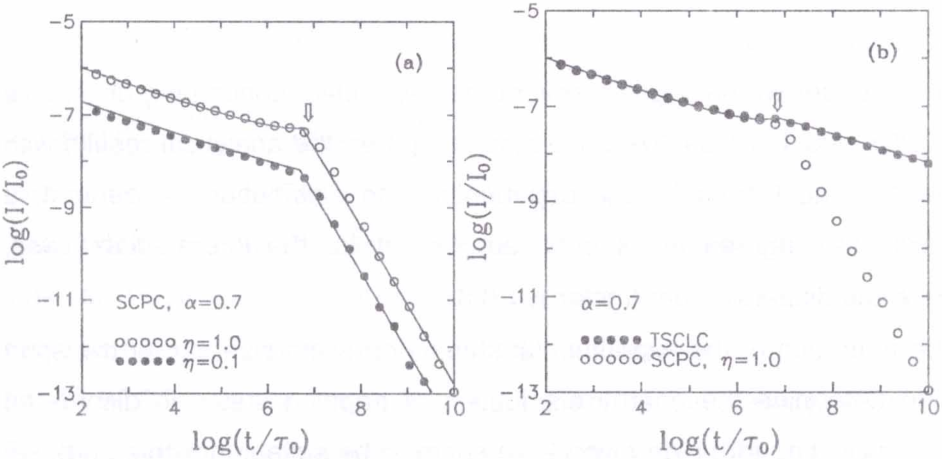


Figure 4. SCPC and TSCLC transients obtained for exponential trap distribution with $\alpha$ = 0.7. $\tau_t/\tau_0 = 2{\cdot}10^{-5}$, $\nu_0\tau_0$ = 1.0. $\alpha$ = 0.5. In this case, the analytical curves computed from the both non-equilibrium and quasi-equilibrium formulae coincide.

Basing on the results given in Figures 1, 4 and 5, one can conclude that the non-equilibrium approximation yields good results for $\alpha \leq 0.3$, whereas the quasi-equilibrium one – for $\alpha \geq 0.7$. For the intermediate values of $\alpha$ none of these approaches is satisfactory enough.

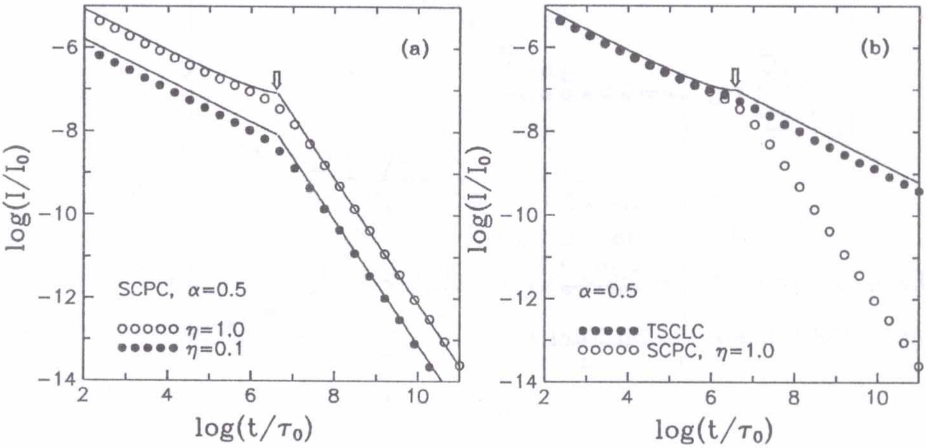


Figure 5. SCPC and TSCLC transients calculated for exponential trap distribution with $\alpha = 0.5$. $\tau_t/\tau_0 = 3\cdot10^{-4}$, $\nu_0\tau_0 = 1.0$.

CONCLUSIONS

In this paper we have given several new formulae, concerning dispersive SCPCs and TSCLCs. We have also compared the analytical results with the numerical ones for the exponential trap distribution. In general, a satisfactory agreement is obtained, except for the intermediate case, when the dispersion parameter $\alpha = 0.5$.

The main aim of the experimental studies on dispersive carrier transport is to determine the energy distribution of trapping states in disordered materials. The formulae given here seem to be suitable for this purpose. In particular, the final decay of SCPC is described, according to Eqs. (12) and (4) by the simple formula $j(t) \propto N_t[\varepsilon_0(t)]/t$, indpendently of the amount of injected charge.

APPENDIX

The quasi-equilibrium approach is based on the assumption that the majority of captured carriers occupies relatively shallow traps of depth $\varepsilon < \varepsilon_0(t)$, being in thermal equilibrium with free carriers. The approximation is justified, if the trap density falls-off sufficiently quickly with increasing energy $\varepsilon$. The corresponding kinetic equation is [10]:

$$n_t(x,t) \approx n(x,t)/\Theta(t), \qquad \text{(A1)}$$

where:

$$\Theta^{-1}(t) \approx C_t \int_0^{\varepsilon_0(t)} N_t(\varepsilon)\tau_r(\varepsilon)\mathrm{d}\varepsilon. \qquad \text{(A2)}$$

Considering the initial time region, $t << \tau_e$, and performing similar calculations as in Section 2, one gets the formula:

$$j(t) = \frac{\kappa\kappa_0 \mu_0 \Theta(t)}{2L}\left[E^2(L,t) - E^2(0,t)\right], \qquad \text{(A3)}$$

which is analogous to Eq. (8). Therefore, the expressions determining quasi-equlibrium current transients for $t << \tau_e$ may be obtained from those given in Section 2 by the substitutions:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{\Phi(t)}\right] \Rightarrow \Theta(t), \qquad \frac{1}{\Phi(t)} \Rightarrow \int_0^t \Theta(t')\mathrm{d}t'.$$

One can prove that above rule holds also in the case of Eqs. (13) and (16), which describe the ultimate current decay and the effective transit time for TSCLCs. The resulting formulae are essentially identical with those derived in [5], except for the formula defining $\Theta(t)$.

As the SCPCs are concerned, the final current decay is always given by Eq. (12), since the carrier retrapping is then negligible. The calculation of the effective transit time encounters some difficulties. We claim that the transit time should not depend on the amount of injected charge. Then the formula obtained for the small signal mode [10] should be valid:

$$\int_0^{\tau_e} \Theta(t)dt = 2^{-1/2}\tau_0. \qquad \text{(A4)}$$

For the exponential trap distribution (5) the function:

$$\Theta(t) \approx \frac{(1-\alpha)\tau_t \nu_0}{\alpha}\left(1.8\nu_0 t\right)^{-(1-\alpha)} \tag{A5}$$

The resulting expressions for the current transients and the effective carrier transit times have form analogous to Eqs. (18) and (20-22), except for the values of some numerical coefficients.

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