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HOW HUNGARIAN TEACHERS THINK ABOUT COMBINATORICS AND ITS TEACHING

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Abstract

Teachers' attitudes and beliefs have a fundamental influence on the way of teaching and the efficiency of learning. This topic was also brought up in the didactic research in recent years, in connection with mathematics teachers as well.– To the best of our knowledge, however, a study that investigates this specific area (combinatorics) was not included. As combinatorics has a special role in connecting the development of mathematical thinking and teaching different areas, our short-time Project (Didactics Grant of the Hungarian Academy: Complex Mathematics Education in the 21st Century) focused on the teaching of combinatorics in Hungary. Our aim was to examine some aspects of this field in connection with Tamás Varga's method and also teachers' beliefs towards combinatorics with the help of a questionnaire. The main results of this survey are discussed below.

1. Introduction

In the last fifty years of Hungarian mathematics education the ideas of Tamás Varga's "Complex Mathematics Education Experiment" have played an important role, and the results of the experiment were realized primarily in the first eight grades, and partly in grades 9 to12. Our research group's[1] goal is to rethink traditions based Tamás Varga's teaching methods, and to

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implement his ideas in secondary schools to a greater extent, first considering the teaching of combinatorics in grades 5 to 12[2].

Our choice for combinatorics as a topic had several reason. Firstly, combinatorics has a specific role in the teaching of mathematics, because it is important in the teaching of thinking methods, and it features in several areas of mathematics. Secondly, a number of combinatorial problems (e.g. ordering), even if at different levels, can be taught to different grades so combinatorics is particularly suitable for comparing the problem solving ability of different age groups see [12]. What is more, this topic also had priority in Tamás Varga's experiment and finally in his teaching method.

Several factors affect the effectiveness of teaching in school. for instance, the mathematics teacher's personal attitude concerning the subject (area) and its teaching has high priority. The inquiry into teachers' beliefs towards combinatorics is part of the research of our research group.

2. Theoretical background

2.1. **Tamás Varga and his teaching method.** After spending a short period working as a schoolteacher, Tamás Varga, a Hungarian mathematics and physics teacher, worked in the National Institute of Education in Budapest, then was a lecturer at Eötvös Loránd University (ELTE) between 1951 and 1967. His thoughts presented at UNESCO's Research Symposium on Mathematics Education (1962, Budapest) resonated well with the reform efforts initiated by the so-called Sputnik Shock in the Western World. The trend represented by him and his colleagues, set off in 1964 and summarized in their Complex Mathematics Teaching Experiment, can be taken as a kind of response to the weaknesses of the New Math.

Tamás Varga had excellent international contacts, among others with Hans Freudenthal. He published and gave lectures in German, English and French as well.

In Varga's method the *genetic approach* based on discovery and prior knowledge and experiments of the students was decisive, see [3].

In 1978 the new mathematics curriculum for primary schools (grades 1 to 8) was introduced on the grounds of Varga's above mentioned experiment, which resulted in a complete change in mathematics teaching see, [13].

Tamás Varga emphasized the importance of the teaching of combinatorics and probability for instance see, [24], [25], [26], [27]. While in the

[2]Now, according to our results in the short-time Project we obtained the opportunity to go on with the research of Tamás Varga's method and extend our research group within the Content Pedagogy Research Program of the Hungarian Academy – 2016–2020, Complex Mathematics Education in the 21st Century – Improving Mathematical Thinking Based on the Most Recent Research Results

curriculums before 1978 the teaching of combinatorics started at the age of 17 years, in the new concept it started as early as in the first class, at the age of 6 years.

In the new mathematics curriculum for primary schools (1978), combinatorics was not a separate unit because combinatorial thinking is basically important in the different subject areas of mathematics with a special role in probability, see [4]. Problems with more or less combinatorial content appeared at every grade in the primary school.

Although the activity of Tamás Varga and his colleagues has had a great impact on Hungarian mathematics teaching since the seventies of the last century, in the last decades this effect seems to be diminishing. The long-term aim of the work of our research group is the extensive investigation and right-on adaptation of Varga's method. First we had the opportunity to begin this work on the subject of the teaching of combinatorics within a didactics grant for a year of the Hungarian Academy of Sciences in 2015. In the relatively short time of our project we tried to prepare and realize our investigation, but the circumstances did not allow us to obtain precise results in some cases.

2.2. **Teacher's attitudes and beliefs in connection with their teaching.** In the definition of Richardson, see [18], "*attitudes and beliefs are a subset of a group of constructs that name, define, and describe the structure and content of mental states that are thought to drive a person's actions. Other constructs in this set include conceptions, perspectives, perceptions, orientations, theories, and stances. (p.3)".*

In a further distinction, concentrating now on a definition in the teaching, attitude means the "affective" side and belief can be considered as a *major construct of interest in studying teachers' ways of thinking and classroom practices (p.3).*

Richardson emphasizes *that the difference between the two terms remained somewhat unclear in the empirical literature (p.3).* For example, Pajares suggests that among others attitudes (and values, preconceptions, theories, and images) are really "beliefs in disguise" see [17].

Considering secondary mathematics teachers' domain specific belief systems, the term *belief is used as an individual's personal conviction*, see [6], we will use belief in the same meaning in the following – and the organisation of beliefs can be considered as a system of individual ideas (belief system). The teacher's ideas (beliefs) appear in the context of "attitude theory" see, [10] (Theorie von Einstellungen, see, [8]).

Hannula states that beliefs about mathematics should be examined in the context of cognition, emotion and motivation. He emphasizes the cognitive aspect of belief as he mention it in the cognitive domain in his three dimensional model see [11].

Although, according, for example, to Pajares see [17] a total separation between the professional knowledge and teaching beliefs is problematic, some authors mention that it is worth to analyze these areas separately see [19], [14].

Teachers' conceptions (beliefs) influence teaching practice. However, the specific relationship between teacher's beliefs and their teaching practice is not known see [15], [22].

It should be emphasized that the teachers' beliefs have a decisive influence on their students' beliefs and what is more, the image about mathematics is largely decided in the school see [10]. So, teachers' beliefs in connection with a mathematical subject (e.g. algebra or geometry) may have a decisive role in the relevant student's beliefs which also emphasize the importance of our investigation, although this question is not directly considered in the present research.

Teacher's ideas have been studied in many practical contexts, for example, on the use of modeling tasks see [16], [1] or on problem solving. It seems that teachers can think differently about mathematics when teaching mathematics in different areas (e.g calculus, geometry, probability or statistics) see [6]. There are teachers' belief research results related e.g. to the teaching of statistics and calculus [5], [6], [7], but the authors are unaware of the existence of such studies specifically on combinatorics.

3. Methodology

In this study, we do not examine teachers' beliefs about mathematics in general, only teachers' ideas about one area, i.e. combinatorics; although it is obvious that the teacher's basic orientation (e.g. static or process-oriented way of thinking, see [10] influences the teaching of this topic as well.

In order to obtain an image of Hungarian mathematics teachers' ideas concerning combinatorics, we prepared a questionnaire consisting of statements on four main issues with four statements each. The four main issues are the following: *1. How positive is the attitude towards combinatorics. 2. Combinatorics problems are difficult. 3. The formulas and standard problems are important in teaching combinatorics. 4. Knowing combinatorics is useful in other areas of life as well.*

The questionnaire – asking for grading statements – is in our opinion capable of measuring emotional attitudes (not only cognitive attitudes), even if only to a limited extent.

While filling out the questionnaire, the teachers had to classify the 16 statements belonging to four topics on a 5-level Likert scale.

In order to better appraise individual teachers' attitudes and orientation about teaching combinatorics, two extra tables were added to the previous 16 questions. In these tables teachers declared in the system of specific topics (Algebraic expressions, Equations, Functions, Sets, Logic, Graphs, Combinatorics, Coordinate geometry, Plane geometry, Sequences, Statistics, Number Theory, Word problems, Spatial Geometry, Trigonometry, Probability) how much they like teaching combinatorics and how successful they are in teaching combinatorics. When specifying the topics, we gave as narrow areas as possible, aiming to gain a more accurate picture. For example, graphs were not included in combinatorics (graphs are mentioned separately) and plane and spatial geometry were also handled separately.

When we assembled the questionnaire, it was important that

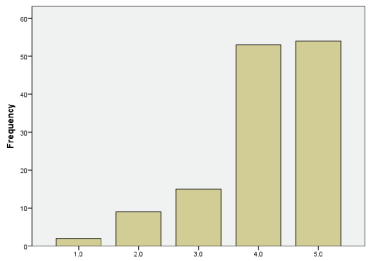
- The questionnaire should not be too long (since we wanted it to be filled out by many teachers)
- Statements should be kept in the shortest possible form and should be as clear as possible in order to get responses that can be evaluated easily
- Statements should be objective, wording should not be only positive (if possible, the wording should not influence the choice of the participant)
- All topics should have the same number of statements (so it can be evaluated easily)
- Preferably there should be statements of varying strength within one topic (so it can be evaluated easily)

The full questionnaire is available for viewing at the authors.

We analyzed the responses and examined some questions in connection with the results. In this study the following questions are in the focus:

- Do teachers like teaching combinatorics?
- Do teachers teach combinatorics willingly?
- How do they relate to some of the fundamental problems in teaching combinatorics (usefulness and problems of the subject, the question of teaching standard problems)?

3.1. **The conditions of the study.** The people who filled out the questionnaire (either on paper or electronically) were teachers who taught in Hungarian middle schools or secondary schools. They could fill out the test anonymously and voluntarily. Until the evaluation date, 156 teachers filled out the test, from which 136 were fully filled out, (31 male and 102 female, and 3 not declared). Analysis was done on these 136 questionnaires. This



		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1.0	2	1.5	1.5	1.5
	2.0	9	6.7	6.8	8.3
	3.0	15	11.1	11.3	19.5
	4.0	53	39.3	39.8	59.4
	5.0	54	40.0	40.6	100.0
	Total	133	98.5	100.0	
Missing	System	2	1.5		
Total		135	100.0		

FIGURE 1. Answers based on the statement "I like teaching combinatorics".

relatively small number set some limitations in our results. Most of the participants had been in teaching for at least 15 to 25 years.

3.2. **The mathematical background.** In the analysis, one important aspect was to assign an objective capability parameter to each teacher. To do so, an extended version of the Rasch model was used as a tool, which assigns a parameter to the individuals and also to the questions. This is important because this way we also find out which issues are more important and less important and which responses indicate similar capabilities.

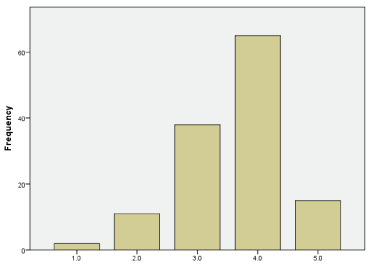
The parameters for the teachers were based on the four statements mentioned before, by which with the help of descriptive statistics and ANOVA we examined how the teachers' opinions related to one another's on the following: usefulness and difficulty of combinatorics, how positive their attitudes are towards the topic and how important they think standard problems are. Of the results of this study we pick only the ones relevant to this paper. (The detailed statistical evaluation of the teacher's answers is available for viewing at the authors in Hungarian.)

4. RESULTS

The results come from the short-time investigation and can be the basis of further and more precise investigations. The following part concentrates on the questions (see Methodology, section 3) and the limits of the evaluation of the teachers' answers are also considered. Looking at the statements' basic statistics we recorded the distribution of the answers.

For example, from the answers to the first question we see that most participants enjoy teaching combinatorics (Figure 1) and most of the time they think they are successful (Figure 2).

Based on the basics statistics we also found that participants think combinatorics is useful and not only the standard problems. The substantial majority of teachers think that combinatorics is not only for individuals



		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1.0	2	1.5	1.5	1.5
	2.0	11	8.1	8.4	9.9
	3.0	38	28.1	29.0	38.9
	4.0	65	48.1	49.6	88.5
	5.0	15	11.1	11.5	100.0
	Total	131	97.0	100.0	
Missing	System	4	3.0		
Total		135	100.0		

FIGURE 2. Answers based on the statement "I teach combinatorics successfully".

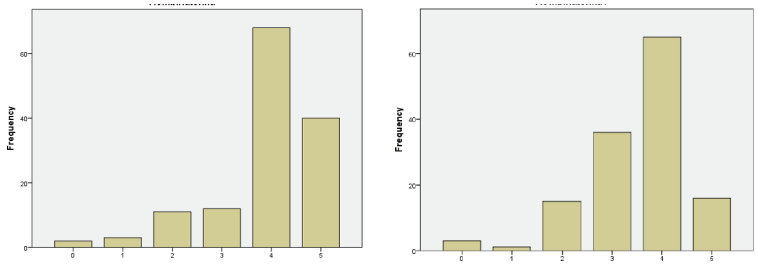


FIGURE 3. How willingly (left figure) and how successfully (right figure) mathematics teachers teach combinatorics "in the context of other areas".

who excel in mathematics, but it is important for every pupil. This is interesting because the majority of the teachers (even though less than before) think that if someone is not good at problem solving, then he/she will not be able to handle combinatorics problems.

Furthermore, we examined how much teachers like to teach combinatorics, compared to 16 other areas of mathematics (see earlier).

It turned out that teachers ***enjoy teaching the most*** *the following four areas: equations, functions, sequences and algebraic expressions, while the four areas which* ***they least enjoy teaching*** *are graphs, logic, trigonometry and probability.* Although we should note that the average was above 3 on the scale of 0-5 (5 is the best rate) for every topic.

The four areas which are ***taught most successfully*** *are equations, functions, sets and statistics, while the ones which are* ***taught least successfully*** *are trigonometry (the only one with an average of below 3), graphs, logic and coordinate geometry.*

Looking at combinatorics compared to other areas of mathematics (Figure 3) the picture is a little different to the one on Figures 1 and 2 where

the teachers had to make statements about combinatorics alone. They do not like it so much compared to other topics but the successfulness of their teaching of combinatorics is considered similarly as before.

We found the following results using the modified *Rasch model in our* four areas.

a) How important formulas and standard problems are in combinatorics: In the course of the person parameters fit test several teachers stood out, and the person- item map show that statements 4 *(It is good in teaching combinatorics that the standard problems get the emphasis.)* and 8 *(It is good for the student to realize clearly which type of combinatorial task is to be solved (permutation, combination or variation), this way he/she can easily find the appropriate term)* stand out of the model. However, statement number 4 is the one which *divides the teachers:* The item's characteristic curves show the same. We can also see that *there is strong agreement among the teachers that the primary goal of teaching combinatorics is not the teaching of the standard problems.*

b) How useful the teachers think combinatorics is: The items fit well into the model, statement 15 stood out the most: *how important combinatorics is in terms of model development.* Item parameters are homogeneous according to the Andersen test, but the person parameters are not, according to the Wald test. The person parameter fit test show that teachers fit better than at the previous area (attitudes towards standard problems), but there are some outstanders here too.

We can see on the person- item map that the items are shifted to the left as *the vast majority of the teachers find the teaching of combinatorics useful.* We can see at the item parameter fit and also on the map, that question 15 does not fit perfectly into the model, and also it can be observed that the individuals do not fit in either.

c) How difficult teachers think combinatorics problems are: While examining the item parameters the fit was not perfect, but all four items were usable. The Andersen test did not show that the items are homogeneous, it left out two statements in the Rasch test because of inappropriate responses, the Wald test is not interpretable either. It can be concluded from the person parameter test and also from person and item maps that several teachers do not fit the model and neither do two statements.

We can see on the person and item maps that *most teachers do not agree with statement 7 (combinatorics is mostly for people who excel in mathematics). Statement 12 (combinatorics problems are often difficult to teachers) is the most divisive statement in this area.*

d) Teachers' attitudes towards combinatorics: Interestingly, the first statement (*how much the teacher likes combinatorics*) stood out the most from the rest. But even this statement fell under the usable category.

The Wald test and the Andersen likelihood test could not be used again as it was in the case of the items 1 and 2. At the same time it cannot be said based on items 3 and 4 either that the item or person parameters would be homogeneous. It can be concluded from the person and item map that *the vast majority of teachers have a positive attitude towards combinatorics because for statement 1 most participants of the test gave high values. Statement 11 (whether the teacher teaches combinatorics successfully), split the teachers and many gave low scores. They similarly gave only few points for statement 2 (how much they bring combinatorics problems into their other lessons).*

5. Discussion

It can be said that the majority of responding teachers have a positive attitude towards combinatorics. They teach combinatorics willingly, but the extent of their willingness varies compared to the other topics. At first sight this may be because of the appearance of the more popular topics as well as the separation of the topics. However, it can be seen that although the teaching of graphs is less popular in Hungary, and this area has been listed as a separate topic, it did not improve further the popularity of combinatorics. It may be that several teachers did not consider graphs as part of combinatorics and this way mentioned separately the topic could not influence the popularity of combinatorics.

In case of combinatorics and also of other topics, the success of teaching does not reach the extent to which they like to teach these topics – according to the responses to the second table of the survey. Combinatorics was not among the best here either.

The opinion about "Teaching formulas and standard problems" divides teachers but most of the teachers consider "learning combinatorics" useful; this can be partly related to the opinion which was chosen by most of them, "the basic aim of teaching this topic is not the teaching of standard problems".

Although it is generally believed by the teachers that combinatorics is not only for those with better performance, still this topic is often labeled as complicated.

Due to the circumstances of completing the survey (i.e. the respondents voluntarily presented their opinions on combinatorics) the responses cannot be considered representative among Hungarian mathematics teachers. In spite of the positive attitude, our findings pointed at several contradictory

(or seemingly contradictory) phenomena in connection with teachers' beliefs and attitudes towards combinatorics and its teaching, which may have their causes mainly in the present state of Hungarian combinatorics teaching. Comparative (similar) and more precise studies among teachers in other countries would be needed to examine this statement.

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