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Realistic Approach Towards Mathematics Teaching

Jan Melichar

„Nothing exists in the reason which has not passed through the senses."
J. A. Komenský

Currently four possible approaches are being presented in the methodology of mathematics teaching:

1. Mechanical – The mechanical approach towards teaching corresponds with understanding the teaching as a system of reactions. By means of drill we can programme the pupil as a computer towards exercising arithmetic, algebraic and geometrical operations and solving problems which can be classified according to certain signs and solved further according to certain patterns.

2. Structuralistic – The structuralistic approach towards teaching can be demonstrated by two examples: the traditional geometry organized on a basis of axiomatic construction and the so called modern mathematics based on a theory of sets and logic. For pupils a structured world of sets and relations has been created.

3. Empirical – The empirical approach comes out of the needs of practise and it is supposed to serve those needs. In the teaching process the experience of the pupils is used, but the pupils are not lead to systematic and rational utilization of this experience.

4. Realistic – The real stimuli from the constantly enlarging world of the pupil are used as well. The pupil becomes a re-discoverer of mathematics which stimulates and develops his ability.

The realistic mathematics education (RME) comes out of the principle that teaching mathematics means constructing mathematics by the own steps of the pupils from informal approaches connected with reality towards something acceptable as formal mathematics.

Let us have a look at an example of the realistic approach used in the subject methodology of mathematics in the third year in the curricula for primary school teachers.

In the 17th to the 19th century the opinion crystallized gradually that the basis of mathematics is the study among objects and the operations exercised with them. In the 20th century mathematics became a summary of theories each of them studying a complex of objects characterized only by precisely formulated, but otherwise completely random relations among them.

As an example from real life students can be shown the following set of objects characterized only by precisely formulated relations among them as observed in a real situation.

Since 1919 four main blood groups have been distinguished in medicine: A, B, AB, 0 (A Czech doctor Jan Jansk¸ contributed greatly to their discovery.) Blood transfusions can only be performed in the following cases:

1. The donor and the recipient have identical blood groups.

2. Provided the donor has a O blood group, the recipient can have any blood group.

3. Provided the recipient has an AB blood group, the donor can have any blood group.

These relations between donors and recipients were observed in real situations and proved in numerous researches.

If we mark a set of blood groups $M = \{A, B, AB, 0\}$ and the relation a person with the blood group x can donate blood to a person with the blood group $y$as xRy, we can write down the laws of blood transfusion as follows:

1. $(\forall x \in M)\ \ xRx$

2. $(\forall x \in M)\ \ 0Rx$

3. $(\forall x \in M)\ \ xRAB$

4. The relations 1) – 3) are valid.

Based on the information 1) – 4), without being a doctor and having to perform a research, a mathematician can verify whether the relation:

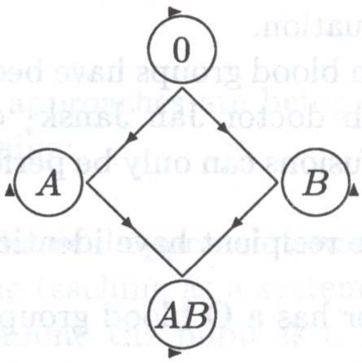
$$(\forall x, y, z \in M)[(xRy) \wedge (yRz)] \Rightarrow (xRz)$$

is valid or not, which means, whether the relations of blood transfusion are transitive.

The information 1) – 4) present the so called axiomatic definition of a given structure. We have reached this definition from a very concrete simple description of relations observed in a real situation. These concrete descriptions of relations observed in real situations are called axioms.

I show the students that similar method is adopted in mathematics. From a simple concrete description of relations in a real situation we come to an abstract concept which we study with the intention to utilize the results of the study in practise.

The information 1) – 4) in a set of blood groups can easily be demonstrated in a graphic form, for example with a knot graph:



We can quite rightly ask ourselves a question what the relation of the mathematic model to reality is. Are the models of theory also models of reality? The models express reality to the same extent to which the axioms expressed the qualities of reality. The practise is the criterion of whether the abstractions exercised reflect those sides of reality which are essential for solving a certain range of problems.

The blood groups can also be used in mathematical statistics for identifying the frequency of genotypes and fenotypes in population.

A chart of genotypes and fenotypes in population:

Genotypes	Fenotypes	Frequencies of genotypes	Frequencies of fenotypes
AA	A	0,08	0,41
$A0$		0,33	
BB	B	0,02	0,18
$B0$		0,16	
AB	AB	0,08	0,08
00	0	0,33	0,33

tab. 1

Before childbirth it is possible with a certain probability to identify the blood group of the child on a basis of the blood group of the father and mother.

Simply speaking, the child „adopts one letter from the genotype of the father and one letter from the genotype of the mother". In mathematical terms: if the mother has a blood genotype group xx and the father has a blood genotype group xy, then the child can have a blood genotype group xx with the probability 0,5 or the blood genotype group xy with the probability 0,5. Obviously, the frequency of genotypes

$$f_{xx} + f_{xy} = 1.$$

Let us form a chart of genotypes and fenotypes of the father, mother and child:

		mother					
		A		*B*			
		AA 0,08	*A*0 0,33	*BB* 0,02	*B*0 0,16	*AB* 0,08	00 0,33
father	*AA* 0,08	*AA*...1	*AA*...0,5 *A*0...0,5	*AB*...1	*AB*...0,5 *A*0...0,5	*AA*...0,5 *AB*...0,5	*A*0...1
A	*A*0 0,33	*AA*...0,5 *A*0...0,5	*AA*...0,25 *A*0...0,5 00...0,25	*AB*...0,5 *B*0...0,5	*AB*...0,25 *A*0...0,25 *B*0...0,25 00...0,25	*AA*...0,25 *AB*...0,25 *A*0...0,25 *B*0...0,25	*A*0...0,5 00...0,5
	BB 0,02	*AB*...1	*AB*...0,5 *B*0...0,5	*BB*...1	*BB*...0,5 *B*0...0,5	*AB*...0,5 *BB*...0,5	*B*0...1
B	*BB* 0,16	*AB*...0,5 *A*0...0,5	*AB*...0,25 *A*0...0,25 *B*0...0,25 00...0,25	*BB*...0,5 *B*0...0,5	*BB*...0,25 *B*0...0,5 00...0,25	*AB*...0,25 *A*0...0,25 *BB*...0,25 *B*0...0,25	*B*0...0,5 00...0,5
AB	0,08	*AA*...0,5 *AB*...0,5	*AA*...0,25 *A*0...0,25 *B*0...0,25 *AB*...0,25	*AB*...0,5 *BB*...0,5	*AB*...0,25 *A*0...0,25 *BB*...0,25 *B*0...0,25	*AA*...0,25 *AB*...0,5 *BB*...0,25	*A*0...0,5 *B*0...0,5
00	0,33	*A*0...1	*A*0...0,5 00...0,5	*B*0...1	*B*0...0,5 00...0,5	*A*0...0,5 *B*0...0,5	00...1

tab. 2

For example from the chart we see: If the father has a blood genotype group *AA* and the mother has a blood genotype group *A*0, the child cannot have blood genotype groups *BB*, *B*0, *AB* and 00. The probability of occurrence of these groups is zero. If the father has a fenotype blood group *A* and the mother also has a group *A*, the child cannot have fenotype groups *B* or *AB*.

It is interesting to calculate the probability of frequency of genotypes in a new generation. We will come to the identical frequency with the previous population. For example the frequency of genotypes AA in population is $0,08$. Let us count the frequency of genotypes in a new generation. These are independent phenomena.

Calculation:

$$\begin{aligned}
f(AA)\cdot f(AA)\cdot 1 &= 0,08\cdot 0,08\cdot 1 = 0,0064\\
f(AA)\cdot f(A0)\cdot 0,5 &= 0,08\cdot 0,33\cdot 0,5 = 0,0132\\
f(AA)\cdot f(AB)\cdot 0,5 &= 0,08\cdot 0,08\cdot 0,5 = 0,0032\\
f(A0)\cdot f(AA)\cdot 0,5 &= 0,33\cdot 0,08\cdot 0,5 = 0,0132\\
f(A0)\cdot f(A0)\cdot 0,25 &= 0,33\cdot 0,33\cdot 0,25 = 0,027225\\
f(A0)\cdot f(AB)\cdot 0,25 &= 0,33\cdot 0,08\cdot 0,25 = 0,0066\\
f(AB)\cdot f(AA)\cdot 0,5 &= 0,08\cdot 0,08\cdot 0,5 = 0,0032\\
f(AB)\cdot f(A0)\cdot 0,25 &= 0,08\cdot 0,33\cdot 0,25 = 0,0066\\
f(AB)\cdot f(AB)\cdot 0,25 &= 0,08\cdot 0,08\cdot 0,25 = 0,0016
\end{aligned}$$

Let us sum up the frequencies of genotypes AA in a new generation, i.e.:

$$0,0064 + 0,0132 + 0,0032 + 0,0132 + 0,027225 + 0,0066 + 0,0032 +$$
$$+ 0,0066 + 0,0016 = 0,081225 \doteq 0,08.$$

In a new generation the frequency of genotype AA is $0,08$ in a round number and is identical with the frequency of the genotype AA in the previous population.

References

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