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# ECONOMIC-MATHEMATICAL MODEL FOR RISK MANAGEMENT OF SALES CHANNELS OF SMALL ENTERPRISES

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## Abstract

The importance of economic and mathematical study of small enterprises is emphasized as an innovative element of economic development. The potential risks appearing during choice of distribution channel of small enterprises are structured, actions for minimizing the risks are indicated. The need to identify the most threaten risks is emphasized, for this purpose the ABC-analysis is recommended. The mathematical tools are developed to minimize risks when choosing sales channels of small enterprises. The model of optimization management of the choice of sales channels based on minimizing risk is proposed. As an example, Svitovyr LLC (Lviv, Ukraine) was considered. The optimal production distribution between the direct (exhibition sales, internet sales) and indirect (specialized hypermarket, distribution network) sales channels was proposed.

## 1. Introduction

Global trends and positive experience of leading European countries give grounds to assert that small business activity is stirred up, has state support, and investment in small business is emphasized as a key area of economic development. In the realization of its economic activity, small business is faced with the choice of effective sales channels influenced by many factors, including risk management and the possibility of timely and flexibly respond to the changing situation. At the same time, identifying the most threatening risks becomes ripe for effective and timely counteraction. The important element of risk management in small business consists in risk reduction through the development of the mathematical tools ensuring optimization of sales channels selection under minimal risks.

## 2. Potential risks

Current economic conditions of small enterprises are characterized by increased uncertainty and risks which are difficult to predict; at the same time they are an integral part of firm functioning. Effective risk-management often becomes the determining factor of success for many small enterprises that use several sales channels in their work. In the conditions of uncertainty, happy choice of mathematical tools for risk minimization acquires particular importance.

Studying and summarizing the literature on the problem of mathematical support of risk analysis [1], [9], [11] gives reason to believe that today there are many approaches and ways to solve this problem, but in practice they are rarely used because of complex mathematical tools, the long duration of collecting primary data for formulation of the original problem as well as the lack of professional experts who would solve the tasks. From the viewpoint of practical applications, the method of ABC-analysis is simple enough to identify groups of phenomena that carry the greatest impact on the state of the object under consideration. To identify risks which should be counteracted in the first place, the approach of Italian scientist Vilfredo Pareto proposed in 1906 will be used. He observed that 80% of the risks are caused by 20% of potential threats.

This principle forms the basis of the sum method, tangent method, empiric method and other modifications of the ABC-analysis, which involve a division of objects into groups A, B, C (or, if necessary, into smaller ones). The objects of the group A have the greatest influence on risks; the objects of the group B exert a medium influence, and the objects of the group C (all other risks) exerts the smallest influence. Many scientists turn their attention to this method (see [2], [4]–[7], [13], [15], among others). It was found that the use of ABC-analysis for identifying the most influential risks (risks with the highest probability of occurrence) will help to identify potential threats and take urgent actions to minimize or eliminate them. This method is simple in practical application, which is a strong argument in favor of its use by small enterprises, where the number of employees is small and the involvement of external experts is expensive.

Now we determine the risk classification based on the structure of potential risks arising in the selection of sales channels by small enterprises (Figs. 1–3).

Interpreting the obtained results, we recommend to prevent the first two types of risks (economic and investment), belonging to the group A. Eliminating or minimizing the impact of these risks will ensure the total threats liquidation by almost 80%.

Having defined the most significant risks appearing during selection of sales channels, we will draw our attention to key indicators which characterize the risk. For economic and investment risks, the sales channel profitability can be considered as such an indicator which may be determined

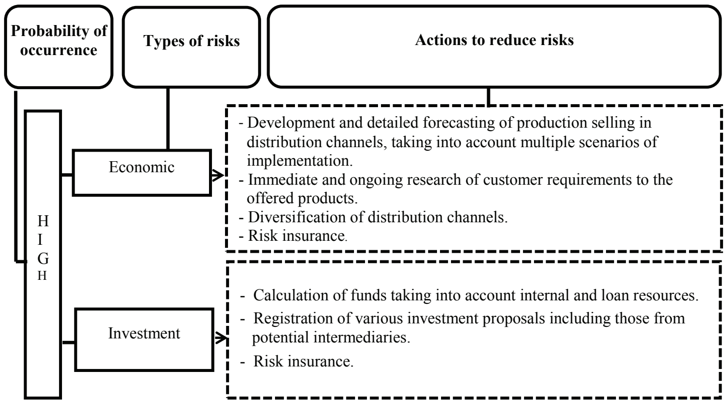


Figure 1. High risks.

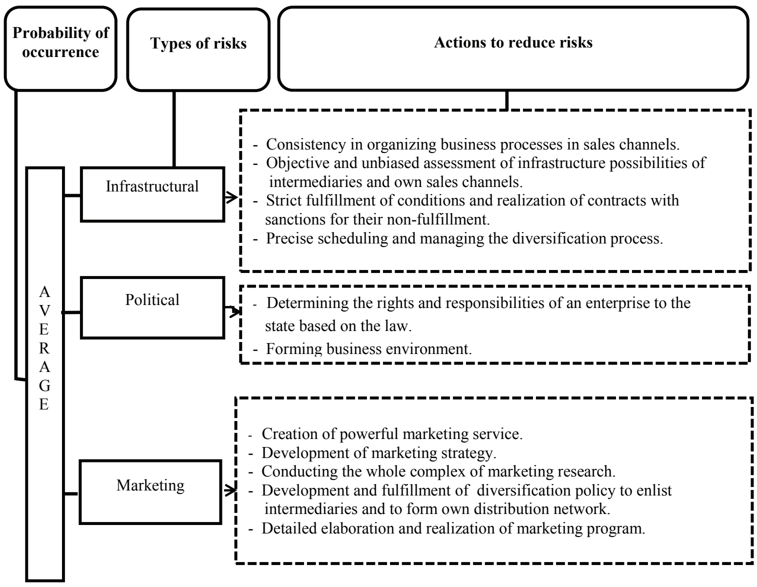


Figure 2. Average risks.

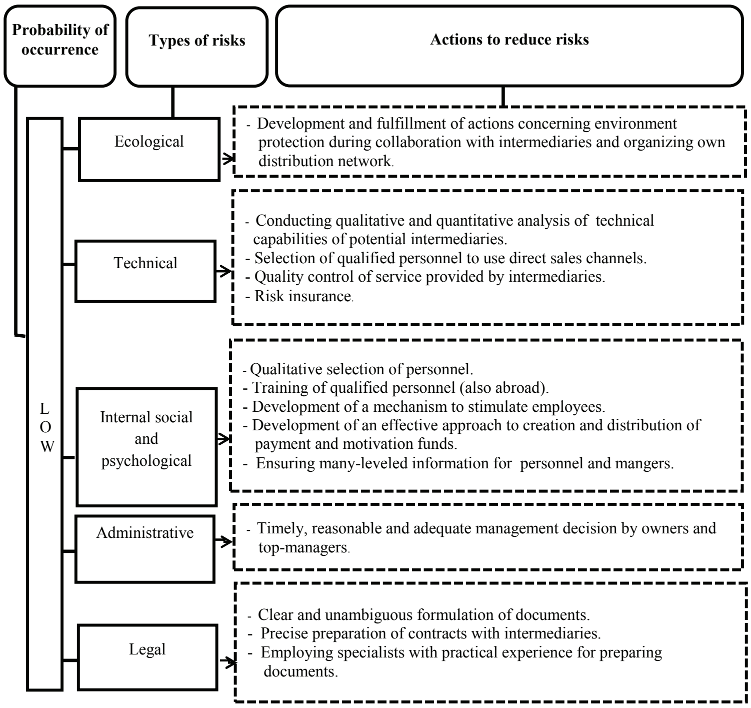


Figure 3. Low risks.

based on the experience of cooperation with the sales channel or based on the statistical data. This approach ensures consideration of the past experience as an accented lever to prevent risk. The current practice of small enterprises shows that their finances as a rule are based on credits. Hence, for an enterprise it is very important to minimize the risks connected with the loss of funds. In the economic literature, this approach is called the strategy "First Security" [10], [14].

To identify the best combination of sales channels of small enterprise, the "portfolio" theory of Harry Markowitz [3], [8] will be used and adapted for optimization of production distribution. The use of this theory allows us not only to compare the riskiness of sales channels but also to determine the risk level under the established profitability of each of the analyzed distribution channels. In this case, the profit level per sale unit for at least five periods creates the necessary input data which permits us to calculate the averaged profit. The "Portfolio" Markowitz theory is based on the principles of risk minimization and expresses the mathematical relationship between risk and profitability. After collecting the primary information concerning the profitability per unit in each of the analyzed sales channels, we calculate the values of pairwise covariation which reflect the relations between profitability of compared distribution channels:

$$\operatorname{cov}(P_i, P_j) = \frac{1}{N-1} \sum_{k=1}^{N} \left(P_{i(k)} - r_i\right) \left(P_{j(k)} - r_j\right), \quad i = 1, \ldots, n, \quad j = 1, \ldots, n, \tag{1}$$

where $P_{i(k)}$ is profitability of the selected assortment item in the $i$-th sales channel within the span of some period, $r_i$ denotes the mean value of profitability of every sales channe, $N$ is the number of years.

The next stage of sales channels optimization on the basis of risk minimization consists in creation of forming a matrix of pairwise covariations of profitability of production unit in each sales channel of an enterprise. This matrix is symmetric with respect to the main diagonal:

$$\mathbf{A}(\operatorname{cov}) = \begin{pmatrix} A_{11} & A_{12} & \ldots & A_{1n} \\ A_{21} & A_{22} & \ldots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & \ldots & A_{nn} \end{pmatrix}, \tag{2}$$

where $A_{ij} = \operatorname{cov}(P_i, P_j)$. After this the inverse matrix $\mathbf{A}(\operatorname{cov})^{\mathbf{-1}}$ is calculated.

The essence of the model of optimal allocation of product sales between distribution channels based on risk minimization consists in finding such a ratio of product which will minimize the risks of using a specific set of sales channels [12]. To do this, we calculate the mean squared deviation based on consideration of the percentage ratio of product distribution between sales channels:

$$\sigma = \mathbf{X} \cdot \mathbf{A}(\operatorname{cov})^{\mathbf{-1}} \cdot \mathbf{X^T}, \tag{3}$$

where $\mathbf{X}$ is a vector with components $x_i$, $\mathbf{X}^T$ is the transposed vector, $x_i$ denotes the part of the production distributed using the $i$-th sales channel, $\mathbf{A}(\operatorname{cov})^{\mathbf{-1}}$ is a matrix inverse to the covariance matrix.

The final stage of problem formulation involves the construction of the objective function and constraints based on the Markowitz model (the mean squared deviation reflects the risk level of the sales channels):

$$\sigma = \mathbf{X} \cdot \mathbf{A}\,(\mathrm{cov})^{\mathbf{-1}} \cdot \mathbf{X}^{\mathbf{T}} \to \min, \tag{4}$$

$$\sum_{i=1}^{n} x_i = 1, \tag{5}$$

$$x_i \geq 0, \quad i = 1, \ldots, n. \tag{6}$$

To illustrate the proposed method, we present the information on profitability of single-phase transformer (Table 1) in direct (exhibition sales, internet sales) and indirect (specialized hypermarket, distribution network) sales channels of Svitovyr LLC (Lviv, Ukraine) during 2010–2014 [12].

Table 1. The values of profit per production unit (UAH) for the single-phase transformer in the sales channels of Svitovyr.

| | Exibition Sales | Internet Sales | Specialized Hipermarket | Distribution Network |
|---|---|---|---|---|
| 2010 | 117.50 | 126.26 | 127.75 | 109.00 |
| 2011 | 124.44 | 162.97 | 154.22 | 105.78 |
| 2012 | 132.15 | 170.16 | 178.40 | 129.65 |
| 2013 | 175.27 | 172.35 | 187.45 | 133.80 |
| 2014 | 161.13 | 152.46 | 166.71 | 101.42 |
| The mean profit value per production unit | 142.10 | 156.83 | 162.91 | 115.93 |

In the considered case, the covariance matrix takes the form:

$$\mathbf{A}\,(\mathrm{cov}) = \begin{pmatrix} 619.6 & 236.0 & 437.6 & 132.5 \\ 236.0 & 353.4 & 398.6 & 168.4 \\ 437.6 & 398.6 & 542.1 & 231.9 \\ 132.5 & 168.4 & 231.9 & 217.3 \end{pmatrix}. \tag{7}$$

The inverse matrix is calculated as

$$\mathbf{A}\,(\mathrm{cov})^{\mathbf{-1}} = \frac{\mathbf{1}}{\mathbf{10^{12}}} \begin{pmatrix} 8.61 & 12.38 & -17.85 & 4.21 \\ 12.38 & 34.41 & -38.02 & 6.35 \\ -17.85 & -38.02 & 49.59 & -12.57 \\ 4.21 & 6.35 & -12.57 & 10.53 \end{pmatrix}. \tag{8}$$

The objective function which should be minimized is written as:

$$\sigma = (x_1, x_2, x_3, x_4) \cdot \begin{pmatrix} 8.61 & 12.38 & -17.85 & 4.21 \\ 12.38 & 34.41 & -38.02 & 6.35 \\ -17.85 & -38.02 & 49.59 & -12.57 \\ 4.21 & 6.35 & -12.57 & 10.53 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

or

$$\begin{aligned} \sigma \;=\; & 8.61x_1^2 + 24.76x_1x_2 - 35.70x_1x_3 + 8.42x_1x_4 \\ & + 34.41x_2^2 - 76.04x_2x_3 + 12.70x_2x_4 + 49.59x_3^2 \\ & - 25.14x_3x_4 + 10.53x_4^2 \to \min \end{aligned} \tag{9}$$

with the constraints

$$x_1 + x_2 + x_3 + x_4 = 1\,, \tag{10}$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0\,. \tag{11}$$

The convexity property of a quadratic form ensures that any local minimum must be a global minimum. A quadratic optimization problem is convex if and only if the inverse covariance matrix in the objective function is positively defined, i.e. its eigenvalues are positive. In our case, the characteristic polynomial of the inverse covariance matrix

$$\lambda^4 - 103.14\lambda^3 + 1271.36\lambda^2 - 3866.86\lambda + 2234.67 = 0 \tag{12}$$

has the following roots

$$\begin{aligned} \lambda_1 = 0.75304 > 0, \quad \lambda_2 = 3.49805 > 0, \\ \lambda_3 = 9.48931 > 0, \quad \lambda_4 = 89.3996 > 0. \end{aligned} \tag{13}$$

Hence, the objective function is positively defined.

The problem is solved using the Lagrange multipliers: to find the minimum of the function

$$\begin{aligned} L \;=\; & 8.61x_1^2 + 24.76x_1x_2 - 35.70x_1x_3 + 8.42x_1x_4 + 34.41x_2^2 \\ & -76.04x_2x_3 + 12.70x_2x_4 + 49.59x_3^2 - 25.14x_3x_4 + 10.53x_4^2 \\ & - \lambda\,(x_1 + x_2 + x_3 + x_4 - 1) \to \min\,. \end{aligned} \tag{14}$$

The extremum conditions are:

$$\frac{\partial L}{\partial x_1} = 0, \qquad \frac{\partial L}{\partial x_2} = 0, \qquad \frac{\partial L}{\partial x_3} = 0, \qquad \frac{\partial L}{\partial x_4} = 0 \tag{15}$$

or

$$
\begin{aligned}
17.22x_1 + 24.76x_2 - 35.70x_3 + \ \ 8.42x_4 - \lambda &= 0, \\
24.76x_1 + 68.82x_2 - 76.04x_3 + 12.70x_4 - \lambda &= 0, \\
-35.70x_1 - 76.04x_2 + 99.18x_3 - 25.14x_4 - \lambda &= 0, \\
8.42x_1 + 12.70x_2 - 25.14x_3 + 21.06x_4 - \lambda &= 0.
\end{aligned} \tag{16}
$$

From system (16) we get:

$$x_1 = 0.7108\lambda, \quad x_2 = 0.5784\lambda, \quad x_3 = 0.8044\lambda, \quad x_4 = 0.3747\lambda \tag{17}$$

with

$$\lambda = 0.4051. \tag{18}$$

Hence, the optimal production distribution between the sales channels will be the following:

$$x_1 = 0.2880, \qquad x_2 = 0.2343, \qquad x_3 = 0.3259, \qquad x_4 = 0.1518, \tag{19}$$

i.e. 28.80% for Exhibition Sales, 23.43% for Internet Sales, 32.59% for Specialized Hypermarket, and 15.18% for Distribution Network.

## 3. Final remarks

We have identified and classified risks arising in small businesses. It was found that the choice of sales channels of small enterprises is connected with the problem of optimizing the combination of distribution channels. As a result, the mathematical model has been formulated to identify the optimal ratio of sales channels based on risk minimization, which is especially important for small businesses that have a high level of dependence on credit. The use of this model will reduce financial expenditures of small businesses when selecting channels of product distribution.

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