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Equations with Parameter Solved in Cabri Geometry

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Abstract:

In this contribution a method for graphical solving any quadratic equation as well as a method for graphing quadratic functions by means of the Cabri Geometry is shown. The goal is to provide high school's students with the visual idea how parameter in both equations and functions is doing.

The notion of a variable is one of the most fundamental in mathematics from elementary school through high school to college. This concept is so important that its invention meant a turning point in the history of mathematics. However, students have problems with the concept of a variable, the reason could be because in mathematics variables are used in many different manners.

One of the difficulties appears when students deal with both a variable and a parameter within one function. The purpose of this article is to consider some of the difficulties that can be created by this fact.

Basic problem can be as follows. Find the solution of the quadratic equation with the parameter m.

Students should distinguish between quantities representing single values, which are called constants, and quantities representing many values, which we call variables. A quantity which value is unchanged is called a constant, a quantity which may assume an unlimited number of values we call a variable. Further, they should know how to work with the parameter – is it a constant or a variable?

The best way how to answer this question is to explain the whole problem (i. e. the task to find the solution of the quadratic equation with the parameter) geometrically in the coordinate plane. Look-out, however. There is no use to present even many static pictures and graphs to explain these concepts, to answer our question. On the other hand, such a computer geometry system as Cabri Geometry allows the dynamic visualizations to help students to understand the problem discussed above.

Mastering time of the geometry system Cabri Geometry (installed either on the PC or on the TI-92 graphing calculator) for the purpose like this is very short – only some half a row has been shown to be enough for the very beginner. Therefore, it depends only on the teacher when his students can look at the dynamic visualizations of the problem concerning the solution of the quadratic equation with the parameter. Why is the role of the teacher in this case so important? Authors' experience indicate it: Students are able to construct the parabola for a chosen value of the parameter m without any problem, but the idea – to construct new axes and place the point (parameter) m on the (for example) x-axis on the purpose to move with it and to demonstrate in this way the dynamic visualization of the moving parabola – is due the teacher.

The following figure shows the result of the previous considerations.

$$x^2(m-1)+2xm+m-3=0.$$

1. $m=1$:

$$\begin{aligned} 2x-2 &= 0 \\ x &= 1 \end{aligned}$$

2. For $m \neq 1$:

$$\begin{aligned} x_{1,2} = \frac{-2m \pm \sqrt{4m^2-4(m-1)(m-3)}}{2(m-1)} &= \\ = \frac{-m \pm \sqrt{4m-3}}{m-1}. \end{aligned}$$

3. For $4m-3=0 \Leftrightarrow m=\frac{3}{4}$ the equation has one double root $x=3$.

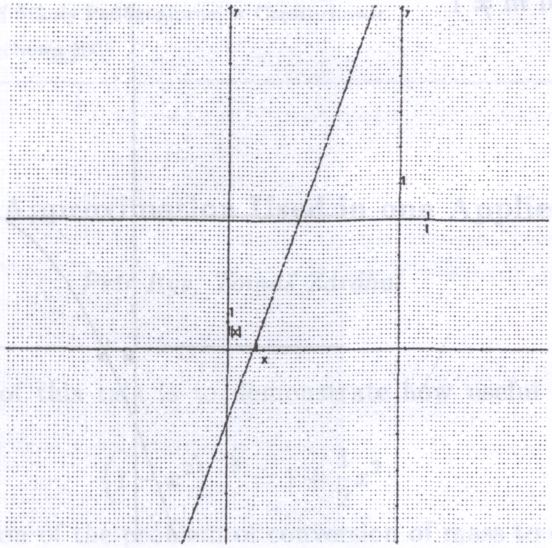
4. For $m>\frac{3}{4} \wedge m \neq 1$ the equation has two different real roots

$$x_{1,2}=\frac{-m \pm \sqrt{4m-3}}{m-1}.$$

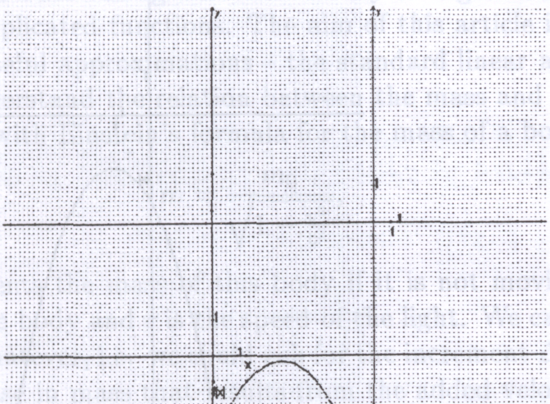
5. For $m<\frac{3}{4}$ the equation has two complex roots

$$x_{1,2}=\frac{-m \pm \mathrm{i}\sqrt{|4m-3|}}{m-1}.$$

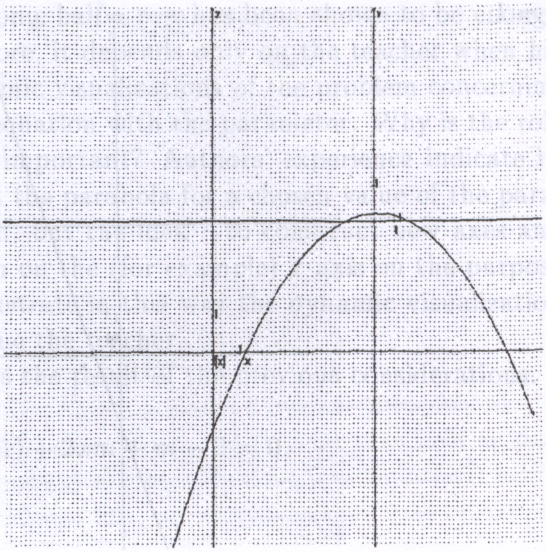
$m = 1$



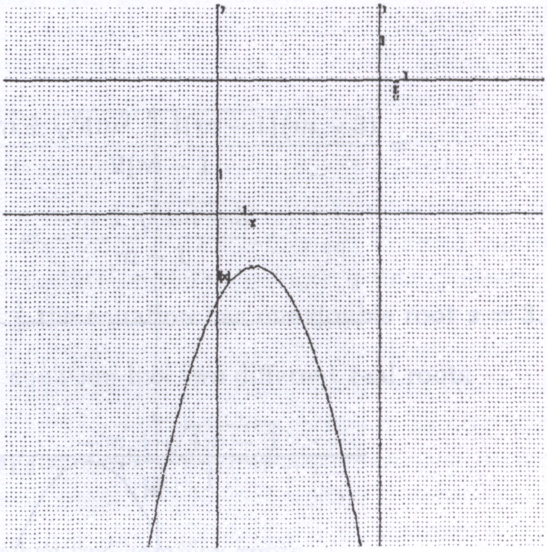
$m = 0,75$



$m > 0,75$ and $m \neq 1$



$m < 0,75$



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