Prace Naukowe Wyższej Szkoły Pedagogicznej w Częstochowie
Matematyka IX, Częstochowa 2003

The Use of the Collocation Method for Solving the Diffusion Equation under Two Mass Specifications

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In this paper the diffusion equation

$$\frac{\partial y(x,t)}{\partial t} = \frac{\partial^2 y(x,t)}{\partial x^2} + u(x,t), \quad -1 < x < 1, \quad 0 < t \leq T \tag{1}$$

with the initial condition

$$y(x,t)\Big|_{t=0} = f(x) \qquad -1 < x < 1 \tag{2}$$

is considered.

Usually the diffusion equation (1) is supplemented by the Dirichlet

$$y(x,t)\Big|_{x=-1} = \phi_1(t), \quad 0 < t \leq T, \tag{3}$$

$$y(x,t)\Big|_{x=+1} = \phi_2(t), \quad 0 < t \leq T \tag{4}$$

or Neumann

$$\frac{\mathrm{d}y(x,t)}{\mathrm{d}x}\Big|_{x=-1} = \phi_3(t), \quad 0 < t \leq T, \tag{5}$$

$$\frac{\mathrm{d}y(x,t)}{\mathrm{d}x}\Big|_{x=+1} = \phi_4(t), \quad 0 < t \leq T \tag{6}$$

boundary conditions.

In equation (1) and conditions (2)–(6) $u(x,t)$, $f(x)$, $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$, $\phi_4(t)$ are known functions.

Instead of the boundary conditions at $x = -1$ and $x = +1$ ((3)–(4) or (5)–(6)), in this paper the diffusion equation (1) is subject to two integral conditions (specifications on mass in a case of diffusion or specifications on energy in a case of heat transfer)

$$\int_{-1}^{v(t)} y(x,t)\,\mathrm{d}x = g_1(t), \qquad 0 < t \leq T, \tag{7}$$

$$\int_{v(t)}^{1} y(x,t)\,\mathrm{d}x = h_1(t), \qquad 0 < t \leq T, \tag{8}$$

where $v(t)$, $g_1(t)$ and $h_1(t)$ are known functions.

Another possibility of integral conditions is the following:

$$\int_{-1}^{1} \varphi(x)\, y(x,t)\,\mathrm{d}x = g_2(t), \qquad 0 < t \leq T, \tag{9}$$

$$\int_{-1}^{1} \psi(x)\, y(x,t)\,\mathrm{d}x = h_2(t), \qquad 0 < t \leq T \tag{10}$$

with $\varphi(x)$, $\psi(x)$, $g_2(t)$ and $h_2(t)$ being known functions.

The diffusion equation subject to mass specification arises from many important applications in mass [1] and heat transfer [2], thermoelasticity [3], pollution of the environment [4], control theory [5], etc.

This paper is a continuation of the previous papers [7–10]. The initial-boundary value problems are solved using the collocation method [6]. According to this method we seek a solution in the form

$$y(x,t) = \sum_{n=0}^{N} a_n(t) T_n(x), \tag{11}$$

where $T_n(x)$ are the Chebyshev polynomials of the first kind.

From the mass specifications (7)–(8) or (9)–(10) we obtain two algebraic equations for unknown coefficients a_n. Next, we introduce the solution (11) into the diffusion equation (1) and demand that this equation is satisfied at $N-1$ collocation points. In this paper the Chebyshev-Gauss-Lobatto points have been chosen as the collocation ones. Numerical examples are analized. A comparison between exact and approximate solutions shows efficiency of the proposed method.

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